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Bianchi type V bulk viscous cosmological models with particle creation in General Relativity

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In this paper, a spatially homogeneous and anisotropic Bianchi type V model filled with an imperfect fluid with bulk viscosity and particle creation, is investigated within the framework of General Relativity. Particle creation and bulk viscosity have been considered as separate irreversible processes. The energy- momentum tensor is modified to accommodate the viscous pressure and creation pressure which is associated with creation of matter out of gravitational field. Exact solutions of the field equations are obtained by applying a special law of variation of Hubble parameter. Using this assumption, we obtain two types of cosmological models. We find a singularity in the first model whereas second model is non-singular. Further we separately study the bulk viscosity and particle creation in each model considering four different cases. The bulk viscosity coefficient ζ is obtained for Truncated, Full Causal and Eckart theories in all cases. All physical parameters are calculated and thoroughly discussed in both models.

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I. INTRODUCTION

Recently, particle creation processes are supposed to play an important role in the early evolution of the universe. Phenomenologically, particle creation has been described in terms of effective bulk viscosity coefficients [1–4]. Prigogine et al. [5, 6] have investigated the first theoretical approach of particle creation which was a macroscopic phenomena allowing for both particle creation and entropy production in the early universe. They have suggested that matter creation take place out of gravitational energy in irreversible process of the non-equilibrium thermodynamics. That leads very naturally to a reinterpretation of the matter stress-energy tensor in the Einstein's general relativity. This type of creation basically corresponds to irreversible energy flow from the gravitational field to the created matter constituent. The rate of particle production can also be described by the quantum-field theory in curved space-time [7]. The idea of elementary particle creation in the expanding universe was given by Parker [8]. Hoyle and Narlikar [9] had presented the idea of continuous creation of matter which was subsequently modified in the form of quasi-steady-state cosmology [10]. Eckart [11] developed the first relativistic theory of non-equilibrium thermodynamics to study the effect of bulk viscosity. Later on, it was pointed out that the Eckart's theory has serious drawbacks concerning causality and stability. Gr⊘n [12] and Maartens [13] presented exhaustive review on cosmological models with non-causal and causal thermodynamics, respectively. Later on, the cosmological models concerning the particle production and bulk viscosity was studied by several researchers [3, 4, 14–22]. Several other authors have also explored the idea of the cosmological models with bulk viscosity and particle creation. Singh and Kale [23] have studied the anisotropic bulk viscous cosmological models with particle creation in general relativity. Further they have solved the field equation of Brans-Dicke theory for Bianchi-type I space model considering the bulk viscosity with particle creation in energy momentum tensor of the cosmic fluid [24]. Recently Chaubey [25] has found the solution for Bianchi type-V bulk viscous cosmological models with particle creation in Brans-Dicke theory.

Now we study the particle creation and bulk viscosity for early universe in details. Let us introduce the effective energy momentum tensor T_{ij} of the standard Einstein's field equation in the presence of creation of particle and bulk viscosity, which includes the creation pressure term p_c and the bulk viscous stress Π as follows:

$$T_{\mu\nu} = (\rho + p + p_c + \Pi)\, u_\mu u_\nu - (p + p_c + \Pi)\, g_{\mu\nu} \tag{1}$$

where ρ is the energy-density, p the pressure. Also the four velocity vector of the fluid following $u^\mu u_\mu = 1$.

The particle number density flow vector N^i $(= \eta u^i)$ and entropy flux vector S^i $(= \nu\eta u^i)$ in second law of thermodynamics suggest the following equations respectively

$$N^i_{;i} = \dot{\eta} + 3\eta H = \Gamma \tag{2}$$

$$S^i_{;i} = \eta\dot{\nu} + \nu\Gamma \geq 0. \tag{3}$$

Here η is particle number density, ν is entropy per particle, H is Hubble parameter and Γ is the source term which will be positive if there is production of particles and it is negative when there is annihilation of particles. But it will vanish if there is no particle production or annihilation. A dot(.) here denotes the differentiation with respect to time.

For an open adiabatic system to cosmology, the supplementary pressure p_c, due to the creation of matter, assumes the following form [6]

$$p_c = -\frac{(\rho + p)}{3H\eta}\Gamma = -\frac{(\rho + p)}{3H}\left(3H + \frac{\dot{\eta}}{\eta}\right). \tag{4}$$

For an open thermodynamical system, the Gibbs equation[13] can be given by

$$\eta T\dot{\nu} = \dot{\rho} - (\rho + p)\frac{\dot{\eta}}{\eta} \tag{5}$$

where T is the temperature of the cosmic fluid. Using the continuity equation (23)[to be introduced in the next section], equation (2) and equation (5), the expression for entropy per particle is

$$\dot{\nu} = -\frac{3Hp_c}{\eta T} - \frac{3H\Pi}{\eta T} - \frac{(\rho + p)}{\eta^2 T}\Gamma. \tag{6}$$

From equations (4) and (6), the entropy per particle can be calculated in more simplified form as

$$\dot{\nu} = -\frac{3H\Pi}{\eta T}. \tag{7}$$

From equations (5) and (7), we have the following relation

$$\frac{\dot{\eta}}{\eta} = \frac{\dot{\rho} + 3H\Pi}{\rho + p}. \tag{8}$$

Using the equation of state $p = \gamma\rho(0 \leq \gamma \leq 1)$ in the above equation (8), the particle number density can be obtained as

$$\eta^{1+\gamma} = \eta_0 \rho \exp\left(\int 3H\Pi\rho^{-1}dt\right) \tag{9}$$

where η_0 is an integration constant. The conventional bulk viscous effect in Bianchi universe can be modeled within the framework of non-equilibrium thermodynamics. Here the transport equation for the bulk viscous pressure Π takes the form[13]

$$\Pi + \tau\dot{\Pi} = -3\zeta H - \frac{\epsilon\tau\Pi}{2}\left(3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\zeta}}{\zeta} - \frac{\dot{T}}{T}\right) \tag{10}$$

where ζ is the bulk viscous coefficient and τ is the relaxation time associated with the dissipative effect. In view of the above relationship, we can study the behaviour of Bulk Viscosity in **Truncated theory**, **Full Causal theory** and **Eckart's theory** by putting $\epsilon = 0$, $\epsilon = 1$ and $\tau = 0$ respectively in the above equation. In truncated theory the above evolution equation (10) reduces to

$$\Pi + \tau\dot{\Pi} = -3\zeta H. \tag{11}$$

Also in this theory, to ensure that the viscous signals do not exceed the speed of light, we consider the following relation

$$\tau = \frac{\zeta}{\rho}. \tag{12}$$

In Full Causal theory, the equation of state for pressure and temperature [13, 15] are taken to be barotropic i.e., $p = \gamma\rho$ and $T = T(\rho)$. Then $T \propto exp\int \frac{dp(\rho)}{\rho + p(\rho)}$, this will reduce to the following equation by using the equation of state for pressure as

$$T = T_0 \rho^{\frac{\gamma}{(1+\gamma)}} \tag{13}$$

where T_0 is a constant. Using equations (12) and (13), the evolution equation (10) will take the form

$$\Pi + \frac{\zeta}{\rho}\dot{\Pi} = -3H\zeta - \frac{\zeta\Pi}{2\rho}\left[3H - \frac{(1+2\gamma)\dot{\rho}}{(1+\gamma)\rho}\right]. \tag{14}$$

In Eckart's non-causal theory, the evolution equation (10) will reduce to

$$\Pi = -3\zeta H. \tag{15}$$

In the following sections, we solve the standard Einstein's field equation for an anisotropic Bianchi type V model in the presence of particle creation and bulk viscosity.

