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EINSTEIN-ÆTHER THEORY*

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We review the status of "Einstein-Æther theory", a generally covariant theory of gravity coupled to a dynamical, unit timelike vector field that breaks local Lorentz symmetry. Aspects of waves, stars, black holes, and cosmology are discussed, together with theoretical and observational constraints. Open questions are stressed.

1. Introduction

Could there be an æther after all and we have just not yet noticed it? By an "æther" of course we do not mean to suggest a mechanical medium whose deformations correspond to electromagnetic fields, but rather a locally preferred state of rest at each point of spacetime, determined by some hitherto unknown physics. Such a frame would not be determined by a circumstance such as the moon's gravitational tidal field, or the thermal cosmic microwave background radiation, but rather would be inherent and unavoidable. Considerations of quantum gravity have in multiple ways led to this question, and it has also been asked in the context of cosmology, where various puzzles hint that perhaps something basic is missing in the standard relativistic framework.

Lorentz symmetry violation by preferred frame effects has been much studied in non-gravitational physics, and is currently receiving attention as a possible window on quantum gravity.[1] But what about gravity itself? General relativity is based on local Lorentz invariance, so if the latter is violated what becomes of the former? It is hard to imagine, both philosophically and technically, how we could possibly give up general covariance, the deep symmetry finally grasped through Einstein's long struggle. Thus the

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question that interests us here is whether a generally covariant effective field theory with a preferred frame could describe nature.[a]

The simplest description of such a frame would appear to be via a scalar field T, a cosmic time function, which has been proposed in various contexts.[3,4,5,6] The gradient $T_{,a}$, if timelike, defines a preferred rest frame, and one can envision dynamics that would force it to be everywhere timelike. But while a scalar field is simplest, the norm of the gradient $|T_{,a}|$ is "extra information", which has nothing to do with specifying a frame *per se* but rather specifies the rate of a particular cosmic clock. It may be that Nature provides such a clock; we just wish to point out that the clock rate is extra information. Constraining the gradient to have fixed norm is not a viable option since, as explained in section 3, this would lead inevitably to caustics where $T_{;ab}$ diverges. Another noteworthy feature of using a scalar is that, by construction, the 4-velocity of the preferred frame is necessarily hypersurface-orthogonal, i.e. orthogonal to the surfaces of constant T. Again, perhaps this is the way Nature works, but it is a presumption not inherent in the notion of a local preferred frame determined by microphysics.

The alternative discussed in this paper is to describe the preferred frame by a vector field constrained kinematically to be timelike and of unit norm, which we call the *æther field* u^a. Such a field is specified by three independent parameters at each point, and generally couples via covariant derivatives, so the theory is far more complicated than that of a scalar time function. It is instinctive to worry about ghost modes given a vector field without gauge invariance. However the unit constraint on the vector renders it an unfamiliar beast. All variations of the vector are spacelike, since they connect two points on the unit hyperboloid, so ghosts need not arise.

There is a sparse history of studies of unit vector fields coupled to gravity.[7,8,9,10,12,13,14] Here we focus on the particular approach and results in which we have been involved.[15,16,17,18,19,20] We begin with the action principle that defines the theory, and then discuss a Maxwell-like special case, linearized waves, PPN parameters, energy, stars and black holes, and cosmology.

2. Einstein-æther action principle

In the spirit of effective field theory, we consider a derivative expansion of the action for the metric g_{ab} and æther u^a. The most general action that

[a]More general sorts of Lorentz violation in the gravitational sector are examined in Ref.2.

is diffeomorphism-invariant and quadratic in derivatives is

$$S = \frac{-1}{16\pi G}\int d^4x\sqrt{-g}\Big(R + K^{ab}{}_{mn}\nabla_a u^m \nabla_b u^n + \lambda(u^a u_a - 1)\Big) \qquad (1)$$

where

$$K^{ab}{}_{mn} = c_1 g^{ab} g_{mn} + c_2 \delta^a_m \delta^b_n + c_3 \delta^a_n \delta^b_m + c_4 u^a u^b g_{mn}. \qquad (2)$$

The coefficients $c_{1,2,3,4}$ are dimensionless constants, R is the Ricci scalar, and λ is a Lagrange multiplier that enforces the unit constraint. The metric signature is $(+---)$, and units are chosen such that the speed of light defined by the metric g_{ab} is unity. The constant G is related to the Newton constant $G_{\rm N}$ by a c_i-dependent rescaling to be discussed below. Other than the signature choice we use the conventions of Ref. 21. The possible term $R_{ab}u^a u^b$ is proportional to the difference of the c_2 and c_3 terms via integration by parts, hence has been omitted. We have also omitted any matter coupling since we are interested here in the dynamics of the metric-æther sector in vacuum. Note that since the covariant derivative of u^a involves the Levi-Civita connection, which involves first derivatives of the metric, the æther part of the action in effect contributes also to the metric kinetic terms. We call the theory with this action *Einstein-æther theory*, and abbreviate using "Æ-theory".

