Spinor field and accelerated regimes in cosmology

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A self-consistent system of interaction nonlinear spinor and scalar fields within the scope of a BI cosmological model filled with perfect fluid is considered. The role of spinor field in the evolution of the Universe is studied. It is shown that the spinor field nonlinearity can generate a negative effective pressure, which can be seen as an alternative source for late time acceleration of the Universe.

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I. INTRODUCTION

The accelerated mode of expansion of the present day Universe encourages many researchers to introduce different kind of sources that is able to explain this. Among them most popular is the dark energy given by a Λ term [1, 2, 3], quintessence [4, 5, 6, 7], Chaplygin gas [8, 9]. Recently cosmological models with spinor field have been extensively studied by a number of authors in a series of papers [10, 11, 12, 13, 14, 15]. The principal motive of the papers [10, 11, 12, 13, 14] was to find out the regular solutions of the corresponding field equations. In some special cases, namely with a cosmological constant (Λ term) that plays the role of an additional gravitation field, we indeed find singularity-free solutions. It was also found that the introduction of nonlinear spinor field results in a rapid growth of the Universe. This allows us to consider the spinor field as a possible candidate to explain the accelerated mode of expansion. Note that similar attempt is made in a recent paper by Kremer *et. al.* [16]. In this paper we study the role of a spinor field in the late-time acceleration of the Universe. To avoid lengthy calculations, we mainly confine ourselves to the study of master equation describing the evolution of BI Universe. We here give the solutions to the spinor and scalar field equations symbolically, for details one can consult [13, 14].

II. BASIC EQUATIONS: A BRIEF JOURNEY

We consider a self consistent system of nonlinear spinor and scalar fields within the scope of a Bianchi type-I gravitational field filled with a perfect fluid. The spinor and the scalar field is given by the Lagrangian

$$\mathscr{L} = \frac{i}{2}\left[\bar{\psi}\gamma^{\mu}\nabla_{\mu}\psi - \nabla_{\mu}\bar{\psi}\gamma^{\mu}\psi\right] - m\bar{\psi}\psi + F + \frac{1}{2}(1+\lambda_1 F_1)\,\varphi_{,\alpha}\varphi^{,\alpha}, \tag{2.1}$$

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where λ is the coupling constant and F and F_1 are some arbitrary functions of invariants generated from the real bilinear forms of a spinor field. Here we assume $F = F(I,J)$ and $F_1 = F_1(I,J)$ with $I = S^2,\ S = \bar{\psi}\psi,\ J = P^2$ and $P = i\bar{\psi}\gamma^5\psi$.

The gravitational field is chosen in the form

$$ds^2 = dt^2 - a_1^2 dx_1^2 - a_2^2 dx_2^2 - a_3^2 dx_3^2, \tag{2.2}$$

where a_i are the functions of t only and the speed of light is taken to be unity. We also define

$$\tau = a_1 a_2 a_3. \tag{2.3}$$

We consider the spinor and scalar field to be space independent. In that case for the spinor, scalar and metric functions we find the following expressions [14].

For $F = F(I)$ we find $S = C_0/\tau$ with C_0 being an integration constant. The components of the spinor field in this case read

$$\psi_{1,2}(t) = (C_{1,2}/\sqrt{\tau})e^{-i\beta}, \quad \psi_{3,4}(t) = (C_{3,4}/\sqrt{\tau})e^{i\beta}, \tag{2.4}$$

with the integration constants obeying C_0 as $C_0 = C_1^2 + C_2^2 - C_3^2 - C_4^2$. Here $\beta = \int (m - \mathscr{D})dt$.

For $F = F(J)$ in case of massless spinor field we find $P = D_0/\tau$. The corresponding components of the spinor field in this case read: with

$$\begin{aligned} \psi_{1,2} &= \left(D_{1,2}e^{i\sigma} + iD_{3,4}e^{-i\sigma}\right)/\sqrt{\tau}, \\ \psi_{3,4} &= \left(iD_{1,2}e^{i\sigma} + D_{3,4}e^{-i\sigma}\right)/\sqrt{\tau}, \end{aligned} \tag{2.5}$$

with $D_0 = 2\,(D_1^2 + D_2^2 - D_3^2 - D_4^2)$.