II. BASIC EQUATIONS

As a gravitational field we consider the Bianchi type V space-time given by

$$ds^2 = dt^2 - A^2dx^2 - e^{2mx}[B^2dy^2 + C^2dz^2]. \tag{16}$$

The Einstein's field equation can be written as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{\mu\nu} \tag{17}$$

all the symbols above have their usual meaning. The Einstein's field equation (17) in Bianchi V line element (16) for the energy-momentum tensor (1) can be translated into the following set of non-linear equations as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -8\pi G(p + p_c + \Pi) \tag{18}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -8\pi G(p + p_c + \Pi) \tag{19}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{m^2}{A^2} = -8\pi G(p + p_c + \Pi) \tag{20}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3m^2}{A^2} = 8\pi G\rho \tag{21}$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - 2\frac{\dot{A}}{A} = 0. \tag{22}$$

The Bianchi identity reads

$$\dot{\rho} + 3(\rho + p)H = -3(p_c + \Pi)H. \tag{23}$$

Here we consider that Hubble parameter H is directly proportional to a negative n power of average scale factor a so that we have a relationship between these parameters given by $H = h_0 a^{-n} = h_0(ABC)^{-n/3}$ where $h_0 > 0$ and $n \geq 0$ are constants. This type of relation

gives a constant value of deceleration parameter which is a very useful tool in solving the field equations. Earlier Berman [26], Berman and Gomide [27] had considered such type of assumption for solving FRW cosmological models. Later on, many workers [28–31] have used this assumption for solving Einstein's field equations in general relativity and different scalar tensor theory of gravitation. The general formulas of certain physical parameters for the metric equation (16) are given as follows:
The expansion scalar is given by

$$\theta = u^{\mu}_{;\mu} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}. \tag{24}$$

The shear scalar has the form

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{6}. \tag{25}$$

We also introduce generalized Hubble parameter H:

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) \tag{26}$$

with $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of x, y and z respectively. Let us introduce the function V as

$$V = ABC. \tag{27}$$

The average scale factor a can be written in terms of metric functions as

$$a = (ABC)^{1/3} = V^{1/3}. \tag{28}$$

It should be noted that the parameters H, V and a are connected by the following relation

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{a}}{a}. \tag{29}$$

The deceleration parameter q is given as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{30}$$

Using the relation $H = h_0 a^{-n}$ and the above equations, the expressions for deceleration parameter q and average scale factor a can be calculated as

$$q = n - 1. \tag{31}$$

$$a = (nh_0 t)^{1/n}, n \neq 0 \tag{32}$$

and

$$a = a_0 \exp(h_0 t), n = 0 \tag{33}$$

where a_0 is an integration constant. Now we follow the approach of Saha and Rikhvitsky [32] and Shri Ram et al. [31]. Hence from equations (18)-(22), the quadrature form of the metric functions can be given as follows:

$$A(t) = a \tag{34}$$

$$B(t) = B_0 a \exp\left(\frac{X}{3}\int \frac{dt}{a^3}\right) \tag{35}$$

$$C(t) = C_0 a \exp\left(-\frac{X}{3}\int \frac{dt}{a^3}\right) \tag{36}$$

where B_0, C_0 and X are arbitrary constants. Taking into account that

$$BC = A^2$$

which is the consequence of the equation(22) [for simplicity we set the integration constant to be unit], we find $B_0 C_0 = 1$.

Now in the following sections we present the exact solutions of the above field equations in view of two different values of average scale factor given by equations (32) and (33). In this way, we can formulate two basic cosmological models. These models can also be studied in context with different physical behaviors of the particle creation and bulk viscosity in the energy momentum tensor of the cosmic fluid. That will create various sub-models.

III. THE COSMOLOGICAL MODELS

A. Model 1

Putting the value of the average scale-factor from equation (32) into equations (34)-(36), the exact values of the metric functions can be obtained as

$$A = (nh_0 t)^{1/n} \tag{37}$$

$$B = B_0 (nh_0 t)^{1/n} \exp\left[\frac{X}{3h_0(n-3)}(nh_0 t)^{\frac{n-3}{n}}\right] \tag{38}$$

$$C = C_0 (nh_0 t)^{1/n} \exp\left[-\frac{X}{3h_0(n-3)}(nh_0 t)^{\frac{n-3}{n}}\right] \tag{39}$$

where $n \neq 3$.

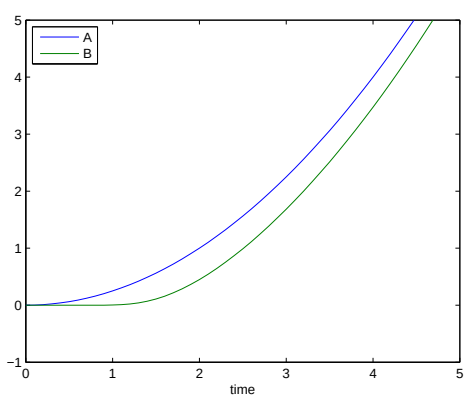

FIG. 1. Variation of scale factors A and B for the parameter $n = 0.5$ with time.

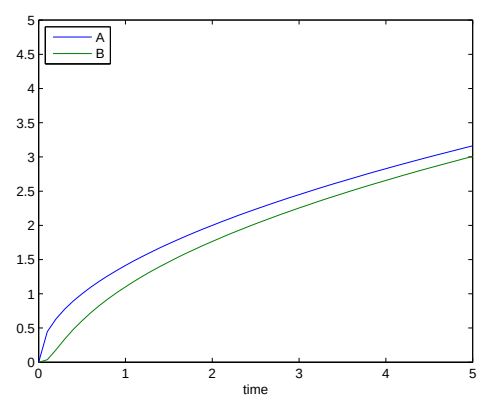

FIG. 2. Variation of scale factors A and B for the parameter $n = 2$ with time.

Hence the Expansion scalar θ, Shear scalar σ^2, generalized Hubble parameter H and the Volume scalar V can be written as

$$\theta = 3h_0(nh_0 t)^{-1} \tag{40}$$

$$\sigma^2 = \frac{X^2}{9}(nh_0t)^{-6/n} \tag{41}$$

$$H = h_0(nh_0t)^{-1} \tag{42}$$

and

$$V = (nh_0t)^{3/n}. \tag{43}$$

The anisotropy parameter A_m can be given by

$$A_m = \frac{2}{27}\frac{X^2}{h_0^2}(nh_0t)^{\frac{2n-6}{n}}. \tag{44}$$

The directional Hubble parameters can be obtained as

$$H_1 = h_0(nh_0t)^{-1} \tag{45}$$

$$H_2 = h_0(nh_0t)^{-1} + \frac{X}{3}(nh_0t)^{-3/n} \tag{46}$$

$$H_3 = h_0(nh_0t)^{-1} - \frac{X}{3}(nh_0t)^{-3/n}. \tag{47}$$

From equation(21), the value of the energy density can be found as

$$\rho = \rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{-6/n} - \rho_2(nh_0t)^{-2/n} \tag{48}$$

where $\rho_0 = \frac{3h_0^2}{8\pi G}$, $\rho_1 = \frac{X^2}{72\pi G}$, $\rho_2 = \frac{3m^2}{8\pi G}$. Due to the barotropic equation $p = \gamma\rho$, we have the following expression for pressure as

$$p = \gamma\left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{-6/n} - \rho_2(nh_0t)^{-2/n}\right]. \tag{49}$$