Another way to express the theory is using a tetrad e^a_A rather than the metric, where A is a Lorentz index. Then the æther can be specified as $u^a = u^A e^a_A$, with a unit Lorentz 4-vector u^A satisfying the constraint $\eta_{AB}u^A u^B = 1$, where η_{AB} is the fixed Minkowski metric. This decouples the normalization condition on u^A from the dynamical metric. The Lagrange density is then of the form $K^{ab}_{AB} D_a u^A D_b u^B$, where D_a is the Lorentz-covariant derivative involving the spin-connection ω^{CD}_a, and K^{ab}_{AB} is a linear combination of the four terms $g^{ab}\eta_{AB}$, $e^a_A e^b_B$, $e^a_B e^b_A$, and $u^C u^D e^a_C e^b_D \eta_{AB}$. This theory has a local Lorentz invariance, which can be used to set the components of u^A to $(1,0,0,0)$. That produces the form of the theory as presented by Gasperini.[7] One can also use a Palatini formalism, in which the spin connection is treated as an independent variable to be determined via its field equation. In this case the spin connection has torsion, because of the coupling to u^A. If the solution $\omega^{CD}_a(e,u)$ is substituted back into the action, one returns to the tetrad form, but with different coefficients for each of the four terms in K^{ab}_{AB}. The relation between these coefficients and the original ones has not yet been worked out.

Given a metric and a unit vector field, there is a one parameter family of metrics that can be constructed (aside from simple rescalings). When

expressed in terms of a different metric in this family, the action changes, but only insofar as the values of the c_i in (2) are concerned. More precisely, consider a field redefinition of the form

$$g'_{ab} = g_{ab} + (B-1)u_a u_b, \qquad u'^a = Cu^a, \tag{3}$$

with $C = [1 + (B-1)u^2]^{-1/2}$ (where $u_a := g_{am}u^m$ and $u^2 = g_{mn}u^m u^n$). Lorentz signature of both g_{ab} and g'_{ab} requires $B > 0$. The coefficient C is chosen such that $g'_{ab}u'^a u'^b = g_{ab}u^a u^b$, so the unit constraint is unchanged, hence in the action we can put simply $C = B^{-1/2}$. The action (1) for (g'_{ab}, u'^a) takes the same form as a functional of (g_{ab}, u^a), but with different values of the constants c_i. The general relation between the c_i and the c'_i has recently been worked out by Foster[11]. His results reveal, for example, that one can arrange for $c_1 + c_3 = 0$ by choosing $B = 1 - c'_1 - c'_3$ (provided $(c'_1 + c'_3) < 1$ for Lorentz signature). A special case previously worked out by Barbero and Villaseñor[10] shows that the æ-theory is equivalent via field redefinition to GR when the parameters satisfy[b] $c_1+c_4 = 0$, $c_1+c_2+c_3 = 0$, and $c_3 = \pm\sqrt{c_1(c_1-2)}$. Lorentz signature implies $c_1 < 0$. Note that if one first makes a field redefinition such that $c_1 + c_3 = 0$, then the Barbero-Villaseñor result reduces to the statement that the theory is equivalent to GR only if all coefficients vanish. Hence with the $c_1 + c_3 = 0$ description we ensure that non-zero coefficients always represent true deviations from GR.

