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WMAP anomaly : Weak lensing in disguise

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Statistical isotropy (SI) has been one of the simplifying assumptions in cosmological model building. Experiments like WMAP and PLANCK are attempting to test this assumption by searching for specific signals in the Cosmic Microwave Background (CMB) two point correlation function. Modifications to this correlation function due to gravitational lensing by the large scale structure (LSS) surrounding us have been ignored in this context. Gravitational lensing will induce signals which mimic isotropy violation even in an isotropic universe. The signal detected in the Bipolar Spherical Harmonic (BipoSH) coefficients A^{20}_{ll} by the WMAP team may be explained by accounting for the lensing modifications to these coefficients. Further the difference in the amplitude of the signal detected in the V-band and W-band maps can be explained by accounting for the differences in the designed angular sensitivity of the instrumental beams. The arguments presented in this article have crucial implications for SI violation studies. Constraining SI violation will only be possible by complementing CMB data sets with all sky measurements of the large scale dark matter distribution. Till that time, the signal detected in the BipoSH coefficients from WMAP-7 could also be yet another suggested evidence of strong deviations from the standard ΛCDM cosmology based on homogeneous and isotropic FRW models.

The Cosmic Microwave Background (CMB) anisotropy measurements are one of the cleanest probes of cosmology. The CMB full sky temperature anisotropy measurements have been used to test the assumption of the isotropy of the universe. Ever since the release of first year data of the Wilkinson Microwave Anisotropy Probe (WMAP), statistical isotropy of the CMB anisotropy has attracted considerable attention. The study of full sky maps from the WMAP 5 year data [1–3] and the very recent WMAP 7 year data [4], has led to some intriguing anomalies that may be interpreted to indicate deviations from statistical isotropy.

The CMB temperature anisotropies are assumed to be Gaussian, which is in good agreement with current CMB observations. Hence the two point correlation function contains complete information about the underlying CMB temperature field. The two point correlation function can be expressed in terms of the spherical harmonic coefficients of CMB temperature maps,

$$\begin{aligned} C(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2) &= \langle \Delta T(\hat{\mathbf{n}}_1) \Delta T(\hat{\mathbf{n}}_2) \rangle \\ &= \sum_{lml'm'} \langle a_{lm} a^*_{l'm'} \rangle Y_{lm}(\hat{\mathbf{n}}_1) Y^*_{l'm'}(\hat{\mathbf{n}}_2) \,. \end{aligned} \tag{1}$$

In the case of statistical isotropy, the correlation function depends only on the angular separation between the two directions and not on the directions $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$ explicitly. This property makes it possible to expand the correlation function in the Legendre polynomial (P_l) basis,

$$\begin{aligned} C(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2) &= C(\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2) \\ &= \sum_l \frac{2l+1}{4\pi} C_l P_l(\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2) \,, \end{aligned} \tag{2}$$

$$C_l = \langle a_{lm} a^*_{l'm'} \rangle \delta_{ll'} \delta_{mm'} \,. \tag{3}$$

The angular power spectrum, C_l, appearing in Eq. 3 encodes all information in the covariance matrix for the statistically isotropic case. This however is only true in the case of an unlensed CMB sky as will be evident from the primary message of the article.

In the absence of statistical isotropy, the correlation function explicitly depends on the two directions $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$. In this case, the covariance matrix has been shown to have non-vanishing off-diagonal elements. This feature of the covariance matrix is captured well in the Bipolar Spherical Harmonic (BipoSH) representation which was introduced by Hajian & Souradeep (HS) [5, 6]. The BipoSH form a complete orthonormal basis for functions defined on $S^2 \times S^2$. The CMB two point correlation function can be expanded in the BipoSH basis in the following manner,

$$\begin{aligned} C(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2) &= \sum_{LMl_1l_2} A^{LM}_{l_1l_2} \left\{ Y_{l_1}(\hat{\mathbf{n}}_1) \otimes Y_{l_2}(\hat{\mathbf{n}}_2) \right\}_{LM} \\ &= \sum_{LMl_1l_2} A^{LM}_{l_1l_2} \sum_{m_1m_2} \mathcal{C}^{LM}_{l_1m_1l_2m_2} Y_{l_1m_1}(\hat{\mathbf{n}}_1) Y_{l_2m_2}(\hat{\mathbf{n}}_2) \,, \end{aligned} \tag{4}$$

where $\mathcal{C}^{LM}_{l_1m_1l_2m_2}$ are the Clebsch-Gordon coefficients, the indices of which satisfy the following relations, $|l_1 - l_2| \leq L \leq l_1 + l_2$ and $m_1 + m_2 = M$.