We now investigate the behavior of the above cosmological model by analyzing the different physical parameters. The behaviours of the scale factors A, B and C can be observed in the figures 1, 2 and 3. The scale factors A and B are increasing function of time. They are accelerated and decelerated for the parameter $n = 0.5$ and $n = 2$ respectively. Above results show that all three scale factors are zero at the initial time $t = 0$. Expansion scalar, Shear scalar, Hubble parameter and the three directional Hubble parameters are all infinite at $t = 0$. It is also observed that the spatial volume is zero at this initial time. The mean anisotropy parameter also diverge at this time for $n < 3$. The energy density and pressure tend to infinity at this epoch. All these values of different physical parameters show that the universe starts evolving with zero volume and expands with cosmic time t. That is the model has point singularity at $t = 0$. Now we also study these parameters for very large time as $t \to \infty$. We find here that A and B tend to zero but C will become indeterminate. Expansion scalar, Shear scalar, Hubble parameter, the three directional Hubble parameters and the mean anisotropy parameter will all become zero for large time. The energy density and pressure tend to zero at $t \to \infty$. All these indicate that the universe is expanding with the increase of cosmic time but the rate of expansion and shear scalar decrease to zero and finally tend to isotropic. This model approaches isotropic during late time of its evolution as $\lim \sigma^2/\theta = 0$ for $t \to \infty$.

Now in the following subsections we study the behavior of particle creation and bulk viscosity of this model in four different physical laws. Therefore, we can create four different sub models out of the above model.

1. Bulk Viscosity Energy-Density Law

Let us assume here that the bulk viscous stress Π be associated with energy density ρ by the following relationship

$$\Pi = \Pi_0 \rho^{\omega} \tag{50}$$

where Π_0 is a constant. The above assumption is motivated by the relation $\zeta = \zeta_0 \rho^{\omega}$ where $\zeta_0 \geq 0, \omega \geq 0$. This expression for ζ was suggested by several researchers [33–35]. Further they added that for $\omega = 1$, this will correspond to radiative fluid and for $\omega = 1.5$, this may represent a string dominated universe [36, 37]. So, here the expression for Π can be written as

$$\Pi = \Pi_0 \left[\rho_0 (nh_0 t)^{-2} - \rho_1 (nh_0 t)^{-6/n} - \rho_2 (nh_0 t)^{-2/n}\right]^{\omega}. \tag{51}$$

The graphical behaviour of the above equation for Π can be observed through the figure 4 in different parameters. The creation pressure can be obtained as

$$\begin{aligned} p_c = {} & p_0 (nh_0 t)^{-2} + p_1 (nh_0 t)^{-2/n} + p_2 (nh_0 t)^{-6/n} \\ & -\Pi_0 \left[\rho_0 (nh_0 t)^{-2} - \rho_1 (nh_0 t)^{-6/n} - \rho_2 (nh_0 t)^{-2/n}\right]^{\omega} \end{aligned} \tag{52}$$

where $p_0 = \left[\frac{2n}{3} - (1+\gamma)\right] \rho_0$, $p_1 = \frac{(3\gamma+1)\rho_2}{3}$ and $p_2 = (\gamma - 1)\rho_1$. The variation of creation pressure in this case can be observed in figures from 5 to 10.

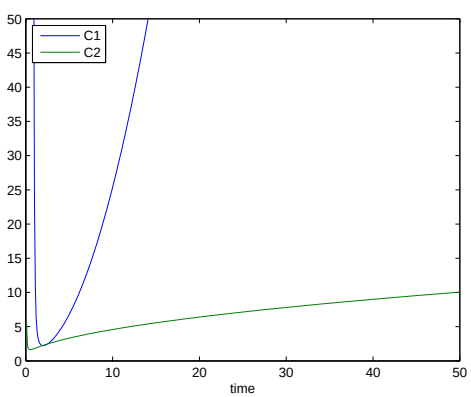

FIG. 3. Variation of the scale factor C for $n = 0.5$ and $n = 2$ with time.

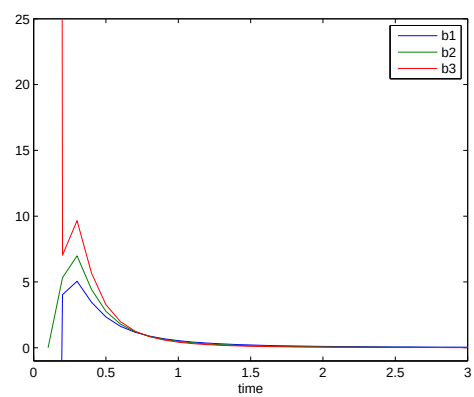

FIG. 4. Variation of bulk viscous stress Π with time for the subcase Bulk Viscosity Energy-Density Law in Model 1. $b1$, $b2$ and $b3$ represent variation for the parameter $\omega = 1.25$, $\omega = 1.5$ and $\omega = 1.75$ respectively.

The value of bulk viscosity coefficient in Truncated theory can be written as

$$\begin{aligned} \zeta = {} & \zeta_1 \left[\rho_0 (nh_0 t)^{-3} - \rho_1 (nh_0 t)^{\frac{-6}{n}-1} - \rho_2 (nh_0 t)^{\frac{-2}{n}-1}\right] \\ & + \zeta_2 \left[\rho_0 (nh_0 t)^{-2} - \rho_1 (nh_0 t)^{\frac{-6}{n}} - \rho_2 (nh_0 t)^{\frac{-2}{n}}\right]^{\omega-1} \\ & \left[\frac{3\rho_1}{n} (nh_0 t)^{\frac{-6}{n}-1} + \frac{\rho_2}{n} (nh_0 t)^{\frac{-2}{n}-1} - \rho_0 (nh_0 t)^{-3}\right]. \end{aligned} \tag{53}$$

Here $\zeta_1 = 3h_0$ and $\zeta_2 = 2nh_0 \omega \Pi_0$.

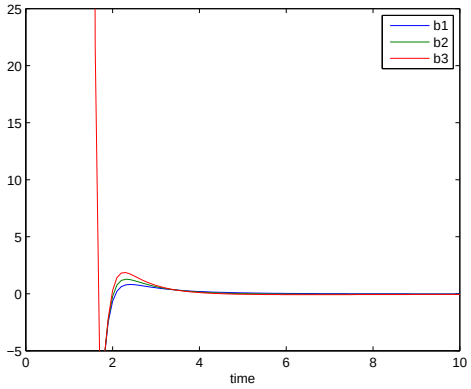

FIG. 5. Variation of creation pressure Pc for $n = 0.5$ and $\gamma = 0$ with time for the subcase Bulk Viscosity Energy-Density Law in Model 1. $b1$, $b2$ and $b3$ represent variation for the parameter $\omega = 1.25$, $\omega = 1.5$ and $\omega = 1.75$ respectively.