The field equation from varying the æther in the action (1) takes the form

$$\nabla_a J^a{}_m - c_4 \dot{u}_a \nabla_m u^a = \lambda u_m \tag{4}$$

where

$$J^a{}_m := K^{ab}{}_{mn} \nabla_b u^n, \tag{5}$$

and $\dot{u}^n = u^b \nabla_b u^n$. The field equation from varying the metric in the action (1) together with a matter action takes the form

$$G_{ab} = T^{(u)}_{ab} + 8\pi G T^{\text{matter}}_{ab}, \tag{6}$$

[b]This corrects our earlier statement of the equivalence in Ref. 18. We thank B.Z. Foster for pointing out this error.

where the æther stress tensor is given by[18]

$$\begin{aligned} T^{(u)}_{ab} = & \nabla_m (J_{(a}{}^m u_{b)} - J^m{}_{(a} u_{b)} - J_{(ab)} u^m) \\ & + c_1 \left[(\nabla_m u_a)(\nabla^m u_b) - (\nabla_a u_m)(\nabla_b u^m) \right] \\ & + c_4 \, \dot{u}_a \dot{u}_b \\ & + \left[u_n (\nabla_m J^{mn}) - c_4 \dot{u}^2 \right] u_a u_b \\ & - \frac{1}{2} g_{ab} L_u . \end{aligned} \tag{7}$$

The constraint has been used in (7) to eliminate the term that arises from varying $\sqrt{-g}$ in the constraint term in (1), and λ has been eliminated by solving for it via the contraction of the æther field equation (4) with u^a. The notation $L_u = -K^{ab}{}_{mn} \nabla_a u^m \nabla_b u^n$ is the æther lagrangian.

Our goal is to determine the theoretical and observational constraints on the parameters c_i, and to identify phenomena whose observation could reveal the existence of the æther field. For such phenomena one can look at post-Newtonian effects, gravitational and æther waves, and cosmology.

3. Maxwell-like simplified theory

Before considering the general, rather complicated, theory it makes good sense to ask if there is a simplification that might serve at least as a decent starting point. A great simplification occurs with the choice $c_1 + c_3 = 0$ and $c_2 = c_4 = 0$, so that the connections drop out of the æther terms in (1). The æther part of the Lagrange density then reduces to

$$2c_1 u_{[a,b]} u^{[a,b]} + \lambda (u^2 - 1). \tag{8}$$

This theory was studied long ago by Nambu [22] in a flat space context. It is almost equivalent to Einstein-Maxwell theory in a gauge with $u^2 = 1$. The difference is that one equation is missing, since the action need only be stationary under those variations of u that preserve $u^2 = 1$. The missing equation is an initial value constraint equation, Gauss' law. If the current to which u^a is coupled is conserved, then Gauss' law holds at all times if it holds at one time.

This theory coupled to dynamical gravity was first examined in Ref. 8, and further studied extensively in Ref. 15 and Ref. 9. In Ref. 15 it was shown to be equivalent to Einstein-Maxwell theory coupled to a charged dust, restricted to the sector in which there exists a gauge such that the vector potential is proportional to the 4-velocity of the dust, i.e. the æther

field u^a. The charge-to-mass ratio of the dust is given by[c] $(8\pi G/c_1)^{1/2}$. The extremal value corresponds to $c_1 = 2$. A number of results were established concerning static, spherically symmetric solutions, black holes, and linearized solutions.

This case is appealing due to its simplicity, however a serious flaw was noticed: solutions can have "shocks" or caustics beyond which the evolution of the æther cannot be extended. In particular[15], consider the æther configurations that can be written as the gradient of a scalar $u_a = T_{,a}$, so that the Maxwell-like "field strength" tensor $u_{[a,b]}$ vanishes and u^a is orthogonal to the surfaces of constant T. Then the field equations reduce to the vacuum Einstein equation, together with the vanishing of the Lagrange multiplier λ and the statement that the gradient of T is a unit vector, $T_{,a}T^{,a} = 1$. Then u^a is necessarily a geodesic:

$$u^a \nabla_a u_b = u^a \nabla_b u_a = 0. \tag{9}$$

The first equality holds since u_b is a gradient, and the second holds since it is a unit vector. If we launch geodesics orthogonal to an initial surface of constant T, they will generically cross after some finite proper time. Where they cross there is no well-defined value of u^a, and the derivative $\nabla_a u_b$ is singular. These are the shock discontinuities. A different demonstration of the existence of shocks appears in Ref. 9.

We note in passing that the preceding demonstration of shocks applies in a very different context, namely the version of k-essence [4] recently called "ghost condensation" [5,6].[d] This is the theory of a scalar field ϕ with Lagrangian density of the form $P(X)$, where $X = \phi_{,a}\phi^{,a}$, where $P(X)$ has a minimum at some value c. Among the solutions to the field equations is the special class which have $X = c$ everywhere. The above argument shows that generically such configurations have caustics where $\phi_{;ab}$ is singular. In the cosmological setting, Hubble friction drives all solutions to the minimum $X = c$. As far as we know it is an open question whether the diverging gradients of the $X = c$ solutions is reflected in the generic cosmological solution.

