Mass Profiles and Shapes of Cosmological Structures
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MOND AS MODIFIED INERTIA

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Abstract. I briefly highlight the salient properties of modified-inertia formulations of MOND, contrasting them with those of modified-gravity formulations, which describe practically all theories propounded to date. Future data (e.g. the establishment of the Pioneer anomaly as a new physics phenomenon) may prefer one of these broad classes of theories over the other. I also outline some possible starting ideas for modified inertia.

1 Modified MOND inertia vs. modified MOND gravity

MOND is a modification of non-relativistic dynamics involving an acceleration constant a_0. In the formal limit $a_0 \to 0$ standard Newtonian dynamics is restored. In the deep MOND limit, $a_0 \to \infty$, a_0 and G appear in the combination (Ga_0). Much of the NR phenomenology follows from this simple prescription, including the asymptotic flatness of rotation curves, the mass-velocity relations (baryonic Tully-fisher and Faber Jackson relations), mass discrepancies in LSB galaxies, etc.. There are many realizations (theories) that embody the above dictates, relativistic and non-relativistic.

The possibly very significant fact that $a_0 \sim cH_0 \sim c(\Lambda/3)^{1/2}$ may hint at the origin of MOND, and is most probably telling us that a. MOND is an effective theory having to do with how the universe at large shapes local dynamics, and b. in a Lorentz universe (with $H_0 = 0,\ \Lambda = 0$) $a_0 = 0$ and standard dynamics holds.

We can broadly classify modified theories into two classes (with the boundary not so sharply defined): In modified-gravity (MG) formulations the field equation of the gravitational field (potential, metric) is modified; the equations of motion of other degrees of freedom (DoF) in the field are not. In modified-inertia (MI) theories the opposite it true. More precisely, in theories derived from an action modifying inertia is tantamount to modifying the kinetic (free) actions of the non-gravitational degrees of freedom. Local, relativistic theories in which the kinetic

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actions are of the standard form with some physical metric are of the MG type; so, relativistic MI theories are non-local or non-metric.

Start, for example, from the standard NR action

$$S = -\frac{1}{8\pi G}\int d^3r\ (\vec{\nabla}\phi)^2 - \sum_i m_i\phi(\mathbf{r}_i) + \sum_i m_i \int dt\ v_i^2(t)/2,$$

which describes a system of masses m_i interacting gravitationally. Modifying gravity would be modifying the free action of the gravitational potential (the first term) into something like $-(a_0^2/8\pi G)\int d^3r\ F(a_0, \phi, \vec{\nabla}\phi, ...)$, where in the deep MOND limit $F \propto a_0^{-3}$ (e.g. the theory of Bekenstein and Milgrom 1984). In MI we replace the particle kinetic action by $\sum m_i S_K[a_0, \{\mathbf{r}_i(t)\}]$, where $\{\mathbf{r}_i(t)\}$ represents the full trajectory of particle i and the kinetic action is a functional of it. In the deep MOND limit $S_K \rightarrow \frac{1}{a_0}s_K[\{\mathbf{r}(t)\}]$. In such theories the equation of motion of a particle in the (unmodified) gravitational potential, ϕ, is of the form $\mathbf{A}[\{\mathbf{r}(t)\}, \mathbf{r}(t), a_0] = -\vec{\nabla}\phi[\mathbf{r}(t)]$, where the inertia functional $\mathbf{A}$, of the dimensions of acceleration, is a functional of the whole trajectory and a function of the instantaneous position; it reduces to the acceleration for $a_0 \rightarrow 0$. For $a_0 \rightarrow \infty$ the equation of motion takes the form $\mathbf{U}[\{\mathbf{r}(t)\}, \mathbf{r}(t)] = -a_0\vec{\nabla}\phi[\mathbf{r}(t)]$.

