Dirac Spinor in Bianchi-I Universe with time dependent Gravitational and Cosmological Constants

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Self-consistent system of nonlinear spinor field and Bianchi I (BI) gravitational one with time dependent gravitational constant (G) and cosmological constant (Λ) has been studied. The initial and the asymptotic behaviors of the field functions and the metric one have been thoroughly investigated. Given $\Lambda = \Lambda_0/\tau^2$, with $\tau = \sqrt{-g}$, G has been estimated as a function of τ. The role of perfect fluid at the initial state of expansion and asymptotical isotropization process of the initailly anisotropic universe has been elucidated.

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1. INTRODUCTION

Einstein's theory of gravity contains two parameters, considered as fundamental constants: Newton's gravitational constant G and the cosmological constant Λ [1]. A possible time variation of G has been suggested by Dirac [2] and extensively discussed in literature [3–7]. The "cosmological constant" Λ as a function of time was studied by many authors. Chen and Wu [8] advocated the possibility that the cosmological constant varies in time as $1/R^2$, with R being the scale factor of Robertson-Walker model. Further Abdel-Rahman [9] considered a model with the same kind of variation, while Berman *et al.* [10–12] stressed that the relation $R \propto t^{-2}$ plays an important role in cosmology. Berman and Gomide [13] also showed that all the phases of the universe, i.e., radiation, inflation, and pressure-free, may be considered as particular cases of the decelaration parameter $q =$ constant type, where

$$q = -R\ddot{R}/\dot{R}^2, \tag{1.1}$$

where dots stand for time derivative. This definition was extended by Singh and Agrawal [14] to the Bianchi cosmological models. Recently we studied the behavior of self-consistent nonlinear spinor field (NLSF) in a Bianchi I (B-I) universe [15] that was followed by the study of the self-consistent system of interacting spinor and scalar fields [16]. These studies were further extended to more general NLSF in presence of perfect fluid [17,18].

The aim of this paper is to extend our study with time dependent gravitational constant G and cosmological constant Λ in Einstein's equation.

2. FUNDAMENTAL EQUATIONS AND GENERAL SOLUTIONS

The Dirac spinor field is given by the Lagrangian

$$L = \frac{i}{2}\left[\bar{\psi}\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar{\psi}\gamma^\mu\psi\right] - m\bar{\psi}\psi + L_N. \tag{2.1}$$

The nonlinear term L_N describes the self-interaction of a spinor field and can be presented as some arbitrary functions of invariants generated from the real bilinear forms of a spinor field having the form

$$S = \bar{\psi}\psi, \quad P = i\bar{\psi}\gamma^5\psi, \quad v^\mu = (\bar{\psi}\gamma^\mu\psi), \quad A^\mu = (\bar{\psi}\gamma^5\gamma^\mu\psi), \quad T^{\mu\nu} = (\bar{\psi}\sigma^{\mu\nu}\psi),$$

where $\sigma^{\mu\nu} = (i/2)[\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu]$. Invariants, corresponding to the bilnear forms, look

$$I = S^2, \quad J = P^2, \quad I_v = v_\mu\, v^\mu = (\bar{\psi}\gamma^\mu\psi)\, g_{\mu\nu}(\bar{\psi}\gamma^\nu\psi),$$

$$I_A = A_\mu\, A^\mu = (\bar{\psi}\gamma^5\gamma^\mu\psi)\, g_{\mu\nu}(\bar{\psi}\gamma^5\gamma^\nu\psi), \quad I_T = T_{\mu\nu}\, T^{\mu\nu} = (\bar{\psi}\sigma^{\mu\nu}\psi)\, g_{\mu\alpha}g_{\nu\beta}(\bar{\psi}\sigma^{\alpha\beta}\psi).$$

According to the Pauli-Fierz theorem [19], among the five invariants only I and J are independent as all other can be expressed by them: $I_v = -I_A = I + J$ and $I_T = I - J$. Therefore we choose the nonlinear term $L_N = \lambda F(I, J)$, thus claiming that it describes the nonlinearity in the most general of its form. Here λ is the coupling constant.