Another flaw with the Maxwell-like case is that it admits negative energy configurations, as shown by Clayton [9] using the Hamiltonian formalism in the decoupling limit where gravity is neglected.[e] The existence of negative energy configurations in this case is related to the fact that the Lagrange

[c]The convention with Maxwell Lagrangian given by $-F^2/16\pi$ is adopted here.
[d]For a discussion of caustics in a more general k-essence scenario see Ref. 23.
[e]The argument in Ref. 9 has a minor flaw, but the conclusion is correct. The negative

multiplier λ can be negative, so in the charged dust interpretation the mass density is negative.

Before returning to the general class of Lagrangians we note that one might also consider the theory where the restriction on the norm of u^a is enforced not rigidly by a constraint but rather by a potential energy term $V(u^a u_a)$ in the action. This approach was discussed by Kostelecký and Samuel [8], and more recently explored by Bjorken[24], Moffat [25], and Gripaios[26]. It has an additional, massive, mode, which should be checked for a possible wrong sign of the kinetic energy.

4. Waves

The spectrum of linearized waves is important for several reasons. First, it can be used to constrain the theory a priori, by rejecting values of the parameters c_i for which waves carry negative energy or for which there are exponentially growing modes. Second, wave phenomena can be used to place observational constraints on the parameters, using radiation from compact objects such as the binary pulsar, as well as cosmological perturbations.

The spectrum of linearized waves around a flat spacetime background was worked out for the general theory defined by the action (1) in Ref. 19.[f] The wave modes in a de Sitter background were found in Ref. 14 (for $c_4 = 0$), which also studied the metric perturbations in inflation interacting with the vector as well as a scalar inflaton.

Here we summarize the results for the modes around flat space. Since the æther has three degrees of freedom, the total number of coupled metric-æther modes is five. There are two purely gravitational (spin-2) modes, two transverse æther (spin-1) modes in which the æther vector wiggles perpendicular to the propagation direction, and one longitudinal or "trace" (spin-0) mode. The waves all have a frequency that is proportional to the wave vector. Hence they are "massless" and have fixed speeds. The speeds for the different types of modes are all different, and each mode has a particular polarization type.

energy configuration described there is a time-independent pure gradient $u_i = \partial_i \phi(\vec{x})$. This initial data with vanishing time derivative $u_{i,t} = 0$ indeed has negative energy, however the equation of motion implies that the time derivative does not remain zero (unless $u_i = 0$).

[f]The Maxwell-like special case was previously treated in Ref. 15, and the case with only c_1 non-zero was treated in Ref. 16. In Ref. 14 the modes were found in the small c_i limit where the æther decouples from the metric (*cf.* section 6.2.2).

Table 1 gives the speeds and polarizations for the spin-2, spin-1, and spin-0 modes, in that order. The metric and æther have been expanded as $g_{ab} = \eta_{ab} + h_{ab}$ and $u^a = \underline{u}^a + v^a$, where η_{ab} is the Minkowski metric and $\underline{u}^a$ is the constant background value that has components $(1, 0, 0, 0)$ in the coordinate system adopted. The gauge conditions $h_{0i} = 0$ and $v_{i,i} = 0$ are imposed, where i stands for the spatial components. The propagation direction corresponds to $i = 3$, and $I = 1, 2$ labels the transverse directions. The notation c_{123} stands for $c_1 + c_2 + c_3$, etc, and s is the wave speed.

Table 1. Wave mode speeds and polarizations.

squared speed	polarization
$1/(1 - c_{13})$	h_{12}, $h_{11} = -h_{22}$
$(c_1 - \frac{1}{2}c_1^2 + \frac{1}{2}c_3^2)/c_{14}(1 - c_{13})$	$h_{I3} = [c_{13}/s(1 - c_{13})]v_I$
$c_{123}(2 - c_{14})/c_{14}(1 - c_{13})(2 + c_{13} + 3c_2)$	$h_{00} = -2v_0$ $h_{11} = h_{22} = -c_{14}v_0$ $h_{33} = (2c_{14}/c_{123})(1 + c_2)v_0$

4.1. *Stability*

The squared speed refers to the squared ratio of frequency to wave-vector, so if it is negative for real wave-vectors the frequency is imaginary, implying the existence of exponentially growing modes.[g] The requirement that no such modes exist restricts the parameters of the theory.[h] For c_i small compared to unity this requirement reduces to the conditions $c_1/c_{14} \geq 0$ for the transverse vector-metric modes and $c_{123}/c_{14} \geq 0$ for the trace mode.