Special relativity entails a familiar example of modified (non-MOND) inertia with the standard NR particle kinetic action being replaced by $S_K = -\int \delta\tau = -\int[1-(v/c)^2]^{1/2}\ dt$ such that the equation of motion becomes $\mathbf{F} = md(\gamma\mathbf{v})/dt = m\mathbf{A} = m\gamma[\mathbf{a} + \gamma^2(\mathbf{a}\cdot\mathbf{v})\mathbf{v}/c^2]$.

With the exception of some heuristic proposals described in Milgrom (1994, 1999), all MOND theories proposed to date are of the MG type (e.g. Bekenstein & Milgrom 1984, Soussa & Woodard 2003, Bekenstein 2004, Sanders 2005).

2 Some properties of non-relativistic modified inertia theories

In Milgrom (1994, 1999) I derived certain general properties of NR MI formulations of MOND for particle dynamics: If we retain Galilei invariance in addition to the requirements of Newtonian and MOND limits, the particle kinetic action has to be non-local in time. For example, an action of the form $\int f(a/a_0)v^2\ dt$ can give the desired MOND dynamics, but is not Galilei invariant. The Lorentz invariant action $-\int F(a^\mu a_\mu/a_0^2)d\tau$ ($a^\mu = d^2x^\mu/d\tau^2$), replacing the Lorentz free particle action $-\int d\tau$, does have a Galilei invariant NR limit, but this is, alas, $-\int F(a^2/a_0^2)dt$, which is not the correct NR action. It seems to me that if we forgo Galilei invariance we should replace it with a more general symmetry, one that involves a_0, and that reduces to Galilei when $a_0 \rightarrow 0$. This must then entail a corresponding extension of Lorentz invariance (see below).

Given a particle kinetic action, S_K, bound trajectories satisfy an integral, virial relation of the form $S_K(1 + \frac{\partial lnS_K}{\partial lna_0}) = \frac{1}{2}\langle\mathbf{r}\cdot\vec{\nabla}\phi\rangle$ ($\langle\rangle$ is the time average). From this follows that for any circular orbit in an axi-symmetric potential we have

$$\mu(g/a_0)g = g_N,$$

where $g = v^2/r$ is the correct (MOND) acceleration, $G_N = -\partial\phi/\partial r$ the Newtonian acceleration, and $\mu(x)$ is simply derived from the action as restricted to circular orbits (we only have to know the action values for circular orbits to get $\mu(x)$).

3 Observable differences

While the most salient aspects of galaxy dynamics are very similar in mondified inertia and mondified gravity, there are important difference that may eventually help reject one in favor of the other.

1. The predictions of the two differ when forces other than gravity are present; e.g., in a Milliken-like experiment where strong gravity is almost balanced by an electric force, resulting in a sub-a_0 acceleration. In MG there should not be a MOND departure, as the gravitational field is large; in MI there should, as the total acceleration is small. Such an experiment does not seem feasible at present.

2. The definition of conserved quantities, and adiabatic invariants, (momentum, angular momentum, etc.) in terms of the non-gravitational degrees of freedom is different in the two approaches: these quantities are derived from the kinetic actions, which are modified in MI, but not in MG (for example, in SR the momentum is $m\gamma\mathbf{v}$). All significant tests of MOND to date concern stationary situations and do not involve the conservation laws. But future studies involving formation, mergers, accretion, relaxation, etc. of and in galaxies may become accurate enough to constrain the type of underlying modification.

3. Even in simple stationary situations, predictions of observables, such as galaxy rotation curves, may differ somewhat in the two classes of theories. For example, we saw above that MI predicts $\mu(g/a_0)g = g_N$ for the rotation curves, while MG (e.g. the NR modified-Poisson theory propound by Bekenstein and Milgrom 1984) give somewhat different results. The differences were considered by Brada & Milgrom (1995); they are not large and are also partly masked by uncertainties in the form of the interpolating function μ. But, with the number of galaxies with good data increasing, time may be ripe for a detailed analysis that might constrain $\mu(x)$ and simultaneously perhaps distinguish between the alternatives (see e.g. Famaey & Binney 2005).