The NLSF equations and components of the energy-momentum tensor for the spinor field corresponding to the Lagrangian (2.1) are

$$i\gamma^\mu\nabla_\mu\psi - \Phi\psi + i\mathcal{G}\gamma^5\psi \;=\; 0, \tag{2.2a}$$
$$i\nabla_\mu\bar{\psi}\gamma^\mu + \Phi\bar{\psi} - i\mathcal{G}\bar{\psi}\gamma^5 \;=\; 0, \tag{2.2b}$$

and

$$T^\rho_\mu = \frac{i}{4}g^{\rho\nu}\Big(\bar{\psi}\gamma_\mu\nabla_\nu\psi + \bar{\psi}\gamma_\nu\nabla_\mu\psi - \nabla_\mu\bar{\psi}\gamma_\nu\psi - \nabla_\nu\bar{\psi}\gamma_\mu\psi\Big) - \delta^\rho_\mu L_{sp} + T^\rho_{\mu\,(m)}, \tag{2.3}$$

where

$$\Phi = m - \mathcal{D} = m - 2\lambda S\frac{\partial F}{\partial I}, \qquad \mathcal{G} = 2\lambda P\frac{\partial F}{\partial J}.$$

Here $T^\rho_{\mu\,(m)}$ is the energy-momentum tensor of a perfect fluid. For a Universe filled with perfect fluid, in the concomitant system of reference ($u^0 = 1,\, u^i = 0, i = 1, 2, 3$) we have

$$T^\nu_{\mu(m)} \;=\; (p+\varepsilon)u_\mu u^\nu - \delta^\nu_\mu p \;=\; (\varepsilon,\, -p,\, -p,\, -p), \tag{2.4}$$

where energy ε is related to the pressure p by the equation of state $p = \zeta\,\varepsilon$. The general solution has been derived by Jacobs [20]. Here ζ varies between the interval $0 \le \zeta \le 1$, whereas $\zeta = 0$ describes the dust Universe, $\zeta = \frac{1}{3}$ presents radiation Universe, $\frac{1}{3} < \zeta < 1$ ascribes hard Universe and $\zeta = 1$ corresponds to the stiff matter. In (2.2) and (2.3) ∇_μ denotes the covariant derivative of spinor, having the form [21]

$$\nabla_\mu\psi = \frac{\partial\psi}{\partial x^\mu} - \Gamma_\mu\psi, \tag{2.5}$$

where $\Gamma_\mu(x)$ are spinor affine connection matrices.

Einstein's field equations with variable cosmological and gravitational "constants" Λ and G are given by

$$R^\mu_\nu - \frac{1}{2}\,\delta^\mu_\nu R = -8\pi G(t)T^\mu_\nu + \Lambda(t)\delta^\mu_\nu \tag{2.6}$$

where R^μ_ν is the Ricci tensor; $R = g^{\mu\,\nu}R_{\mu\,\nu}$ is the Ricci scalar; and T^μ_ν is the energy-momentum tensor of matter field given by (2.3). From the divergence of (2.6) we get

$$8\pi G_{,\mu}T^\mu_\nu + 8\pi G\big(T^\mu_{\nu;\mu}\big) - \Lambda,\mu\delta^\mu_\nu = 0, \tag{2.7}$$

The Bianchi I model is given by

$$ds^2 = dt^2 - a^2(t)\,dx^2 - b^2(t)\,dy^2 - c^2(t)\,dz^2. \tag{2.8}$$

We study the space-independent spinor fields, hence T^μ_ν is the function of t alone. Taking this into account for the metric (2.8), the Einstein's equations (2.6) and (2.7) reduces to

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}}{b}\frac{\dot{c}}{c} = 8\pi GT^1_1 - \Lambda, \tag{2.9a}$$
$$\frac{\ddot{c}}{c} + \frac{\ddot{a}}{a} + \frac{\dot{c}}{c}\frac{\dot{a}}{a} = 8\pi GT^2_2 - \Lambda, \tag{2.9b}$$
$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} = 8\pi GT^3_3 - \Lambda, \tag{2.9c}$$
$$\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{\dot{b}}{b}\frac{\dot{c}}{c} + \frac{\dot{c}}{c}\frac{\dot{a}}{a} = 8\pi GT^0_0 - \Lambda, \tag{2.9d}$$

$$8\pi\dot{G}T^0_0 + 8\pi G\Big[\dot{T}^0_0 + T^0_0\Big(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c}\Big) + T^1_1\frac{\dot{a}}{a} + T^2_2\frac{\dot{b}}{b} + T^3_3\frac{\dot{c}}{c}\Big] - \dot{\Lambda} = 0, \tag{2.10}$$