For the scalar field we find

$$\varphi = C\int \frac{dt}{\tau(1+2\lambda_1 F_1)}, \quad C = \text{const.} \tag{2.6}$$

Solving the Einstein equation for the metric functions we find

$$a_i(t) = D_i[\tau(t)]^{1/3}\exp\left[X_i \int_0^t [\tau(t')]^{-1}dt'\right], \tag{2.7}$$

with the integration constants obeying

$$D_1 D_2 D_3 = 1, \qquad X_1 + X_2 + X_3 = 0,$$

As one sees, the spinor, scalar and metric functions are in some functional dependence of τ. It should be noted that besides these, other physical quantities such as spin-current, charge etc. and invariant of space-time are too expressed via τ [13, 14]. It should be noted that at any space-time points where $\tau = 0$ the spinor, scalar and gravitational fields become infinity, hence the space-time becomes singular at this point [14]. So it is very important to study the equation for τ (which can be viewed as master equation) in details, exactly what we shall do in the section to follow. In doing so we analyze the role of spinor field in the character of evolution.

III. EVOLUTION OF BI UNIVERSE AND ROLE OF SPINOR FIELD

The equation for τ is found from the Einstein one:

$$R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = \kappa T_\mu^\nu + \delta_\mu^\nu \Lambda. \tag{3.1}$$

The details can be found in [13]. This equation indeed describes the evolution of the universe and has the following general form:

$$\frac{\ddot{\tau}}{\tau} = \frac{3}{2}\kappa\Big(T_1^1 + T_0^0\Big) + 3\Lambda, \tag{3.2}$$

where Λ is the cosmological constant, T_μ^ν is the energy-momentum tensor. Note also that here a positive Λ corresponds to the universal repulsive force, while a negative one gives an additional gravitational force. Note that a positive Λ is often considered to be a form of dark energy. Though our main object is to verify the role of spinor field in the evolution of the Universe, we include the Λ term in order to explain some results obtained later. For this purpose we recall that the Bianchi identity $G_{\mu;\nu}^\nu = 0$ gives

$$\dot{T}_0^0 = -\frac{\dot{\tau}}{\tau}\big(T_0^0 - T_1^1\big). \tag{3.3}$$

After a little manipulation from (3.2) and (3.3) one finds the following expression for T_0^0:

$$\kappa T_0^0 = 3H^2 - \Lambda - C_{00}/\tau^2, \tag{3.4}$$

where the definition of the generalized Hubble constant H as

$$H = \frac{1}{3}\dot{\tau}/\tau, \tag{3.5}$$

Let us now stop here for a while. Consider the case when $\Lambda = 0$. At the moment when expansion rate is zero (it might be at a time prior to the "Big Bang", or sometimes in the far future when the universe cease to expand we have $H = 0$. Then the nonnegativity of T_0^0 suggests that $C_{00} \leq 0$. Let us now consider another case when τ is large enough for the term $1/\tau^2$ to be omitted. As we know T_0^0 (the energy density), decreases with the increase of τ. If τ is big enough for T_0^0 to be neglected, from (3.4) we find

$$3H^2 - \Lambda \to 0.$$

It means for τ to be infinitely large, $\Lambda \geq 0$. In case of $\Lambda = 0$ we find that beginning from some value of τ the rate of expansion of the Universe becomes trivial, that is the universe does not expand with time. Whereas, for $\Lambda > 0$ the expansion process continues forever. As far as negative Λ is concerned, its presence imposes some restriction on the energy density T_0^0, namely, T_0^0 can never be small enough to be ignored. It imposes some restrictions on τ, precisely, there exists some upper limit for τ (note that τ is essentially nonnegative, i.e. bound from below). Thus we see that a negative Λ, depending on the choice of parameters can give rise to an oscillatory mode of expansion. Thus we come to the following conclusion:

Let T_μ^ν be the source of the Einstein field equation; T_0^0 is the energy density and T_1^1, T_2^2, T_3^3 are the principal pressure and $T_1^1 = T_2^2 = T_3^3$. An ever-expanding BI Universe may be obtained if and only if the Λ term is positive (describes a repulsive force and can be viewed as a form of dark energy) and is introduced into the system as in (3.1).