4. With MG we still have in the NR regime $\mathbf{a} \equiv \dot{\mathbf{v}} = -\vec{\nabla}\phi$; so all test bodies have the same acceleration at the same position in the modified potential ϕ irrespective of their trajectory. With MI, the inertial force per unit mass $\mathbf{A}$ is not the acceleration anymore; so the measured acceleration depends not only on position but on details of the trajectory as well. (In SR, e.g., electrons running perpendicular or parallel to an electric field have the same $d(\gamma\mathbf{v})/dt$, but undergo different accelerations.) In particular, the function $\mu(a/a_0)$ appearing above in the description of circular orbits in MI is not relevant for other trajectories, for which we do not even have a simple relation between the MOND and Newtonian accelerations. For instance, a term in the action of the form $\int\ dt\ f(a/a_0)(\mathbf{a}\cdot\mathbf{v}/a_0)^2$ enters strongly for linear trajectories, but does not affect circular trajectories at all (since it vanishes for them). The fact that in MI we have to specify an action that is a functional of the trajectory permits us an infinitely larger freedom then in

MG. So we can make the modification strongly dependent on orbital eccentricity, or on the degree of binding of the orbit, etc. etc..

In galaxies, one measures instantaneous velocities and distances, assumes an orbit, and deduces the acceleration from these. If it were possible to directly measure the accelerations of bodies in the same position but on different orbits they should agree in MG but may differ in MI. It is difficult to estimate the expected differences without a specific theory. In the Newtonian regime the differences are small, of course, whereas in the NR MOND regime we saw that the equation of motion is of the form $\mathbf{U}[\{\mathbf{r}(t)\},\mathbf{r}(t)] = -a_0\vec{\nabla}\phi[\mathbf{r}(t)]$, where $\mathbf{U}$ has dimension of acceleration^2, and has the same value for all particles at the same position. The differences in the actual accelerations might then not be so strongly dependent on the orbit if, for example, $\mathbf{U}$ is dominated by a^2. Perhaps a comparison between the behavior of massive bodies and light rays will enlighten us on this point, but for that we would need a relativistic version of MI.

Closer to home, the Pioneer anomaly, if verified as a new-physics effect (Anderson et al. 2002), might provide a decisive test. It can be naturally explained in the context of MOND as MI but is difficult to explain in the context of a MG theory (Milgrom 2002): The Pioneer anomaly has no match in planetary motions for which a constant, unmodelled acceleration of the magnitude shown by the spacecraft is ruled out by a large margin. The planets probe heliocentric radii smaller than where the Pioneer anomaly has been found. So a MG theory may still have a little leeway by having the anomaly set in rather abruptly with distance just at the interim heliocentric radii (e.g., Sanders 2005). A MI explanation will build on the fact that the orbits of the spacecraft differ greatly from those of the planets: the former are close to linear and unbound, the latter quasi circular and bound. It is intriguing in this connection that the analysis for Pioneer 11 (Anderson et al. 2002) shows an onset of the anomaly just around the time where the spacecraft was kicked from a bound, nearly elliptical orbit to the unbound, almost linear orbit on which it is now (the corresponding event for Pioneer 10 is not covered). The onset still wants verification, but if real, it would be a signature of MI.

In the sense discussed here, the dark matter doctrine is a kind of MG; so any indication that the mass discrepancy in galactic systems is due to MI will also argue against DM.

4 Possible approaches to MOND inertia

We do not have a MI theory for MOND at the level of satisfaction achieved for for MG formulations. This line of inquiry has attracted relatively little attention, perhaps because MI is technically more difficult to implement as a fundamental theory. But, instances of MI in effective theories are rife in physics, from the kinematics of electrons in solids and bodies in fluids, to mass renormalization and the Higgs mechanism in field theory. MOND too could result as such an effective theory. Special Relativity is another possible source of inspiration in seeking to modify inertia. It entails a modification of newtonian inertia, brought about by the imposition of a new symmetry: Lorentz invariance. Whichever idea we follow

we should be guided by the cosmological connection of a_0, hinting that MOND might result only in the context of a non-Minkowskian universe, with a_0 reflecting the departure from flatness of space time.