Lim argued[14] that one should additionally demand that the modes propagate subluminally (relative to the metric g_{ab}). Although there is nothing wrong with local superluminal propagation in a Lorentz-violating theory, he pointed out that the vector field (in an inhomogeneous background) might tilt in such a way as to allow energy on such locally superluminal paths to flow around a closed spacetime curve. It is not clear to us that it is really necessary to impose this extra demand, since even in general relativity the *classical* field equations do not forbid the formation of closed timelike curves, around which relativistic fields could propagate. In any

[g]The factor s in the polarization h_{I3} of the transverse æther mode implies that when $s^2 < 0$ there is a $\pi/2$ phase shift of the metric perturbation relative to that of the æther.
[h]This is not yet an *observational* constraint since it has not been shown that the growing modes are not stabilized before their effects become apparent.

case, if we do make this demand, then in the case of small c_i it implies $c_{13} \leq 0$, $c_1/c_{14} \leq 1$, and $c_{123}/c_{14} \leq 1$.

On the other hand, if gravitational waves propagate subluminally relative to the "speed of light" for matter, then matter can emit gravitational Čerenkov radiation. Using this phenomenon, a very tight constraint on the difference between the maximum speed of high energy cosmic rays and that of gravitational waves was derived by Moore and Nelson [27]. For cosmic rays of galactic origin, the constraint is $\Delta c/c < 2 \times 10^{-15}$, while for extragalactic cosmic rays it is $\Delta c/c < 2 \times 10^{-19}$. Čerenkov radiation in the additional, æther-metric modes has not been examined. Constraints could conceivably eventually be obtained using these processes.

The requirement that all the modes propagate on the light cone of g_{ab} is satisfied if and only if $c_4 = 0$, $c_3 = -c_1$, and $c_2 = c_1/(1 - 2c_1)$.

4.2. *Astrophysical radiation*

Since there are three additional modes, as well as a modified speed for the usual gravitational waves, one expects that the energy loss rate for orbiting compact binaries will be affected. Not only are there additional modes to carry energy, but also lower multipole moments may act as sources, for example there may well be a monopole moment generating the spin-0, "trace" mode. These phenomena have not yet been studied. Agreement with observations of binary pulsars should yield constraints on the c_i parameters.

Another potential source of constraints is the primordial perturbation spectrum in cosmology. Lim[14] has begun a study of this.

5. Newtonian limit and PPN parameters

Carroll and Lim[13] have examined the Newtonian limit. They adopt the ansatz of a static metric, with the æther vector parallel to the timelike Killing vector. Restricting to the linearized field equations they recover the Poisson equation $\nabla^2 U_N = 4\pi G_{\rm N} \rho_{\rm m}$ for the gravitational potential U_N, where $\rho_{\rm m}$ is the usual matter energy density and

$$G_{\rm N} = \frac{G}{1 - c_{14}/2}, \tag{10}$$

where G is the parameter appearing in the action (1). Actually c_4 was set to zero in Ref. 13, but it can be restored without calculation by using the fact[18] that in spherical symmetry the effect of the c_4 term can be generated by the replacements $c_1 \to c_{14}$ and $c_3 \to c_3 - c_4$.

The PPN formalism[31,32] can be applied to Æ-theory since it is a metric theory, at least in the approximation (which is observationally known to be very accurate) that the matter is minimally coupled in the usual way to the metric.[i] In the general setting there are ten PPN parameters, but five of them vanish automatically in any theory that is based on an invariant action principle, so we need consider only the five remaining parameters, $\beta, \gamma, \xi, \alpha_1, \alpha_2$.

The two PPN parameters β and γ, known as the Eddington-Robertson-Schiff (ERS) parameters, are defined by the PPN expansion for the metric coefficients,

$$g_{00} = 1 - 2U_N + 2\beta U_N^2 + \cdots \tag{11}$$

$$g_{ij} = (1 + 2\gamma U_N + \cdots)\delta_{ij} \tag{12}$$

where U_N is the Newtonian gravitational potential. Thus β controls the non-linearity and γ the space curvature due to gravity. In general relativity $\beta = \gamma = 1$. The field equations are apparently too complicated to solve analytically, even assuming static, spherical symmetry. By numerical solution of the weak field equations in a $1/r$ expansion it was found[18] that the metric takes the form (11,12), with U_N is proportional to $1/r$, consistent with the Newtonian limit. Moreover, the ERS parameters take precisely the same values as in general relativity in the generic case $c_{123} \neq 0$. (For the special cases with $c_{123} = 0$ see Ref. 18.) Hence the theory is indistinguishable from GR at the static, post-Newtonian level.