These BipoSH coefficients can be expressed in terms of the covariance matrix derived from observed CMB maps,

$$A^{LM}_{l_1l_2} = \sum_{m_1m_2} \langle a_{l_1m_1} a_{l_2m_2} \rangle \mathcal{C}^{LM}_{l_1m_1l_2m_2} \,. \tag{5}$$

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In the case of statistical isotropy the only non-vanishing BipoSH coefficients are A^{00}_{ll} and they can be expressed in terms of the CMB angular power spectrum C_l,

$$A^{00}_{ll} = (-1)^l \Pi_l C_l, \tag{6}$$

where $\Pi_l = \sqrt{2l+1}$. We reiterate that the above discussion is true only for the case of unlensed statistically isotropic CMB temperature fields.

Weak lensing of the CMB photons due to scalar perturbations leave measurable imprints on the CMB two point correlation function. The lensing modification to the CMB angular power spectra have been well studied [7, 8]. It is well known that lensing introduces coupling between different multipole moments of the temperature field, which otherwise are not expected to be present in an unlensed statistically isotropic CMB sky (See Eq. 3). This coupling between the various multipole moments arises due to the coupling between the projected lensing potential $\Psi(\hat{n})$ [9] and the unlensed CMB temperature field. This feature again can be most generally captured in the BipoSH representation.

In the discussion that follows we have assumed that the unlensed CMB temperature field is statistically isotropic and that the projected lensing potential and the CMB temperature field are uncorrelated. Lensing remaps the temperature field. To leading order terms in the deflection field $\mathbf{\Delta}$, the lensed temperature field $\tilde{T}(\hat{\mathbf{n}})$ can be expressed in terms of the unlensed temperature field T,

$$\begin{aligned} \tilde{T}(\hat{\mathbf{n}}) &= T(\hat{\mathbf{n}} + \mathbf{\Delta}) \\ &\approx T + \Delta^a \nabla_a T + \frac{1}{2}\Delta^a \Delta^b \nabla_a \nabla_b T\,, \end{aligned} \tag{7}$$

where the lensing deflection field on the sky can be expressed in terms of the gradient of the projected lensing potential field on the sphere as,

$$\Delta_a = \nabla_a \Psi(\hat{\mathbf{n}}). \tag{8}$$

The two point correlation of the lensed temperature field can be expressed as follows,

$$\begin{aligned} \langle \tilde{T}(\hat{\mathbf{n}}_1)\tilde{T}(\hat{\mathbf{n}}_2)\rangle &= \langle T(\hat{\mathbf{n}}_1 + \mathbf{\Delta_1}) T(\hat{\mathbf{n}}_2 + \mathbf{\Delta_2})\rangle \\ &= \langle T(\hat{\mathbf{n}}_1) T(\hat{\mathbf{n}}_2)\rangle \\ &+ \langle \nabla^a \psi(\hat{\mathbf{n}}_1) \nabla_a T(\hat{\mathbf{n}}_1) T(\hat{\mathbf{n}}_2)\rangle \\ &+ \langle \nabla^a \psi(\hat{\mathbf{n}}_2) \nabla_a T(\hat{\mathbf{n}}_2) T(\hat{\mathbf{n}}_1)\rangle + O(\psi^2)\,. \end{aligned} \tag{9}$$

The corrections to the CMB angular power spectrum due to lensing arise only due to terms which are of $O(\psi^2)$, whereas the corrections to the BipoSH coefficients can be shown to be only due to terms linear in the lensing deflection field [10, 11].

Without making assumptions about the isotropy of the lensed temperature field, the two point correlation can be most generally expressed in terms of the BipoSH coefficients and the spherical harmonic coefficients of the lensing deflection field ψ_{lm},

$$\begin{aligned} \tilde{A}^{LM}_{l_1 l_2} = A^{LM}_{l_1 l_2} &+ {}_1\alpha^{LM}_{l_1 l_2}(\psi_{lm}, A^{L'M'}_{l'_1 l'_2}) \\ &+ {}_2\alpha^{LM}_{l_1 l_2}(\psi_{lm}, A^{L'M'}_{l'_1 l'_2})\,. \end{aligned} \tag{10}$$