where points denote differentiation with respect to t. If we suppose the energy conservation law $T^\mu_{\nu;\mu} = 0$ to hold, then (2.10) reduces to

$$\dot{T}_0^0 + T_0^0\Big(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c}\Big) + T_1^1\frac{\dot{a}}{a} + T_2^2\frac{\dot{b}}{b} + T_3^3\frac{\dot{c}}{c} = 0, \tag{2.11a}$$

$$8\pi\dot{G}T_0^0 - \dot{\Lambda} = 0, \tag{2.11b}$$

Let us now go back to the spinor field equations (2.2). Using the equalities [4,22]

$$g_{\mu\nu}(x) = e_\mu^a(x)e_\nu^b(x)\eta_{ab}, \qquad \gamma_\mu(x) \;=\; e_\mu^a(x)\bar{\gamma}^a,$$

where $\eta_{ab} = \mathrm{diag}(1,-1,-1,-1)$, $\bar{\gamma}_\alpha$ are the Dirac matrices of Minkowski space and $e_\mu^a(x)$ are the set of tetradic four-vectors, we obtain the Dirac matrices $\gamma^\mu(x)$ of B-I space-time

$$\gamma^0 = \bar{\gamma}^0, \quad \gamma^1 = \bar{\gamma}^1/a(t), \quad \gamma^2 = \bar{\gamma}^2/b(t), \quad \gamma^3 = \bar{\gamma}^3/c(t),$$

$$\gamma_0 = \bar{\gamma}_0, \quad \gamma_1 = \bar{\gamma}_1 a(t), \quad \gamma_2 = \bar{\gamma}_2 b(t), \quad \gamma_3 = \bar{\gamma}_3 c(t).$$

The $\Gamma_\mu(x)$ matrices are defined by the equality

$$\Gamma_\mu(x) = \frac{1}{4}g_{\rho\sigma}(x)\big(\partial_\mu e_\delta^b e_b^\rho - \Gamma_{\mu\delta}^\rho\big)\gamma^\sigma\gamma^\delta,$$

which gives

$$\Gamma_0 = 0, \quad \Gamma_1 = \frac{1}{2}\dot{a}(t)\bar{\gamma}^1\bar{\gamma}^0, \quad \Gamma_2 = \frac{1}{2}\dot{b}(t)\bar{\gamma}^2\bar{\gamma}^0, \quad \Gamma_3 = \frac{1}{2}\dot{c}(t)\bar{\gamma}^3\bar{\gamma}^0. \tag{2.12}$$

Flat space-time matrices we choose in the form [23]

$$\bar{\gamma}^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \bar{\gamma}^1 \;=\; \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},$$

$$\bar{\gamma}^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \quad \bar{\gamma}^3 \;=\; \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

Defining γ^5 as follows,

$$\gamma^5 = -\frac{i}{4}E_{\mu\nu\sigma\rho}\gamma^\mu\gamma^\nu\gamma^\sigma\gamma^\rho, \quad E_{\mu\nu\sigma\rho} = \sqrt{-g}\varepsilon_{\mu\nu\sigma\rho}, \quad \varepsilon_{0123} = 1,$$
$$\gamma^5 = -i\sqrt{-g}\gamma^0\gamma^1\gamma^2\gamma^3 \;=\; -i\bar{\gamma}^0\bar{\gamma}^1\bar{\gamma}^2\bar{\gamma}^3 = \bar{\gamma}^5,$$

we obtain

$$\bar{\gamma}^5 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}.$$

Defining

$$\tau(t) = a(t)b(t)c(t) = \sqrt{-g}, \tag{2.13}$$

we rewrite the equation (2.2a) together with (2.5) and (2.12)

$$i\bar{\gamma}^0\Big(\frac{\partial}{\partial t} + \frac{\dot{\tau}}{2\tau}\Big)\psi - \Phi\psi + i\mathcal{G}\gamma^5\psi = 0. \tag{2.14}$$

Defining $U_j(v) = \sqrt{\tau}\,\psi_j(t)$, where $j = 1,2,3,4$, and $v = \int \mathcal{G}dt$ from (2.14) one deduces the following system of equations

$$\left.\begin{array}{l} U_1' + i(\Phi/\mathcal{G})U_1 - U_3 = 0, \\ U_3' - i(\Phi/\mathcal{G})U_3 + U_1 = 0, \end{array}\right\} \tag{2.15a}$$

$$\left.\begin{array}{l} U_2' + i(\Phi/\mathcal{G})U_2 - U_4 = 0, \\ U_4' - i(\Phi/\mathcal{G})U_4 + U_2 = 0, \end{array}\right\} \tag{2.15b}$$

where prime denotes differentiation with respect to v.