To expose the post-Newtonian differences from GR it is necessary to examine the remaining PPN parameters, but these have not yet been computed for the Æ-theory. The parameter ξ is related to preferred location effects, and likely vanishes in Æ-theory. The parameters $\alpha_{1,2}$ are related to preferred frame effects and almost surely do not vanish. They will presumably arise when motion of the gravitating system relative to the asymptotic æther frame is allowed for.[j]

One can hazard a guess based on the PPN parameters that were calculated for the vector-tensor theory without the unit constraint.[31] In that case, β and γ are also unity in the case that corresponds most closely to Æ-theory, namely $\omega = 0$ in the vector-tensor parameters of Ref. 31 together

[i]Strictly speaking, if the matter couples also to the æther, producing local Lorentz violating effects in the matter sector, then the PPN formalism must be modified to allow for dependence of the PPN parameters on the composition of the matter sources.
[j]A recent preprint of Sudarsky and Zloshchastiev[33] addresses this point.

with $c_4 = 0$ in our parameters[18]. If this agreement persists, one would have in this case $\xi = \alpha_1 = 0$, but $\alpha_2 \neq 0$ generically. However, in the special case when $c_{13} = 0$, for which the spin-2 waves propagate on the light cone of g_{ab}, the vector-tensor parameters satisfy $\tau = \eta$, and the result of Ref. 31 yields $\alpha_2 = 0$. Thus perhaps in this special case *all* the PPN parameters of Einstein-Æther theory agree with those of GR.

Current limits on α_1 and α_2 are of order 10^{-4} and 10^{-7} respectively, so constraints of this order on the parameters of Æ-theory might be expected, at least for generic parameters.

6. Energy

6.1. *Total energy*

As discussed in section PPN, the Newtonian potential satisfies the Poisson equation with source term $8\pi G_{\rm N}\rho_{\rm m}$. This implies that in terms of the metric coefficient $g_{00} \sim 1 - r_0/r$, the source mass is given by $m = r_0/2G_{\rm N}$. Assuming the source couples minimally to the metric g_{ab}, this mass corresponds to an energy m (since the speed of light defined by g_{ab} is one in our units). Accordingly, one can infer that the energy of any isolated gravitating system is given by the same formula. This differs from the ADM mass $r_0/2G$ that one would have inferred from the action (1). The reason is that the æther stress tensor (7) adds $1/r$ terms to the Newtonian field equation.

Another path to the same conclusion is to examine the energy via its definition as the value of the Hamiltonian that generates asymptotic time translations. Using Wald's Noether charge formalism, Foster[28] showed that when one takes into account the falloff behavior of the fields at spatial infinity, the æther contribution to the energy flux integral takes the form

$$E^{(\text{æther})} = -\frac{1}{8\pi G}\oint d^2S\; K^{ta}{}_{rb}\,\nabla_a u^b, \tag{13}$$

where $K^{mn}{}_{ab}$ is the tensor defined in (2). An equivalent result was found by Eling[29] using the Einstein pseudotensor. It was shown in Ref. 18 that for a spherically symmetric, static solution the line element has the asymptotic form

$$ds^2 = (1 - \frac{r_0}{r} + \cdots)\,dt^2 - (1 + \frac{r_0}{r} + \cdots)\,dr^2 - r^2 d\Omega^2 \tag{14}$$

and the æther has a t-component of the form

$$u^t = (1 - \frac{r_0}{2r}) + \cdots\,. \tag{15}$$

The r-component of the æther starts at $O(1/r^2)$, hence does not contribute to (13). Using this form in (13) one finds for the total energy

$$E = \frac{r_0}{2G}(1 - \frac{c_{14}}{2}). \tag{16}$$

Using the relation (10) between G and $G_{\rm N}$, this can be re-expressed as $r_0/2G_{\rm N}$. Thus the total energy is related to the ADM mass $M_{\rm ADM} = r_0/2G$ by a constant rescaling.