Since we have already assumed the unlensed temperature field to be isotropic, the above equation can be further simplified and expressed (Using Eq. 6) in terms of the unlensed CMB angular power spectrum,

$$\begin{aligned} \tilde{A}^{LM}_{l_1 l_2} = C_{l_1}\delta_{l_1 l_2}\delta_{L0}\delta_{M0} &+ {}_1\alpha^{LM}_{l_1 l_2}(\psi_{lm}, C_{l'_1}, C_{l'_2}) \\ &+ {}_2\alpha^{LM}_{l_1 l_2}(\psi_{lm}, C_{l'_1}, C_{l'_2})\,. \end{aligned} \tag{11}$$

Evaluating Eq. 9 in the harmonic space allows us to obtain the following expression for the coefficient $\alpha^{LM}_{l_1 l_2}$,

$$\begin{aligned} \alpha^{LM}_{l_1 l_2} &= {}_1\alpha^{LM}_{l_1 l_2} + {}_2\alpha^{LM}_{l_1 l_2} \\ &= \psi_{LM} \frac{C_{l_1} F(l_2, L, l_1) + C_{l_2} F(l_1, L, l_2)}{\sqrt{4\pi}} \\ &\times \frac{\Pi_{l_1}\Pi_{l_2}}{\Pi_L}\, \mathcal{C}^{L0}_{l_1 0 l_2 0}\,, \end{aligned} \tag{12}$$

where,

$$F(l_1, L, l_2) = \frac{[l_2(l_2+1) + L(L+1) - l_1(l_1+1)]}{2}\,.$$

Note that lensing by scalar density perturbations necessarily generates only the even parity (i.e. $l_1 + l_2 + L$ is even) BipoSH coefficients which is apparent from Eq. 12 due to the presence of $\mathcal{C}^{L0}_{l_1 0 l_2 0}$ which vanishes for odd parity ($l_1 + l_2 + L$ is odd).

The above discussion of BipoSH coefficients is in terms of the HS estimator which differs from the estimator used by the WMAP team. The WMAP estimator is valid only for even parity Bipolar coefficients unlike the HS estimator. For even parity coefficients, the two estimators are related by the following expression,

$$(A^{LM}_{l_1 l_2})_{\rm WMAP} = \frac{\Pi_L}{\Pi_{l_1}\Pi_{l_2}} \frac{1}{\mathcal{C}^{L0}_{l_1 0 l_2 0}} (A^{LM}_{l_1 l_2})_{HS}\,. \tag{13}$$

In discussions that follow we use the WMAP estimator for the BipoSH coefficients.

Motivated by the WMAP detections of isotropy violation [4] in the V-band and W-band maps in ecliptic coordinates, we study the corresponding BipoSH coefficients A^{20}_{ll} and $A^{20}_{l,l+2}$ that arise due to lensing. These coefficients take up an extremely simple form proportional to the angular power spectrum given by,

$$\tilde{A}^{20}_{ll} = \alpha^{20}_{ll} = \frac{3\psi_{20}}{\sqrt{\pi}} C_l W_l^2, \tag{14a}$$

$$\begin{aligned} \tilde{A}^{20}_{l,l+2} &= \alpha^{20}_{l,l+2} \\ &= \frac{\psi_{20}}{\sqrt{\pi}} \left[(l+3) C_{l+2} W^2_{l+2} - l C_l W_l^2\right], \end{aligned} \tag{14b}$$

where W_l^2 corrects the CMB angular power spectrum C_l for convolution of the instrumental response beam function.

We repeat the WMAP analysis to measure the BipoSH coefficients from the measured V-band and W-band maps. In our analysis we account for the effects of

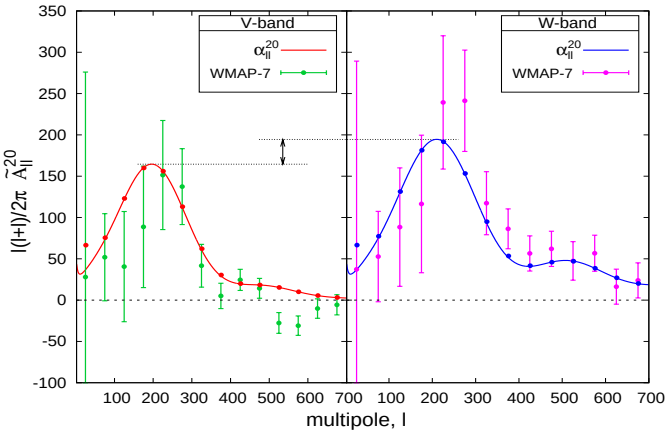

FIG. 1: The amplitude of the signal detected in the BipoSH coefficient A^{20}_{ll} in the V-band map is less than the amplitude of the signal in the W-band map exactly corresponding to the expected difference due to the two instrumental beam widths. It is remarkable and intriguing that it is possible to explain the V-band and W-band detections with a common, consistent value for the quadrupolar component of the projected lensing potential ψ_{20}. See Table I for the beam specifications of WMAP and the best fit value of the parameter ψ_{20}.