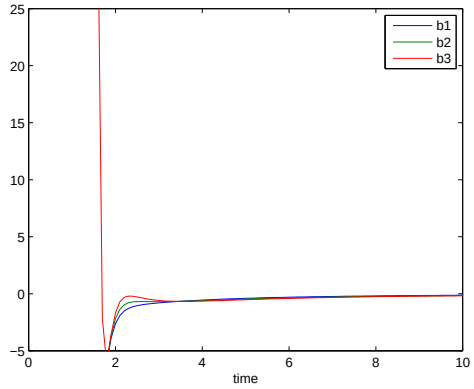

FIG. 6. Variation of creation pressure Pc for $n = 0.5$ and $\gamma = 1$ with time for the subcase Bulk Viscosity Energy-Density Law in Model 1. $b1$, $b2$ and $b3$ represent variation for the parameter $\omega = 1.25$, $\omega = 1.5$ and $\omega = 1.75$ respectively.

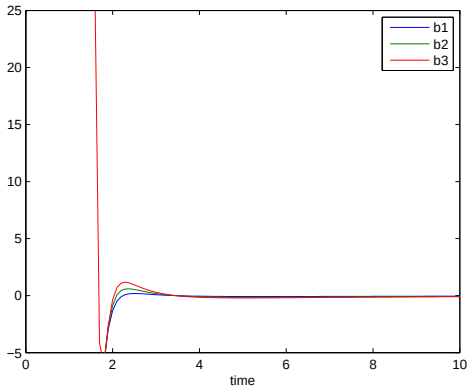

FIG. 7. Variation of creation pressure Pc for $n = 0.5$ and $\gamma = 1/3$ with time for the subcase Bulk Viscosity Energy-Density Law in Model 1. $b1$, $b2$ and $b3$ represent variation for the parameter $\omega = 1.25$, $\omega = 1.5$ and $\omega = 1.75$ respectively.

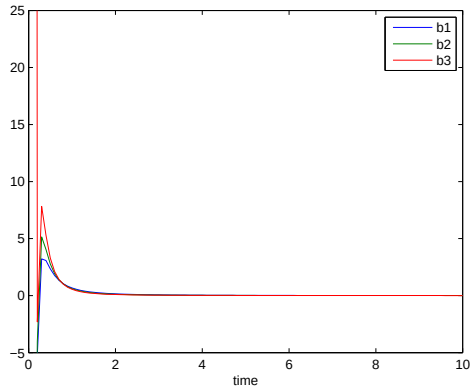

FIG. 8. Variation of creation pressure Pc for $n = 2$ and $\gamma = 0$ with time for the subcase Bulk Viscosity Energy-Density Law in Model 1. $b1$, $b2$ and $b3$ represent variation for the parameter $\omega = 1.25$, $\omega = 1.5$ and $\omega = 1.75$ respectively.

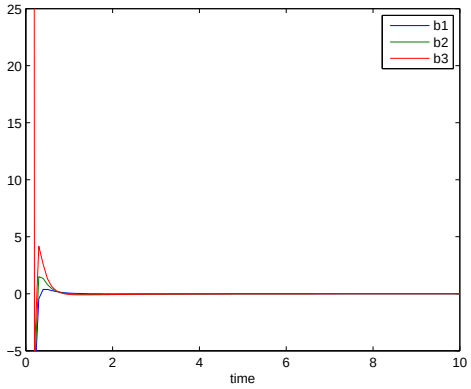

FIG. 9. Variation of creation pressure Pc for $n = 2$ and $\gamma = 1$ with time for the subcase Bulk Viscosity Energy-Density Law in Model 1. $b1$, $b2$ and $b3$ represent variation for the parameter $\omega = 1.25$, $\omega = 1.5$ and $\omega = 1.75$ respectively.

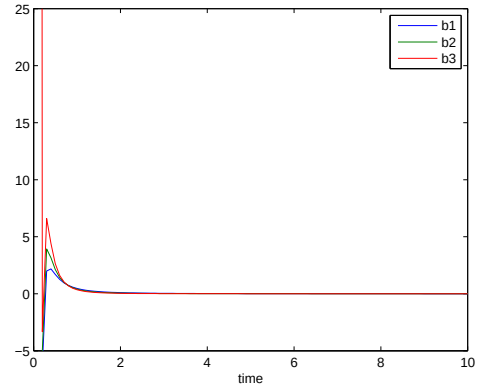

FIG. 10. Variation of creation pressure Pc for $n = 2$ and $\gamma = 1/3$ with time for the subcase Bulk Viscosity Energy-Density Law in Model 1. $b1$, $b2$ and $b3$ represent variation for the parameter $\omega = 1.25$, $\omega = 1.5$ and $\omega = 1.75$ respectively.

The bulk viscosity coefficient in Full Causal theory can be obtained as

$$\zeta = \frac{\zeta_3\left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}\right]^{\omega+2}}{\begin{array}{c}6h_0(nh_0t)^{-1}\left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}\right]^2 \\ +\zeta_4\left[\rho_0(nh_0t)^{-3} - \frac{3\rho_1}{n}(nh_0t)^{\frac{-6}{n}-1} - \frac{\rho_2}{n}(nh_0t)^{\frac{-2}{n}-1}\right] \\ +\zeta_5(nh_0t)^{-1}\left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}\right]^{\omega+1} \\ +\zeta_6\left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}\right]^{\omega-1} \\ \left[-\rho_0(nh_0t)^{-3} + \frac{3\rho_1}{n}(nh_0t)^{\frac{-6}{n}-1} + \frac{\rho_2}{n}(nh_0t)^{\frac{-2}{n}-1}\right]\end{array}} \tag{54}$$

where $\zeta_3 = -2\Pi_0$, $\zeta_4 = \frac{2nh_0(1+2\gamma)}{(1+\gamma)}$, $\zeta_5 = 3\Pi_0 h_0$ and $\zeta_6 = 4nh_0\Pi_0\omega$.
Similarly the bulk viscosity coefficient in Eckart theory can be given as

$$\zeta = \frac{-\Pi_0\left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}\right]^{\omega}}{3h_0(nh_0t)^{-1}}. \tag{55}$$

2. *Uniform Particle Number Density* ($\dot{\eta} = 0$)

In this case of study we consider the particle number density to be uniform during the evolution of the universe, i.e., ($\dot{\eta} = 0$). This assumption gives the following values of the particle production term Γ and the creation pressure p_c as

$$\Gamma = 3H\eta \tag{56}$$

$$p_c = -(1+\gamma)\rho. \tag{57}$$

So here the value of bulk viscous stress and the creation pressure can be obtained as

$$\Pi = \frac{2\rho_0 n}{3}(nh_0t)^{-2} - 2\rho_1(nh_0t)^{-6/n} - \frac{2\rho_2}{3}(nh_0t)^{-2/n}. \tag{58}$$

$$p_c = -(1+\gamma)\left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{-6/n} - \rho_2(nh_0t)^{-2/n}\right]. \tag{59}$$

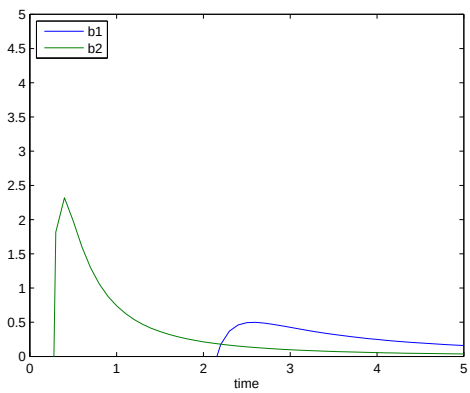

FIG. 11. Variation of Π with time for the subcase Uniform Particle Number Density in Model 1. $b1$, $b2$ represent variation for the parameter $n = 0.5$ and $n = 2$.