6.2. *Positivity*

For coefficients c_i such that $2 - c_{14}$ is positive, positivity of the energy is thus equivalent to positivity of the ADM energy. The usual positive energy theorem[30] for GR assumes the dominant energy condition holds for the matter stress tensor, and proves that the total energy-momentum 4-vector of the spacetime is future timelike. The æther stress tensor (7) does not appear to satisfy the dominant energy condition (for any choice of the c_i), so the proof does not go through as usual. Nevertheless, as discussed below, the energy of the linearized theory is positive for certain ranges of c_i. Perhaps a total divergence term that leaves the energy unchanged must be removed before positivity can be seen. Also, since the asymptotic value of the unit vector selects a preferred frame, it might be that the energy is always positive only in that particular frame. We can make no definite statement about the non-linear energy at this time, based on general formal grounds.

6.2.1. *Linearized wave energy*

It is useful to examine the linearized theory to begin with. The energy density of the various wave modes has been found[20] using the Einstein or Weinberg pseudotensors, averaging over oscillations to arrive at a constant average energy density for each mode. The energy density for the transverse traceless metric mode is always positive. For the vector modes it is positive provided $(2c_1 - c_1^2 + c_3^2)/(1 - c_{13}) > 0$, and for the trace mode it is positive provided $c_{14}(2 - c_{14}) > 0$. For small c_i these energy positivity conditions reduce to $c_1 > 0$ and $c_{14} > 0$, respectively.

In the Maxwell-like case $c_{13} = c_2 = c_4 = 0$ the linearized energy positivity requirement reduces to $0 < c_1 < 2$. The negative energy configurations discussed in section 3 do not show up in the linearized limit. Their energy density is proportional to $-(\nabla u^0)^2$, which is quartic in the perturbation u^i.

6.2.2. *Gravitational decoupling limit*

The linearized waves are coupled metric-æther modes. A simpler limit to consider is a decoupling limit in which gravity is turned off. To access this limit formally we can let G and c_i tend to zero, while the ratios $c_i' = c_i/G$ are held fixed. If the metric is expanded as $g = \eta + \sqrt{G}h$ in the action (1), and the limit $G, c_i \to 0$ is taken, then one is left with just the action for linear gravitons and a decoupled æther action where all metrics are replaced by η and all covariant derivatives are replaced by ordinary partial derivatives. This limit was studied by Lim[14] in the case $c_4 = 0$. Restoring the c_4 dependence one finds perfect agreement between his results and the decoupling limit of the coupled linearized waves.

As mentioned in section 3, Clayton showed that the energy can be negative in the Maxwell-like special case. He also claimed that this remains true for more general choices of the coefficients c_i. However, the case where only c_1 is non-zero corresponds in the decoupling limit to a nonlinear sigma model (NLSM) on the unit hyperboloid which, like all NLSM's, has a stress tensor satisfying the dominant energy condition.

6.3. *Summary of constraints on the parameters*

So far we have discussed constraints from requirements in the linearized theory of positive energy, stability (no exponentially growing modes), and subluminal propagation (not necessarily required). Taken together, the constraints of positive energy and stability in the linearized regime imply $c_1 > 0$, $c_{14} > 0$, and $c_{123} > 0$ (for small parameters). These are likely necessary for a viable theory. The requirement of no superluminal propagation (which we do not feel is necessary) would additionally imply $c_{13} \leq 0$, $c_1 \leq c_{14}$, and $c_{123} \leq c_{14}$.

There is plenty of parameter space in which all the linearized constraints one might think of imposing are satisfied. It remains an important open question to determine whether energy positivity can be ensured beyond the linearized limit. But one thing is already notable, namely that this provides examples of a theory of a vector field which has no standard gauge symmetry and yet which has only stable, positive energy modes. The key factor making this possible is the constraint on the norm of the vector.

7. Stars and black holes?

In static, spherical symmetry the æther vector must be a linear combination of the time-translation Killing vector and the radial vector. Solutions were

found in Ref. 18 for which the æther field has a radial component that falls off as $1/r^2$. The question arises as to what happens to this vector in the near field region of a star. Symmetry and regularity imply that the radial component vanishes at the origin of spherical symmetry, so if indeed a regular solution exists, u^a must be parallel to the Killing vector at the origin and at infinity but not in between. Such solutions have recently been studied perturbatively by Sudarsky and Zloshchastiev[33]. We have recently shown numerically that solutions with no radial component of the æther also exist[34].