It should be noted that the other types of dark energy such as quintessence, Chaplygin gas enters into the system as a part of T_μ^ν and corresponding energy density decreases with the increase of the Universe, hence cannot be considered as source for ever-expanding Universe.

Let us now go back to the Eq. (3.2). The components of the energy-momentum tensor read:

$$\begin{aligned} T_0^0 &= mS - F + \frac{1}{2}(1+2\lambda_1 F_1)\dot{\varphi}^2 + \varepsilon_{pf}, \\ T_1^1 &= T_2^2 = T_3^3 = \mathscr{D}S + \mathscr{G}P - F - \frac{1}{2}(1+2\lambda_1 F_1)\dot{\varphi}^2 - p_{pf}, \end{aligned} \tag{3.6}$$

where, $\mathscr{D} = 2SdF/dI + \lambda_1 S\dot{\varphi}^2 dF_1/dI$ and $\mathscr{G} = 2PdF/dJ + \lambda_1 P\dot{\varphi}^2 dF_1/dJ$. In (3.6) ε_{pf} and p_{pf} are the energy density and pressure of the perfect fluid, respectively and related by the equation of state $p_{pf} = \zeta\varepsilon_{pf}$, where $\zeta \in [0,1]$.

Let us now study the equation for τ in details and clarify the role of material field in the evolution of the Universe. For simplicity we consider the case when both F and F_1 are the functions of $I(S)$ only. For simplicity we set $C = 1$ and $C_0 = 1$. Note that from the Bianchi identity for ε_{pf} and p_{pf} we find $\varepsilon_{pf} = \varepsilon_0/\tau^{1+\zeta}$ and $p_{pf} = \zeta_0\varepsilon_0/\tau^{1+\zeta}$. Further we set $\varepsilon_0 = 1$. Assuming that $F = \lambda S^q$ and $F_1 = S^r$, for the effective energy density and effective pressure we find

$$\begin{aligned} T_0^0 &= \frac{m}{\tau} - \frac{\lambda}{\tau^q} + \frac{\tau^{r-2}}{2(2\lambda_1+\tau^r)} + \frac{1}{\tau^{1+\zeta}} + \Lambda \equiv \varepsilon \\ T_1^1 &= \frac{(q-1)\lambda}{\tau^q} - \frac{[(2-r)\lambda_1+\tau^r]\tau^{r-2}}{2(2\lambda_1+\tau^r)^2} - \frac{\zeta}{\tau^{1+\zeta}} - \Lambda \equiv p. \end{aligned} \tag{3.7}$$

Taking into account that T_0^0 and T_1^1 are the functions of τ, only, the Eq. (3.2) can now be presented as

$$\ddot{\tau} = \mathscr{F}(q_1,\tau), \tag{3.8}$$

where we define

$$\mathscr{F}(q_1,\tau) = (3/2)\kappa\Big(m + \lambda(q-2)\tau^{1-q} + \lambda_1 r\tau^{r-1}/2(2\lambda_1+\tau^r)^2 + (1-\zeta)/\tau^\zeta\Big) + 3\Lambda\tau, \tag{3.9}$$

where $q_1 = \{\kappa, m, \lambda, \lambda_1, q, r, \zeta\}$ is the set of problem parameters. The En. (3.8) allows the following first integral:

$$\dot{\tau} = \sqrt{2[E - \mathscr{U}(q_1,\tau)]} \tag{3.10}$$

where we denote

$$\mathscr{U}(q_1,\tau) = -\frac{3}{2}\Big[\kappa\Big(m\tau - \lambda/\tau^{q-2} - \lambda_1/2(2\lambda_1+\tau^r) + \tau^{1-\zeta}\Big) - \Lambda\tau^2\Big]. \tag{3.11}$$