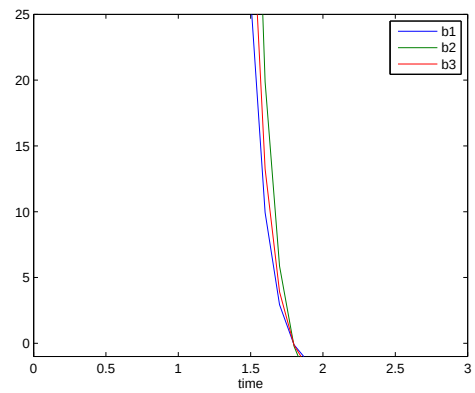

FIG. 12. Variation of creation pressure Pc for $n = 0.5$ with time for the subcase Uniform Particle Number Density in Model 1. $b1$, $b2$ and $b3$ represent variation for the parameter $\gamma = 0$, $\gamma = 1$ and $\gamma = 1/3$ respectively.

The behavior of Π and Pc in Uniform Particle Number Density for Model 1 can be seen in figures from 11 to 13. The expression for bulk viscous coefficient in all three cases are as follows:
Truncated theory:

$$\zeta = \frac{\left[2\rho_1(nh_0t)^{\frac{-6}{n}} + \frac{2\rho_2}{3}(nh_0t)^{\frac{-2}{n}} - \frac{2\rho_0 n}{3}(nh_0t)^{-2}\right]\left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}\right]}{\zeta_7(nh_0t)^{-3} + \zeta_8(nh_0t)^{\frac{-6}{n}} - \zeta_9(nh_0t)^{\frac{-2}{n}}}. \tag{60}$$

Here $\zeta_7 = \left(3h_0\rho_0 - \frac{4h_0\rho_0 n^2}{3}\right)$, $\zeta_8 = 9\rho_1 h_0$ and $\zeta_9 = \frac{5}{3}\rho_2 h_0$.
Full Causal theory:

$$\zeta = \frac{\begin{array}{c}\left[2\rho_1(nh_0t)^{\frac{-6}{n}} + \frac{2\rho_2}{3}(nh_0t)^{\frac{-2}{n}} - \frac{2\rho_0 n}{3}(nh_0t)^{-2}\right] \\ \left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}\right]^2\end{array}}{\begin{array}{c}\left[\frac{2\rho_0 n}{3}(nh_0t)^{-2} - 2\rho_1(nh_0t)^{\frac{-6}{n}} - \frac{2\rho_2}{3}(nh_0t)^{\frac{-2}{n}}\right] \\ \left[\zeta_{10}(nh_0t)^{-3} - \zeta_{11}(nh_0t)^{\frac{-2}{n}-1} - \zeta_{12}(nh_0t)^{\frac{-6}{n}-1}\right] \\ + \left[-\zeta_{13}(nh_0t)^{-3} + \zeta_{14}(nh_0t)^{\frac{-6}{n}-1} + \zeta_{15}(nh_0t)^{\frac{-2}{n}-1}\right] \\ \left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{-6/n} - \rho_2(nh_0t)^{-2/n}\right] \\ +3h_0(nh_0t)^{-1}\left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{-6/n} - \rho_2(nh_0t)^{-2/n}\right]^2\end{array}} \tag{61}$$

where $\zeta_{10} = \frac{3}{2}\rho_0 h_0 + \frac{n(1+2\gamma)}{(1+\gamma)}\rho_0 h_0$, $\zeta_{11} = \frac{3}{2}h_0\rho_2 + \frac{(1+2\gamma)}{(1+\gamma)}h_0\rho_2$, $\zeta_{12} = \frac{3}{2}h_0\rho_1 + \frac{3(1+2\gamma)}{(1+\gamma)}h_0\rho_1$, $\zeta_{13} = \frac{4\rho_0 n^2 h_0}{3}$, $\zeta_{14} = 12\rho_1 h_0$ and $\zeta_{15} = \frac{4h_0\rho_2}{3}$.
Eckart theory:

$$\zeta = \frac{2\rho_1}{3h_0}(nh_0t)^{\frac{-6}{n}+1} + \frac{2\rho_2}{9h_0}(nh_0t)^{\frac{-2}{n}+1} - \frac{2\rho_0 n}{9h_0}(nh_0t)^{-1}. \tag{62}$$

3. Ideal Gas

The conservation of total particle number in standard cosmology can be given as

$$N^i_{;i} = \dot{\eta} + 3\eta H = 0 \tag{63}$$

which leads to $\Gamma = 0$ and $p_c = 0$. Hence this model will only have bulk viscosity. The expression for particle number density and bulk viscous stress can be obtained as

$$\eta = \eta_1 t^{\frac{-3}{n}} \tag{64}$$

and

$$\Pi = \zeta_{16}(nh_0t)^{-2} + \zeta_{17}(nh_0t)^{-6/n} + \zeta_{18}(nh_0t)^{-2/n} \tag{65}$$

where η_1 is an integration constant. Here $\zeta_{16} = \left[\frac{2n}{3} - (1+\gamma)\right]\rho_0$, $\zeta_{17} = (\gamma - 1)\rho_1$ and $\zeta_{18} = \frac{(1+3\gamma)}{3}\rho_2$. The figures from 14 to 16 show the graphical behaviour of Π for Ideal Gas in Model 1.
In this case, the value of bulk viscous coefficient in different theories can be obtained as
Truncated theory:

$$\zeta = \frac{\begin{array}{c}\left[\zeta_{19}(nh_0t)^{-2} + \zeta_{20}(nh_0t)^{\frac{-6}{n}} - \zeta_{21}(nh_0t)^{-2/n}\right] \\ \left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}\right]\end{array}}{\zeta_{22}(nh_0t)^{-3} + \zeta_{23}(nh_0t)^{\frac{-6}{n}-1} - \zeta_{24}(nh_0t)^{\frac{-2}{n}-1}}. \tag{66}$$

Here $\zeta_{19} = \left[(1+\gamma) - \frac{2n}{3}\right]\rho_0$, $\zeta_{20} = (1-\gamma)\rho_1$, $\zeta_{21} = \frac{(1+3\gamma)}{3}\rho_2$, $\zeta_{22} = \left[\left(1+\gamma-\frac{2n}{3}\right)2n+3\right]h_0\rho_0$, $\zeta_{23} = 3(1-2\gamma)h_0\rho_1$ and $\zeta_{24} = \frac{(11+6\gamma)}{3}h_0\rho_2$.
Full Causal theory:

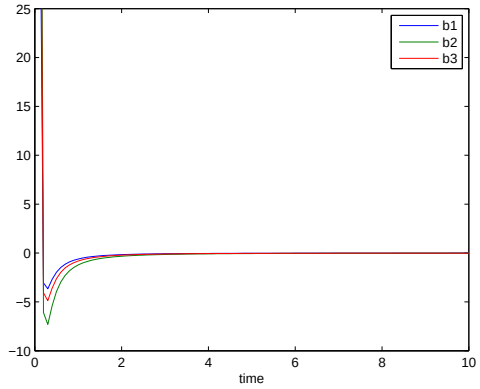

FIG. 13. Variation of creation pressure Pc for $n = 2$ with time for the subcase Uniform Particle Number Density in Model 1. $b1$, $b2$ and $b3$ represent variation for the parameter $\gamma = 0$, $\gamma = 1$ and $\gamma = 1/3$ respectively.