4.1 derived, effective inertia

It is well known that objects moving in a medium with which they interact (electrons in solids, photons in refractive media, bodies in fluids) may exhibit a revised form of inertia. Surprisingly, it often happens that the interactions with the medium can all be encapsuled, at some level of approximation, as a reshaping of the inertial properties of the object: its motion is governed by a modified, effective "free" action with the degrees of freedom of the medium disappearing from the problem. MOND inertia, or indeed the whole of inertia, may result in a similar way. We then have to find an appropriate omnipresent medium, describe the interaction of all known physical DoF with it, and show that to a sufficient approximation this interaction can lead to inertia as we know it (with MOND). In other words, we want to show that the known actions of all DoF result as effective actions from such a mechanism. This would shed new light on Mach's principle because MOND brings into account a new connection between the universe at large and inertia.

An effective theory can violate some of the hallowed principles of relativity even though the fundamental theory from which it is derive does not: Effective theories may be non-local, violate the equivalence principle at different levels, etc.. An effective theory also has a more limited applicability than its parent theory. So, if we derive an effective MOND inertia as we now apply it to galactic systems, with the acceleration constant and the interpolating function coming out of the model in the context of cosmology, this theory need then not be applicable to cosmology itself (perhaps not even to local systems involving strong gravitational fields). The hope is, however, that when we understand the origin of MOND in such terms, the role played by the inertia-modifying medium and the way it affects cosmology and other strong-field systems can also be understood.

As discussed in Milgrom (1999), the vacuum might constitute an appropriate medium: we know it can define an inertial frame since an accelerated observed can detect it's acceleration with respect to the vacuum through the Unruh effect. And the vacuum is also affected by the cosmological state of the universe (e.g., the Gibbons Hawking effect) so it has the potential to explain the nonzero a_0 as a result of non-Minkowskian cosmology. (The field or medium responsible for the observed acceleration of the universe is also a potential candidate: its deduced present day density is numerically related to a_0, which could underlie the link.) I presented in Milgrom (1999) a heuristic argument showing how a MOND-like inertia could follow in this context. There are also pieces of evidence suggesting that kinetic actions can form spontaneously solely through interaction of DoF with the vacuum. For example, the mere interaction of the electromagnetic field with the charged DoF of the vacuum produces a contribution (of the standard form) to its kinetic action–the so called Heisenberg-Euler action (see e.g. Itzykson and

Zuber 1980). But we are still a far cry from having a theory based on this idea.

Some general questions arise when one embarks on such a program: The known instances of derived inertia start from standard physics; so all degrees of freedom start with their standard inertia, which is then modified by the interaction with the medium. Is MOND then also a correction on a preexisting inertia? Are there two contributions to inertia, one the standard, whose origin is just assumed by fiat, and another that modifies it into the MOND form? Or is there only one origin to inertia giving the standard form at $a_0 \to 0$ and MOND at the other end? I suspect the latter because in the formal limit $a_0 \to \infty$ inertia disappears; so; it may require fine tuning to have the two contributions to inertia cancel in the limit, standard inertia being independent of a_0. (But the MOND correction could also be multiplicative, in which case this argument is neutralized.)

And, if inertia is to be produced totally from scratch, does that include the purported inertia-endowing medium itself? In the instances we have of derived inertia, the Newtonian inertial law is still obeyed exactly, and the difference between the effective inertial force and the actual rate of change of momentum of the object is taken up by the medium. This means that the medium itself can have momentum, hence must have inertia to begin with. It remains to be seen whether real-world inertia can be produced with a medium itself devoid of it.