From a mechanical point of view Eq. (3.8) can be interpreted as an equation of motion of a single particle with unit mass under the force $\mathscr{F}(q_1,\tau)$. In (3.10) E is the integration constant which can be treated as energy level, and $\mathscr{U}(q_1,\tau)$ is the potential of the force $\mathscr{F}(q_1,\tau)$. We solve the Eq. (3.8) numerically using Runge-Kutta method. The initial value of τ is taken to be a reasonably small one, while the corresponding first derivative $\dot{\tau}$ is evaluated from (3.10) for a given E. As one sees, the positivity of the radical imposes some restriction on the value of τ, namely in case of $\lambda > 0$ and $q \geq 2$ the value of τ cannot be too close to zero at any space-time point. It is

clearly seen from the graphical view of the potential [cf. Fig. 1]. Thus we can conclude that for some special choice of problem parameters the introduction of nonlinear spinor field given by a self-action provides singularity-free solutions. For numerical solutions we set $\kappa = 1$, spinor mass $m = 1$, the power of nonlinearity we choose as $q = 4$, $r = 4$ and for perfect fluid we set $\zeta = 1/3$ that corresponds to a radiation. Here, in the figures we use the following notations:
1 corresponds to the case with self-action and interaction, i.e., $\lambda = 1$, $\lambda_1 = 1$;
2 corresponds to the case with self-action only, i.e., $\lambda = 1$, $\lambda_1 = 0$;
3 corresponds to the case with interaction only, i.e., $\lambda = 0$, $\lambda_1 = 1$.

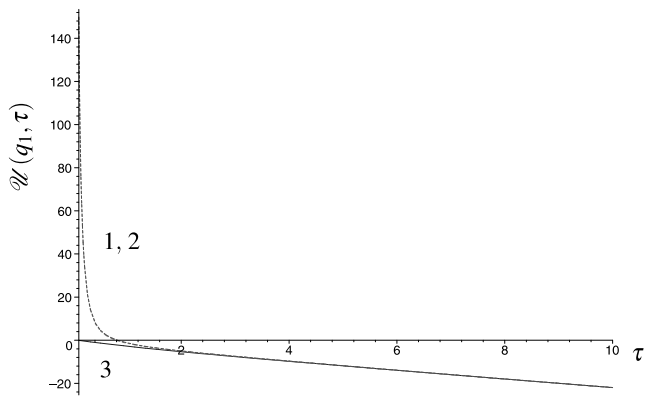

FIG. 1: View of the potential $\mathscr{U}(\tau)$ [Eq. (3.11)] as a function of τ corresponding to three different cases.

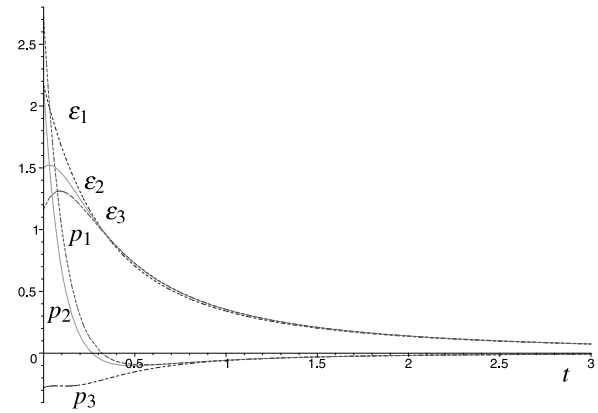

FIG. 2: Effective energy density and effective pressure corresponding to three different cases.