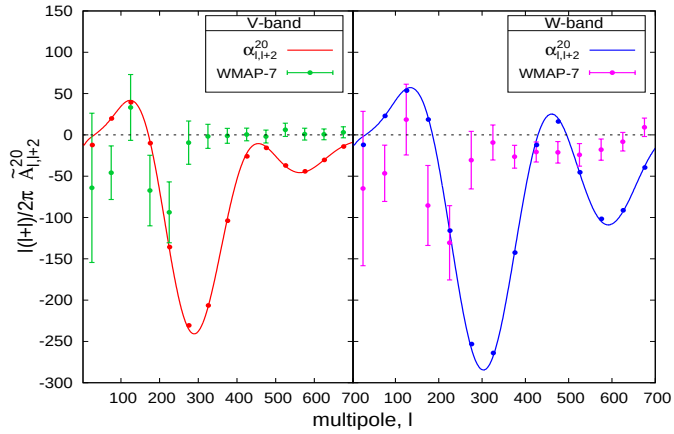

FIG. 2: The signal in the BipoSH coefficients $A^{20}_{l,l+2}$ predicted due to lensing (See Eq. 14b) plotted against the detected signal from WMAP 7 year maps. The value of the parameter ψ_{20} is set to the best fit value derived from the fit obtained to the BipoSH coefficients A^{20}_{ll} and can be found in Table I .

the anisotropic noise, instrument beam, foregrounds and masking, each of which can mimic signals of isotropy violation. We estimate the bias introduced due to all the above mentioned effects (except for the non-circularity of the instrument beam) through simulations. This bias is then subtracted from the signal inferred from the analysis of the WMAP V-band and W-band maps to isolate the systematic effects. We do not perform the inverse noise weighting that results in a much larger estimate for the errors here and lowers the significance of the detections in our analysis. The broad features of the detections are however retained and suffice for the arguments presented in this article. More elaborate analysis incorporating inverse noise weighting on harmonic coefficients a_{lm} of the temperature field and the non-circularity of the beam is underway and will be reported in a more detailed publication.

In our analysis we use the best-fit ΛCDM CMB angular power spectrum C_l generated using the publicly available Boltzmann code CAMB [12]. We use the lensing field harmonic coefficient ψ_{20} which appears in Eq. 14a as a free parameter, as currently no measurements of the projected lensing potential exist at the largest angular scales. We perform a simple χ^2 minimization technique to estimate the bestfit value of this free parameter. Note that we carry out the fitting procedure only with the BipoSH coefficients A^{20}_{ll}, since the detections in these coefficients are highly significant. Our analysis yields a value of $\psi_{20} = 2.25 \times 10^{-2}$ with which we are able to fit the detected signal in the BipoSH coefficients A^{20}_{ll} in both V-band and W-band maps equally well, after accounting for the effects of the respective instrumental beams. Since the harmonic coefficients are coordinate dependent quantities, we specifically mention that this value of ψ_{20} inferred is in ecliptic coordinates. The results of the whole analysis are summarized in Table I and Figure 1. The exceedingly good fit to the data as suggested by the reduced χ^2 values in the above table are due to over estimates of the standard deviation on the data points in absence of inverse noise weighting. The best fit value of the parameter ψ_{20} is then used to predict the signal due to lensing in the BipoSH coefficients $A^{20}_{l,l+2}$ (See Eq. 14b). The results are plotted against the detections found from WMAP 7 year maps and are depicted in Figure 2. In this case the predicted signal does not seem to match the detections, however this could be because of other systematic effects which remain unaccounted, like the non-circular beam. Nevertheless it is interesting that the predicted signal for these BipoSH coefficients follows the trend seen in the detections particularly at low multipoles.

	FWHM (in degrees)	χ^2 per d.o.f	ψ_{20}
V-band	0.326	0.818	2.25×10^{-2}
W-band	0.202	0.854	

TABLE I: WMAP beam specifications from LAMBDA site. The χ^2 per degree of freedom (d.o.f) for the best-fit ψ_{20} parameter that is obtained by fitting Eq. 14a to the BipoSH coefficient A^{20}_{ll} detections in WMAP 7 year maps.