From (2.2) one can write the equations for $S = \bar{\psi}\psi, \quad P = i\bar{\psi}\gamma^5\psi$ and $A = \bar{\psi}\bar{\gamma}^5\bar{\gamma}^0\psi$

$$\dot{S}_0 - 2\mathcal{G}\, A_0 = 0, \tag{2.16a}$$

$$\dot{P}_0 - 2\Phi\, A_0 = 0, \tag{2.16b}$$

$$\dot{A}_0 + 2\Phi\, P_0 + 2\mathcal{G} S_0 = 0, \tag{2.16c}$$

where $S_0 = \tau S, \quad P_0 = \tau P$, and $A_0 = \tau A$, leading to the following relation

$$S^2 + P^2 + A^2 = C^2/\tau^2, \qquad C^2 = \text{const.} \tag{2.17}$$

Let us now solve the Einstein's equations. To do it we first write the expressions for the components of the energy-momentum tensor explicitly. Using the property of flat space-time Dirac matrices and the explicit form of covariant derivative ∇_μ, one can easily find

$$T_0^0 = m\, S - \lambda F(I,\, J) + \varepsilon, \quad T_1^1 = T_2^2 = T_3^3 = \mathcal{D}S \,+\, \mathcal{G}P - \lambda F(I,\, J) -\, p. \tag{2.18}$$

Summation of Einstein equations (2.9a), (2.9b),(2.9c) and (2.9d) multiplied by 3 gives

$$\frac{\ddot{\tau}}{\tau} = 12\pi G\Big(T_1^1 + T_0^0\Big) - 3\Lambda = 12\pi G\left(mS + \mathcal{D}S + \mathcal{G}P - 2\lambda F(I,\, J) \,+ \varepsilon -\, p\right) - 3\Lambda. \tag{2.19}$$

Let us express a, b, c through τ. For this we notice that subtraction of Einstein equations (2.9b) and (2.9a) leads to the equation

$$\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{c}}{ac} - \frac{\dot{b}\dot{c}}{bc} = \frac{d}{dt}\left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right) + \left(\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right)\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} + \frac{\dot{c}}{c}\right) = 0. \tag{2.20}$$

with the solution

$$\frac{a}{b} = D_1 \exp\left(X_1 \int \frac{dt}{\tau}\right), \quad D_1 = \text{const.}, \quad X_1 = \text{const.} \tag{2.21}$$

Analogically, one finds

$$\frac{a}{c} = D_2 \exp\left(X_2 \int \frac{dt}{\tau}\right), \quad \frac{b}{c} = D_3 \exp\left(X_3 \int \frac{dt}{\tau}\right), \tag{2.22}$$

where D_2, D_3, X_2, X_3 are integration constants. In view of (2.13) we find the following functional dependence between the constants $D_1,\, D_2,\, D_3,\, X_1,\, X_2,\, X_3$:

$$D_2 = D_1\, D_3, \qquad X_2 = X_1 \,+\, X_3.$$

Finally, from (2.21) and (2.22) we write $a(t), b(t)$, and $c(t)$ in the explicit form

$$a(t) = (D_1^2 D_3)^{1/3} \tau^{1/3} \exp\left[\frac{2X_1 + X_3}{3} \int \frac{dt}{\tau(t)}\right], \tag{2.23a}$$

$$b(t) = (D_1^{-1} D_3)^{1/3} \tau^{1/3} \exp\left[-\frac{X_1 - X_3}{3} \int \frac{dt}{\tau(t)}\right], \tag{2.23b}$$

$$c(t) = (D_1 D_3^2)^{-1/3} \tau^{1/3} \exp\left[-\frac{X_1 + 2X_3}{3} \int \frac{dt}{\tau(t)}\right]. \tag{2.23c}$$