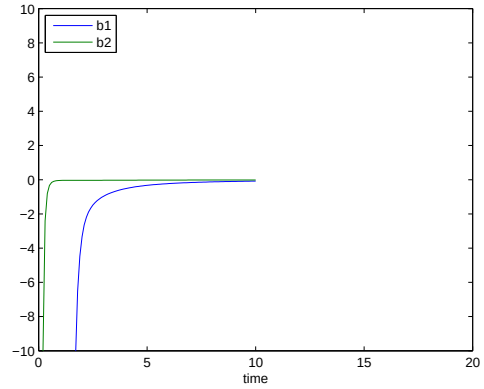

FIG. 14. Variation of Π for $\gamma = 0$ with time for the subcase Ideal Gas in Model 1. $b1$ and $b2$ represent variation for the parameter $n = 0.5$ and $n = 2$ respectively.

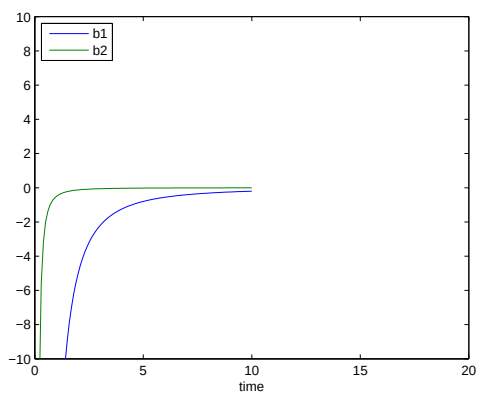

FIG. 15. Variation of Π for $\gamma = 1$ with time for the subcase Ideal Gas in Model 1. $b1$ and $b2$ represent variation for the parameter $n = 0.5$ and $n = 2$ respectively.

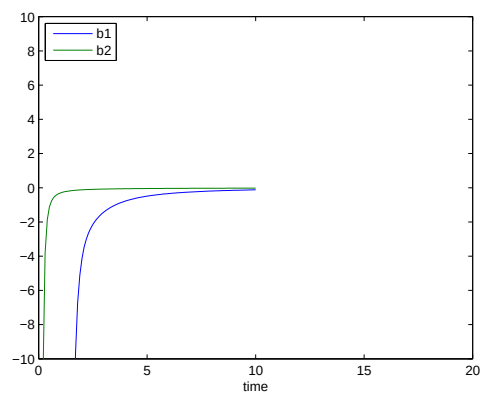

FIG. 16. Variation of Π for $\gamma = 1/3$ with time for the subcase Ideal Gas in Model 1. $b1$ and $b2$ respectively.

$$\zeta = \frac{\begin{array}{c}\left[\zeta_{25}(nh_0t)^{-2} + \zeta_{26}(nh_0t)^{\frac{-6}{n}} - \zeta_{27}(nh_0t)^{-2/n}\right] \\ \left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}\right]^2\end{array}}{\begin{array}{c}\left[\zeta_{28}(nh_0t)^{-3} + \zeta_{29}(nh_0t)^{\frac{-6}{n}-1} - \zeta_{30}(nh_0t)^{\frac{-2}{n}-1}\right] \\ \left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}\right] \\ +3h_0(nh_0t)^{-1}\left[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}\right]^2 \\ +\left[\zeta_{16}(nh_0t)^{-2} + \zeta_{17}(nh_0t)^{\frac{-6}{n}} + \zeta_{18}(nh_0t)^{\frac{-2}{n}}\right] \\ \left[\zeta_{31}(nh_0t)^{-3} - \zeta_{32}(nh_0t)^{\frac{-2}{n}-1} - \zeta_{33}(nh_0t)^{\frac{-6}{n}-1}\right]\end{array}} \tag{67}$$

where $\zeta_{25} = \left[(1+\gamma) - \frac{2n}{3}\right]\rho_0$, $\zeta_{26} = (1-\gamma)\rho_1$, $\zeta_{27} = \frac{(1+3\gamma)}{3}\rho_2$, $\zeta_{28} = \left[(1+\gamma) - \frac{2n}{3}\right]2nh_0\rho_0$, $\zeta_{29} = 6(1-\gamma)h_0\rho_1$, $\zeta_{30} = \frac{2(1+3\gamma)h_0\rho_2}{3}$, $\zeta_{31} = \left[\frac{3}{2} + \frac{n(1+2\gamma)}{(1+\gamma)}\right]\rho_0 h_0$, $\zeta_{32} = \left[\frac{3}{2} + \frac{(1+2\gamma)}{(1+\gamma)}\right]\rho_2 h_0$ and $\zeta_{33} = \left[\frac{3}{2} + \frac{3(1+2\gamma)}{(1+\gamma)}\right]\rho_1 h_0$.
Eckart theory:

$$\zeta = \zeta_{34}(nh_0t)^{-1} + \zeta_{35}(nh_0t)^{\frac{-6}{n}+1} - \zeta_{36}(nh_0t)^{\frac{-2}{n}+1}. \tag{68}$$

Here $\zeta_{34} = \left[(1+\gamma) - \frac{2n}{3}\right]\frac{\rho_0}{3h_0}$, $\zeta_{35} = \frac{(1-\gamma)\rho_1}{3h_0}$ and $\zeta_{36} = \frac{(1+3\gamma)\rho_2}{9h_0}$.

4. *Creation with Second Order Correction in H*

The conservation of total particle number in standard cosmology can be generalized by using the Taylor expansion of $\frac{\dot{\eta}}{\eta} = f(H)$ up to second order in H. This idea was given by Triginer and Pavon in 1994 [21]. So here we have

$$\frac{\dot{\eta}}{\eta} = -3H + dH^2 \tag{69}$$

where d is a constant. The particle number density η also satisfies the balance equation

$$\dot{\eta} + 3\eta H = \Gamma. \tag{70}$$

So from above equations

$$\Gamma = d\eta H^2. \tag{71}$$

Here creation, no creation and annihilation of particles are decided by the conditions $d > 0$, $d = 0$ and $d < 0$ respectively. Using equation (69), creation pressure takes the form

$$p_c = -\frac{(1+\gamma)d}{3}H\rho. \tag{72}$$

Hence the expression for creation pressure and bulk viscous stress can be given as

$$p_c = -\frac{(1+\gamma)dh_0}{3}\left[\rho_0(nh_0t)^{-3} - \rho_1(nh_0t)^{\frac{-6}{n}-1} - \rho_2(nh_0t)^{\frac{-2}{n}-1}\right] \tag{73}$$

$$\begin{aligned}\Pi = {} & \zeta_{37}(nh_0t)^{-2} - \zeta_{38}(nh_0t)^{\frac{-6}{n}} + \zeta_{39}(nh_0t)^{\frac{-2}{n}} \\ & +\zeta_{40}(nh_0t)^{-3} - \zeta_{41}(nh_0t)^{\frac{-6}{n}-1} - \zeta_{42}(nh_0t)^{\frac{-2}{n}-1}.\end{aligned} \tag{74}$$

Here $\zeta_{37} = \left[\frac{2n}{3} - (1+\gamma)\right]\rho_0$, $\zeta_{38} = (1-\gamma)\rho_1$, $\zeta_{39} = \gamma$, $\zeta_{40} = \frac{d(1+\gamma)h_0\rho_0}{3}$, $\zeta_{41} = \frac{d(1+\gamma)h_0\rho_1}{3}$ and $\zeta_{42} = \frac{d(1+\gamma)h_0\rho_2}{3}$. The behaviour of Pc and Π in this case can be observed in the following figures from 17 to 22.