We have clearly established that gravitational lensing can introduce significant corrections to the BipoSH coefficients rendering them non-zero even in an isotropic universe. We have also demonstrated that the difference in amplitude of the signal detected from V-band and W-

band maps can be explained by accounting for the respective beam angular sensitivities.

The best fit ΛCDM cosmology predicts the quadrupolar power in the projected lensing potential power spectrum $C_2^{\psi\psi}$ to be of the order of $\sim 10^{-8}$ requiring the corresponding harmonic coefficients ψ_{2m} to be of order $\sim 10^{-4}$. This suggest that the value of the quadrupole estimated in our analysis would be highly improbable. Hence in the realm of standard ΛCDM cosmology the observed signal in the BipoSH coefficients may not be completely explained by accounting for modifications due to lensing. Hanson et. al. [13] have argued that the detections found in the BipoSH coefficients may be explained by accounting for the non-circular nature of the WMAP instrumental beam. The detections of the BipoSH coefficients in the WMAP 7 year data may be completely explained by accommodating the total contribution from both these effects.

After correcting for the systematic effects of the beam, it will be important to account for the lensing modifications to the CMB two point correlation function as has been discussed throughout this article. It is also pertinent to note that odd parity BipoSH of HS that may be associated with the effect non-circular beam with a specific scan strategy could possibly distinguish between the two effects. Otherwise, this would require a completely independent measurement of the lensing potential field and can be only provided by LSS measurements. In the following section we discuss the LSS measurements that will be essential to estimate the projected lensing potential Ψ.

The projected lensing potential is constructed by forming a linear weighted sum of the gravitational potentials along the line of sight. Since the dominant contribution to the gravitational potentials is due to dark matter, an all sky map of the large scale distribution of dark matter will be needed. This will be made possible through gravitational lensing surveys. A significant contribution to the projected lensing potential power spectra comes from LSS below redshift $z \lesssim 5$ [9]. However to estimate the low multipole moments of the projected lensing potential, it will suffice to map the dark matter distribution upto redshift $z \lesssim 2$. This will allow a reasonable estimate of the lensing contribution to the detections in the BipoSH coefficients.

The Cosmic Evolution Survey COSMOS has mapped the dark matter distribution out upto redshift $z = 5$, however only on a small patch of the sky covering 2 square degrees. Similar deep surveys with much larger sky coverage will be required to achieve this goal. This will be made partially possible with upcoming surveys like Large Synoptic Survey Telescope LSST, Dark Energy Survey DES and EUCLID.

Next, we discuss the possibility of completely explaining the detection of the BipoSH coefficients, as arising from the lensing modifications alone. The statistically significant detections of the quadrupolar bipolar power spectrum in the recent WMAP 7 year results [4] can be explained by accounting for the lensing corrections to the BipoSH coefficients. This explanation would imply that the large scale distribution of dark matter surrounding us happens to have an extremely high quadrupole moment.

There have been other anomalies, like the cold spot observed in the CMB maps, which may not be explained in the realm of the standard ΛCDM cosmology. It has been suggested that this anomaly in the CMB maps may be explained by correcting the observed CMB maps for integrated Sachs-Wolfe (ISW) effect due to an immensely large void in the LSS surrounding us [14, 15]. Further it has been argued that the presence of such large voids may only be explained by invoking non-Gaussianity in the primordial density fluctuations [16]. The best fit value of ψ_{20} obtained in our analysis may not seem as improbable if specific non-Gaussian initial conditions are invoked. More follow up work is motivated by our results.

We have demonstrated that the difference in the amplitude of the signal detected in WMAP V-band and W-band maps can be explained by accounting for the beam angular sensitivities. Further, Groeneboom et. al. [17] assess that the non-circular beam cannot explain the non-zero detections of the BipoSH coefficients, contesting the claims made by Hanson et. al. [13]. These arguments seem to allow for the possibility of the detection in the BipoSH coefficients having a cosmological origin. A weak violation of isotropy may result in a relatively large value for the quadrupole of the projected lensing potential which in effect could magnify the SI violating signal through lensing. In such a case the detections may actually suggest a violation of statistical isotropy [18–21].

Finally, the value of the parameter ψ_{20} inferred could be interpreted as first explicit measurement of projected lensing potential at the largest angular scales. Gravitational lensing has been proposed as one of the tests for modified gravity models [22], suggesting yet another possibility of constraining these models through measurements of the lensed BipoSH coefficients.

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