Thus the system of Einstein's equations is completely integrated. Let us now go back to the equations (2.11a) and (2.11b). In view of (2.18) and (2.13) we find from (2.11a) we find

$$\dot{\varepsilon} + (\varepsilon + p)\frac{\dot{\tau}}{\tau} + \Phi\dot{S}_0 - \mathcal{G}\dot{P}_0 = 0. \tag{2.24}$$

On the other hand from (2.16a) and (2.16b) we have

$$\Phi\dot{S}_0 - \mathcal{G}\dot{P}_0 = 0.$$

Taking this into account and also the equation of state $p = \zeta\,\varepsilon, \qquad 0 \le \zeta \le 1$ from (2.24) we find

$$\varepsilon = \frac{\varepsilon_0}{\tau^{1+\zeta}}, \quad p = \frac{\zeta\,\varepsilon_0}{\tau^{1+\zeta}}, \tag{2.25}$$

where ε_0 is the integration constant. Let us now define G. Taking into account that $\dot{G} = \dot{\tau}\partial G/\partial\tau$ and $\dot{\Lambda} = \dot{\tau}\partial\Lambda/\partial\tau$ we rewrite (2.11b) as

$$8\pi T_0^0\frac{\partial G}{\partial\tau} = \frac{\partial\Lambda}{\partial\tau}. \tag{2.26}$$

On the other hand, inserting a, b, c from (2.23) into (2.9d) we obtain

$$8\pi T_0^0 G = \frac{\dot{\tau}^2}{3\tau^2} - \frac{\mathcal{X}}{3\tau^2} + \Lambda. \tag{2.27}$$

where $\mathcal{X} = X_1^2 + X_1X_3 + X_3^2$. Dividing (2.26) by (2.27) we find the following equation for G

$$\frac{\partial G/\partial\tau}{G} = \frac{3\tau^2\partial\Lambda/\partial\tau}{\dot{\tau}^2 - \mathcal{X} + 3\tau^2\Lambda}. \tag{2.28}$$

Now, Λ is a given function of τ, namely, $\Lambda = \Lambda_0/\tau^2$ as well as T_1^1 and T_0^0. Then (2.19), multiplied by $2\dot{\tau}$ can be written as

$$2\dot{\tau}\,\ddot{\tau} = \big[2\big(12\pi G(T_1^1 + T_0^0) - 3\Lambda\big)\tau\big]\dot{\tau} = \Psi(\tau)\dot{\tau} = \dot{\Psi}(\tau) \tag{2.29}$$

Solution to the equation (2.29) we write in quadrature

$$\int \frac{d\tau}{\sqrt{\Psi(\tau)}} = t. \tag{2.30}$$

Giving the explicit form of $F(I,\,J)$, from (2.30) one finds concrete function $\tau(t)$. Once the value of τ is obtained, one can get expressions for components $\psi_j(t), \quad j = 1, 2, 3, 4$.

In what follows, we analyze the solutions obtained previously. In [17] we gave a detailed analysis of the problem for different $F(I,\,J)$. Here we give a brief account of that.

Setting $F = F(I)$, i.e. when $\mathcal{G} = 0$ from (2.16a) one finds

$$S = \frac{C_0}{\tau}, \quad C_0 = \text{const}. \tag{2.31}$$

For the spinor field in this case we obtain

$$\psi_r(t) = (C_r/\sqrt{\tau})\,e^{-i\Omega}, \quad r = 1, 2, \quad \psi_l(t) = (C_l/\sqrt{\tau})\,e^{i\Omega}, \quad l = 3, 4. \tag{2.32}$$

where $\Omega = \int \Phi dt$ and C_r and C_l are integration constants such that $C_0 = C_1^2 + C_2^2 - C_3^2 - C_4^2$.

If one sets $F = F(J)$ and puts $m = 0$, i.e. when $\Phi = 0$ from (2.16b) one finds

$$P(t) = \frac{D_0}{\tau}, \quad D_0 = \text{ const}. \tag{2.33}$$

In this case for spinor field we obtain

$$\left.\begin{array}{ll} \psi_1 = (1/\sqrt{\tau})(D_1e^{iv} + iD_3e^{-iv}), & \psi_2 = (1/\sqrt{\tau})(D_2e^{iv} + iD_4e^{-iv}), \\ \psi_3 = (1/\sqrt{\tau})(iD_1e^{iv} + D_3e^{-iv}), & \psi_4 = (1/\sqrt{\tau})(iD_2e^{iv} + D_4e^{-iv}). \end{array}\right\} \tag{2.34}$$

where $D_0 = 2\,(D_1^2 + D_2^2 - D_3^2 - D_4^2)$.