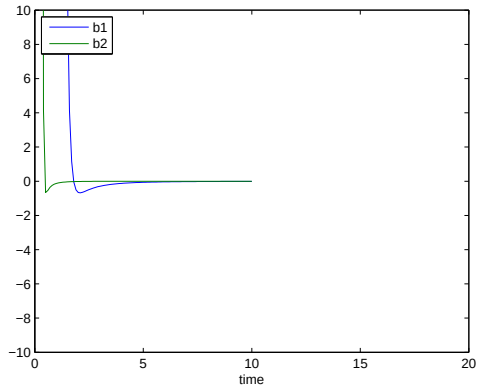

FIG. 17. Variation of creation pressure Pc for $\gamma = 0$ with time for the subcase Creation with Second Order Correction in H in Model 1. $b1$ and $b2$ represent variation for the parameter $n = 0.5$ and $n = 2$ respectively.

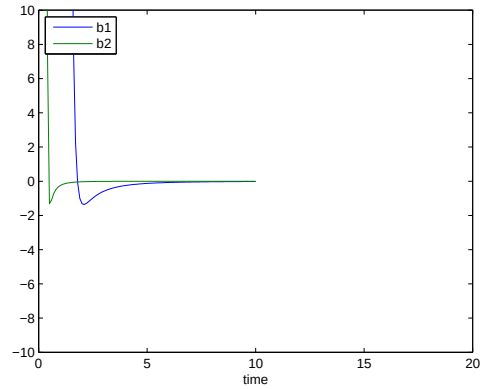

FIG. 18. Variation of creation pressure Pc for $\gamma = 1$ with time for the subcase Creation with Second Order Correction in H in Model 1. $b1$ and $b2$ represent variation for the parameter $n = 0.5$ and $n = 2$ respectively.

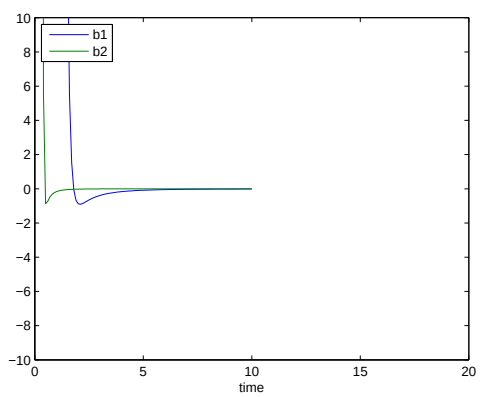

FIG. 19. Variation of creation pressure Pc for $\gamma = 1/3$ with time for the subcase Creation with Second Order Correction in H in Model 1. $b1$ and $b2$ represent variation for the parameter $n = 0.5$ and $n = 2$ respectively.

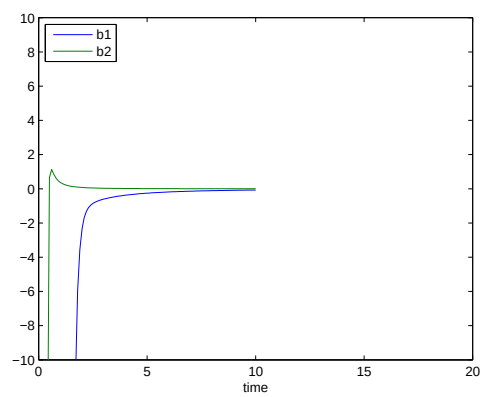

FIG. 20. Variation of Π for $\gamma = 0$ with time for the subcase Creation with Second Order Correction in H in Model 1. $b1$ and $b2$ represent variation for the parameter $n = 0.5$ and $n = 2$ respectively.

In this case, the value of bulk viscous coefficient in different theories can be obtained as Truncated theory:

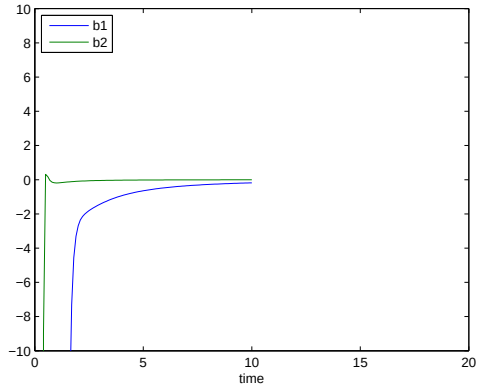

FIG. 21. Variation of Π for $\gamma = 1$ with time for the subcase Creation with Second Order Correction in H in Model 1. $b1$ and $b2$ represent variation for the parameter $n = 0.5$ and $n = 2$ respectively.

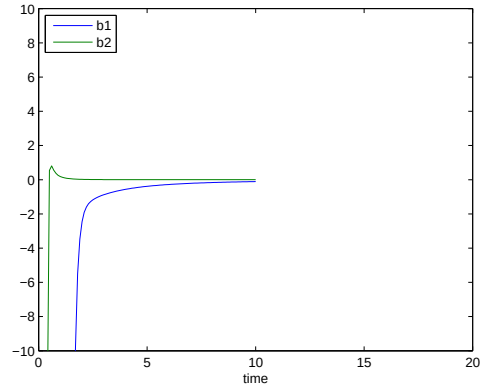

FIG. 22. Variation of Π for $\gamma = 1/3$ with time for the subcase Creation with Second Order Correction in H in Model 1. $b1$ and $b2$ represent variation for the parameter $n = 0.5$ and $n = 2$ respectively.

$$\zeta = \frac{\begin{array}{c}[-\zeta_{37}(nh_0t)^{-2} + \zeta_{38}(nh_0t)^{\frac{-6}{n}} - \zeta_{39}(nh_0t)^{\frac{-2}{n}} \\ -\zeta_{40}(nh_0t)^{-3} + \zeta_{41}(nh_0t)^{\frac{-6}{n}-1} + \zeta_{42}(nh_0t)^{\frac{-2}{n}-1}] \\ \times[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}]\end{array}}{\begin{array}{c}[\zeta_{43}(nh_0t)^{-3} + \zeta_{44}(nh_0t)^{\frac{-6}{n}-1} - \zeta_{45}(nh_0t)^{\frac{-2}{n}-1} \\ -\zeta_{46}(nh_0t)^{-4} + \zeta_{47}(nh_0t)^{\frac{-6}{n}-2} + \zeta_{48}(nh_0t)^{\frac{-2}{n}-2}]\end{array}} \tag{75}$$

where $\zeta_{43} = \left[2n(1+\gamma) + 3 - \frac{4n^2}{3}\right]$, $\zeta_{44} = 3(1-2\gamma)\rho_1 h_0$, $\zeta_{45} = (2\gamma+3)h_0\rho_2$, $\zeta_{46} = d(1+\gamma)h_0^2\rho_0 n$, $\zeta_{47} = \left[\frac{(n+6)d(1+\gamma)h_0^2\rho_1}{3}\right]$ and $\zeta_{48} = \left[\frac{d(n+2)(1+\gamma)h_0^2\rho_2}{3}\right]$.