Another course of research in this vein is to construct mechanical models for inertia based on well understood physics, such as the inertia that is acquired by bodies moving in fluids; then to see in this framework whether MOND-like behavior can result in a context resembling cosmology. If successful this will tell us at least that the above program is feasible, and will perhaps teach us how to go about it.

4.2 New symmetries

In another approach we may try to construct MOND inertia on lines similar to those of special relativistic inertia, which follows from Lorentz invariance of the kinetic action. We could then seek a new symmetry that forces a form of the free actions compatible with MOND. (See, for example, an attempt by Kowalski-Glikman & Smolin 2004 along such lines, using an extension of SR having two more constants beside the speed of light–so called "triply special relativity".)

What is the space on which this new symmetry acts? Is it still space-time or a larger one? The extended, or modified, symmetry should appear, according to the cosmological connection of MOND, because we live in a cosmologically curved space-time; it should then disappear or return to Lorentz invariance when $a_0 \to 0$. Presumably a_0 is to play the role similar to that of of the speed of light in SR whose appearance as a limiting speed has to do with the Minkowskian signature of space-time. But, in contrast, a_0 is not a limiting acceleration and there are no discontinuities as we cross it. This may be telling us that we should be looking for rotations between axes that span a manifold with Riemannian signature.

Without having a concrete application in mind, I am personally intrigued by the following observations, which may give some reader a clue in the right direction. A de Sitter Universe (dSU), which approximates our universe as it is

at present, is a maximally symmetric space time with positive curvature and Minkowskian signature. It can be viewed as a 4-D pseudo-sphere embedded in a flat 5-D Minkowski space, M^5, centered at the origin, say. Consider an arbitrary, time-like world line $x^\mu(\tau)$ in the dSU having a local acceleration $a^\mu \equiv D^2x^\mu/D\tau^2$, of magnitude $a = (-a^\mu g_{\mu\nu} a^\nu)^{1/2}$. Then the acceleration in the M^5 embedding space $a_5^A \equiv d^2x^A/d\tau^2$ has magnitude $a_5 = (-a_5^A \eta_{AB} a_5^B)^{1/2}$, which can be shown to be related to a by $a_5 = (a^2 + c^2\Lambda/3)^{1/2}$. Above, $g_{\mu\nu}$ is the metric in the dSU, η_{AB} that of M^5, and $\Lambda = 3/R^2$ the cosmological constant specifying the curvature radius, R, of the dSU. So, if we make the connection with MOND by defining $\hat{a}_0 = c(\Lambda/3)^{1/2}$ to play a similar role to a_0, we can write $a_5 = (a^2 + \hat{a}_0^2)^{1/2}$.

Inertial world lines, with $a^\mu = 0$, are time-like geodesics of the dSU: great pseudo circles, which are the intersects of the dSU with (2-D) planes though the origin in the embedding space. It can be shown that world lines of finite, constant acceleration a are the intersects of the dSU with planes at a (Minkowskian) distance d from the origin with $d/R = a/(a^2 + \hat{a}_0^2)^{1/2} \equiv \lambda(a/\hat{a}_0)$. For a body at some point p on its world line compare two observers whose world lines go through p and are tangent there to the body's world line, one is inertial, the other has the same acceleration, a, as our body at p. These two reference world lines correspond to two planes one through the origin and one a distance $R\lambda(a/\hat{a}_0)$ from the origin (p itself is by definition a distance R from the origin). We can transform one plane to the other by a rotation through p by an angle θ with $sin\theta = \lambda(a/\hat{a}_0)$. So, kinematic factors such as $\lambda(a/\hat{a}_0)$ resembling MOND's $\mu(a/\hat{a}_0)$ appear in this contexts as geometrical quantities: matrix elements of a rotation taking one from an inertial observer to an accelerated one, just as the Lorentz factor γ appears in the context of Lorentz transformations. Perhaps, in a similar manner, such factors can find their way into the equation of motion of particles to give a desired MOND behavior.

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