As one sees from Fig. 1, in presence of a self-action of the spinor field, there occurs an infinitely high barrier as $\tau \to 0$, it means that in the case considered here τ cannot be trivial [if treated classically, the Universe cannot approach to a point unless it stays at an infinitely high energy level]. Thus we see, the nonlinearity of the spinor field provided by the self-action generates singularity-free evolution of the Universe. But, as it was shown in [13], this regularity can be achieved only at the expense of dominant energy condition in Hawking-Penrose theorem. It is also clear that if the nonlinearity is induced by a scalar field, τ may be trivial as well, thus giving rise to space-time singularity. It should be noted that introduction of a positive Λ just accelerates the speed of expansion, whereas, a negative Λ depending of the choice of E generates oscillatory or non-periodic mode of evolution. These cases are thoroughly studied in [13, 14]. As it was shown in [14] the regular solution obtained my means of a negative Λ is case of interaction does not result in broken dominant energy condition. In Fig. 2 we plot the effective energy density and effective pressure of the matter field. In case of self-action pressure is initially positive, but with the expansion of the Universe it becomes negative. In case of interaction field the pressure is always negative. It means, the models with nonlinear spinor field and interacting spinor and scalar fields can to some extent explain the late time acceleration of the Universe. As one sees, the corresponding quantities (potential, energy density and pressure) differs only at the initial stage depending on the type of nonlinearity.

In Fig. 3 we graphically justified our assumptions about the role of Λ term, namely, in absence

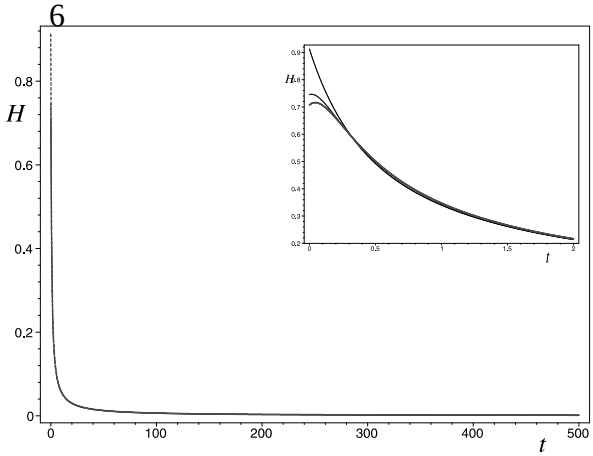

FIG. 3: Evolution of the Hubble constant as the Universe expands.

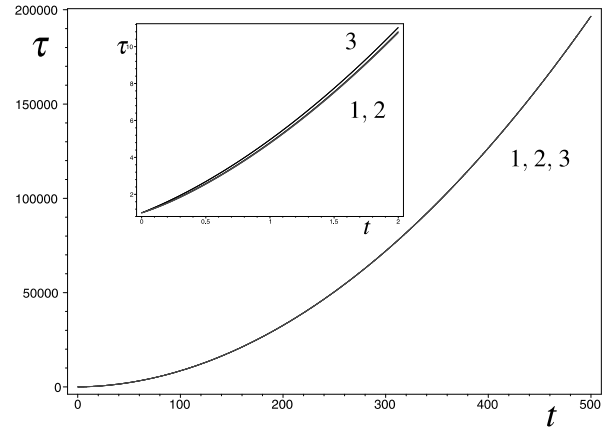

FIG. 4: Expansion of the BI Universe with time for three different cases.

of the cosmological constant, $H \to 0$ as the Universe expands. Finally, in Fig. 4 we illustrate the evolution of the Universe. As one sees, the character of evolution differs only at the initial stage depending on the choice of nonlinearity.

Finally we would like to emphasize that here we restrict within three cases only. Cases with nontrivial Λ term is not considered, since they were thoroughly studied in previous papers [3, 13, 14]. Our main aim here was to emphasize the new role of spinor field to explain the late time acceleration of the Universe.

IV. CONCLUSION

We considered a system of interaction nonlinear spinor and scalar fields within the scope of a BI cosmological model filled with perfect fluid. It is shown that the spinor field nonlinearity can generate a negative effective pressure, which can be seen as an alternative source for late time acceleration of the Universe. Here, beside spinor and scalar fields, we consider usual perfect fluid obeying $p_{pf} = \zeta \varepsilon_{pf}$. We plan to consider a few other fluids in near future that can provide an initial inflation as well.

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