In the case of a black hole there is no regular origin of spherical symmetry, but the question arises as to what happens to u^a on the horizon. The vector u^a cannot exist at a bifurcation surface (where the Killing vector vanishes, like the 2-sphere at the origin $U = V = 0$ of Kruskal coordinates). The reason is that the Killing flow acts there as a Lorentz boost in the tangent space of any point on the bifurcation surface, hence would act non-trivially on u^a. Thus u^a could not be invariant under the Killing flow. Since u^a is constrained to be a unit vector it cannot vanish, hence we infer that it must blow up as the bifurcation surface is approached.

One might think that the impossibility of an invariant æther at the bifurcation surface implies there is no regular black hole solution in this theory, since regularity on the future horizon is somehow connected to regularity at the bifurcation surface. It seems this is not necessarily the case. A result of Rácz and Wald[35] establishes, independent of any field equations, conditions under which a stationary spacetime with a regular Killing horizon can be globally extended to a spacetime with a regular bifurcation surface, and conditions under which matter fields invariant under the Killing symmetry can also be so extended. In spherical symmetry the conditions on the metric are met for a compact Killing horizon with constant, non-vanishing surface gravity, so the result of Ref. 35 indicates that an extension to a regular bifurcation surface must exist. However, one of the conditions on the matter (i.e. æther) field is not met, namely, it is not invariant under the time reflection isometry. This is because the timelike vector u^a obviously breaks the local time reflection symmetry. Thus the æther vector need not be regular at the bifurcation surface (although all invariants must be regular and, given the Einstein equations, the æther stress-tensor must remain regular in the limit that the bifurcation surface is approached). Hence, as far as we know, there is no argument forbidding regular black hole solutions.

In fact, we have expanded the field equations about a regular, static fu-

ture event horizon and shown[34] that locally regular solutions exist. Using a shooting method, we have shown that the free parameters can be chosen so that the solutions extend to asymptotically flat metrics at spatial infinity. Alternatively, it seems an attractive idea to numerically study the spherically symmetric time-dependent collapse scenario. The collapsing matter could be a scalar field, but more simply it could just be a spherical æther wave.

8. Cosmology

Finally we turn to the role of the æther in cosmological models. Assuming Robertson-Walker (RW) symmetry, u^a necessarily coincides with the 4-velocity of the isotropic observers, so it is entirely fixed by the metric. The æther field equation has but one non-trivial component, which simply determines the Lagrange multiplier field λ. Therefore the entire æther stress tensor is also determined by the metric. Like any matter field, when the æther satisfies its equation of motion, its stress tensor is automatically conserved. Hence, in RW symmetry, the æther stress tensor must be a conserved tensor constructed entirely from the spacetime geometry. One such tensor is the Einstein tensor itself. Another is the stress tensor of a perfect fluid with equation of state $p = -\frac{1}{3}\rho$, whose energy density varies with the scale factor a as does the spatial curvature, i.e. as $1/a^2$. The æther stress tensor is just a certain combination of these two conserved tensors[16,13], namely

$$T_{ab}^{\text{æther}} = -\frac{c_{13} + 3c_2}{2}\Big[G_{ab} - \frac{1}{6}{}^{(3)}R(g_{ab} + 2u_a u_b)\Big]. \tag{17}$$

This is written using the conventions of Refs. 18, 19 in which the field equations take the form (6).

The effect of the cosmological æther is thus to renormalize the gravitational constant and to add a perfect fluid that renormalizes the spatial curvature contribution to the field equations. The renormalized, cosmological gravitational constant is given by[13]

$$G_{\text{cosmo}} = \frac{G}{1 + (c_{13} + 3c_2)/2}. \tag{18}$$

Carroll and Lim[13] note that, since this is not the same as G_{N}, the expansion rate of the universe differs from what would have been expected in GR with the same matter content. The ratio is constrained by the observed primordial ^{4}He abundance to satisfy $|G_{\text{cosmo}}/G_{\text{N}} - 1| < 1/8$. They assume

the positive energy, stability, and subluminality constraints discussed above, which imply $G_{\rm cosmo} < G < G_{\rm N}$, so the universe would have been expanding more slowly than in GR. In our notation, the resulting helium abundance constraint can be written as $15c_1 + 21c_2 + 7c_3 + 8c_4 < 2$, where we have included the c_4 dependence omitted in Ref. 13.

Dedication

This paper is dedicated by TJ to Stanley Deser, with admiration and gratitude for his friendship, support, and guidance in gravitational exploration.

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