Let us note that, in the unified nonlinear spinor theory of Heisenberg, the massive term remains absent, and according to Heisenberg, the particle mass should be obtained as a result of quantization of spinor prematter [24]. In the nonlinear generalization of classical field equations, the massive term does not possess the significance that it possesses in the linear one, as it by no means defines total energy (or mass) of the nonlinear field system. So our consideration massless spinor field is justified.

Another choice of nonlinear term is $F = F(K_\pm), \quad K_+ = I + J = I_v = -I_A, \quad K_- = I - J = I_T$. In the case of massless NLSF one finds

$$S^2 \pm P^2 = \frac{D_\pm}{\tau}. \tag{2.35}$$

In all the cases mentioned above we mainly found $\tau = \alpha t$ for small t guarantying anisotropic behavior of the universe at initial state, while $\tau = \beta t^2$ as $t \to \infty$ which is in accord with present day isotropic state. Note that this result was obtained for G constant. Here α and β are constants. As one sees, for $\tau = \alpha t$ as $t \to 0$, the solutions of spinor field are initially singular. But for some special cases, it is possible to obtain the solutions those are initially regular [17,18], but it violates the dominant energy condition in the Hawking-Penrose theorem [25]. Note that one comes to the analogical conclusion choosing $F = F(K), \quad K = IJ$.

Now, setting $\Lambda = \Lambda_0/\tau^2$ from (2.28) we find

$$G = C/\tau^{6\Lambda_0/(\alpha^2 - \mathcal{X} + 3\Lambda_0)}, \qquad \tau = \alpha t, \quad C = \text{const.},$$

and

$$G = D\Big(\frac{4\beta\tau}{4\beta\tau - \mathcal{X} + 3\Lambda_0}\Big)^{6\Lambda_0/(\mathcal{X} - 3\Lambda_0)}, \qquad \tau = \beta t^2, \quad D = \text{const.}.$$

If we consider $\Lambda = \Lambda_0/\tau^2$ and G= constant, then the conservation law $T^\mu_{\nu;\mu} = 0$ doesn't hold separately, as in that case (2.11b) leads to $\Lambda =$ const., which contradicts our assumption. In this case from (2.10) we find

$$\dot{\varepsilon} + (1+\zeta)\frac{\dot{\tau}}{\tau} = -\frac{2\Lambda_0\dot{\tau}}{\tau^3} \tag{2.36}$$

with the solution

$$\varepsilon = \frac{2\Lambda_0}{1-\zeta}\frac{1}{\tau^2} \tag{2.37}$$

Setting $F = K^n$ with $K = \{I,\, J,\, (I \pm J),\, IJ\}$ from (2.30) we conclude that even in presence of time dependent Λ in the Einstein's equation perfect fluid plays no role at the early stage of expansion as well as isotropization of BI universe leaving it to the nonlinear spinor term in (2.1) which confirms our claim made in [17,18].

Finally, we see what heppens with the system in absence of spinor field. As one sees, in this case the relation $\varepsilon = \varepsilon_0/\tau^{1+\zeta}$ takes place. Given $\Lambda = \Lambda_0/\tau^2$ from (2.11b) for G one finds

$$G = \frac{\Lambda_0}{4\pi\varepsilon_0(1-\zeta)}\,\frac{1}{\tau^{(1-\zeta)}} \tag{2.38}$$

3. CONCLUSIONS

Exact solutions to the NLSF equations have been obtained for the nonlinear terms being arbitrary functions of the invariant $I = S^2$ and $J = P^2$, where $S = \bar{\psi}\psi$ and $P = i\bar{\psi}\gamma^5\psi$ are the real bilinear forms of spinor field, for B-I space-time. It has been shown that introduction of time dependent Λ term in Einstein's equation and consideration of gravitational constant to be a function of time do not effect the initial singularity and asymptotic isotropization process which is dominated by the nonlinear spinor term in the Lagrangian. It has also been shown that the results remain unchanged even in the case when the B-I space-time is filled with perfect fluid.

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