Full Causal theory:

$$\zeta = \frac{\begin{array}{c}[-\zeta_{37}(nh_0t)^{-2} + \zeta_{38}(nh_0t)^{\frac{-6}{n}} - \zeta_{39}(nh_0t)^{\frac{-2}{n}} \\ -\zeta_{40}(nh_0t)^{-3} + \zeta_{41}(nh_0t)^{\frac{-6}{n}-1} + \zeta_{42}(nh_0t)^{\frac{-2}{n}-1}] \\ \times[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}]^2\end{array}}{\begin{array}{c}[\zeta_{49}(nh_0t)^{-3} + \zeta_{50}(nh_0t)^{\frac{-6}{n}-1} - \zeta_{51}(nh_0t)^{\frac{-2}{n}-1} \\ -\zeta_{52}(nh_0t)^{-4} + \zeta_{53}(nh_0t)^{\frac{-6}{n}-2} + \zeta_{54}(nh_0t)^{\frac{-2}{n}-2}] \\ \times[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}] \\ +3h_0(nh_0t)^{-1}[\rho_0(nh_0t)^{-2} - \rho_1(nh_0t)^{\frac{-6}{n}} - \rho_2(nh_0t)^{\frac{-2}{n}}]^2 \\ +[\zeta_{37}(nh_0t)^{-2} - \zeta_{38}(nh_0t)^{\frac{-6}{n}} + \zeta_{39}(nh_0t)^{\frac{-2}{n}} \\ +\zeta_{40}(nh_0t)^{-3} - \zeta_{41}(nh_0t)^{\frac{-6}{n}-1} - \zeta_{42}(nh_0t)^{\frac{-2}{n}-1}] \\ \times[\zeta_{55}(nh_0t)^{-3} - \zeta_{56}(nh_0t)^{\frac{-6}{n}-1} - \zeta_{57}(nh_0t)^{\frac{-2}{n}-1}]\end{array}} \tag{76}$$

where $\zeta_{49} = \left[2(1+\gamma) - \frac{4n}{3}\right] nh_0\rho_0$, $\zeta_{50} = 6(1-\gamma)\rho_1 h_0$, $\zeta_{51} = 2h_0\rho_2\gamma$, $\zeta_{52} = d(1+\gamma)h_0^2\rho_0 n$, $\zeta_{53} = \left[\frac{(n+6)d(1+\gamma)h_0^2\rho_1}{3}\right]$, $\zeta_{54} = \left[\frac{d(n+2)(1+\gamma)h_0^2\rho_2}{3}\right]$, $\zeta_{55} = \left[\frac{3}{2}h_0 - \frac{(1+2\gamma)}{2(1+\gamma)}\right]\rho_0$, $\zeta_{56} =$

$\left[\frac{3}{2}+\frac{3(1+2\gamma)}{(1+\gamma)}\right]\rho_1 h_0$, $\zeta_{57}=\left[\frac{3}{2}+\frac{(1+2\gamma)}{(1+\gamma)}\right]h_0\rho_2$.
Eckart theory:

$$\zeta=\frac{\begin{array}{c}[-\zeta_{37}(nh_0t)^{-2}+\zeta_{38}(nh_0t)^{\frac{-6}{n}}-\zeta_{39}(nh_0t)^{\frac{-2}{n}}\\ -\zeta_{40}(nh_0t)^{-3}+\zeta_{41}(nh_0t)^{\frac{-6}{n}-1}+\zeta_{42}(nh_0t)^{\frac{-2}{n}-1}]\end{array}}{3h_0(nh_0t)^{-1}}. \tag{77}$$

B. Model 2

In this case, putting the value of the average scale-factor from equation (33) into equations (34)-(36), the exact values of the metric functions can be obtained as

$$A=a_0\exp\left(h_0t\right) \tag{78}$$

$$B=B_0a_0\exp\left[h_0t-\frac{X}{3a_1h_0}\exp\left(-h_0t\right)\right] \tag{79}$$

$$C=C_0a_0\exp\left[h_0t+\frac{X}{3a_1h_0}\exp\left(-h_0t\right)\right]. \tag{80}$$

Hence the Expansion scalar θ, Shear scalar σ^2, generalized Hubble parameter H and the Volume scalar V can be written as

$$\theta=3h_0 \tag{81}$$

$$\sigma^2=\frac{X^2}{a_1^2}\exp\left(-2h_0t\right) \tag{82}$$

$$H=h_0 \tag{83}$$

and

$$V=V_0\exp\left(3h_0t\right). \tag{84}$$

The anisotropy parameter A_m can be given by

$$A_m=\frac{2}{3}\frac{X^2}{h_0^2a_1^2}\exp\left(-2h_0t\right). \tag{85}$$

The directional Hubble parameters can be obtained as

$$H_1=h_0 \tag{86}$$

$$H_2=h_0+\frac{X}{3a_1}\exp\left(-h_0t\right) \tag{87}$$

$$H_3=h_0-\frac{X}{3a_1}\exp\left(-h_0t\right). \tag{88}$$

Here the value of the energy density can be calculated as

$$\rho=\rho_3-\rho_6\exp\left(-2h_0t\right) \tag{89}$$

where $\rho_3=\frac{3h_0^2}{8\pi G}$, $\rho_6=\rho_4+\rho_5$, $\rho_4=\frac{X^2}{72\pi Ga_1^2}$, $\rho_5=\frac{3m^2}{8\pi Ga_1^{2/3}}$. So, we have the equation for pressure as

$$p=\gamma\left[\rho_3-\rho_6\exp\left(-2h_0t\right)\right]. \tag{90}$$

Here we observe that all three scale factors A, B, C, the shear scalar σ^2, the volume scalar V, the anisotropy parameter A_m, the energy density ρ and the pressure p are all constant at the initial time $t = 0$. The variation of A, B and C can be observed graphically in figure 23. The expansion scalar θ and the generalized Hubble parameter H are constants through out the evolution of the universe. The directional Hubble parameters H_1, H_2 and H_3 are constant at this time $t = 0$. These results show that the universe starts evolving with constant volume and expands with exponential rate. Thus the model represents uniform expansion due to the constant expansion scalar and volume grows exponentially with time. Similarly we can study this model for very large time as $t \to \infty$. We find that for large time all three scale factors and the volume scalar will become infinity. The mean anisotropy parameter and shear scalar decrease with time and tend to zero at $t \to \infty$ which means that the universe is accelerating in later stage of its evolution. Also the directional Hubble parameters become constant and uniform. Also the energy density so as pressure become constant for large time. Now in this case also we study in the following subsections the behaviour of particle creation and bulk viscosity of this model in four different physical laws. That will give four different sub models out of the above model.

1. *Bulk Viscosity Energy-Density Law*

The bulk viscous stress and creation pressure in this case can be obtained as

$$\Pi = \Pi_0[\rho_3 - \rho_6 \exp(-2h_0 t)]^{\omega}. \tag{91}$$

$$p_c = p_3 + p_4 \exp(-2h_0 t) - \Pi_0[\rho_3 - \rho_6 \exp(-2h_0 t)]^{\omega}. \tag{92}$$

Here $p_3 = -(1+\gamma)\rho_3$, $p_4 = \left[(1+\gamma)\rho_6 - \frac{2}{3}\rho_6\right]$. The following figures from 24 to 27 show the variation of Π and Pc in this case for Model 2.

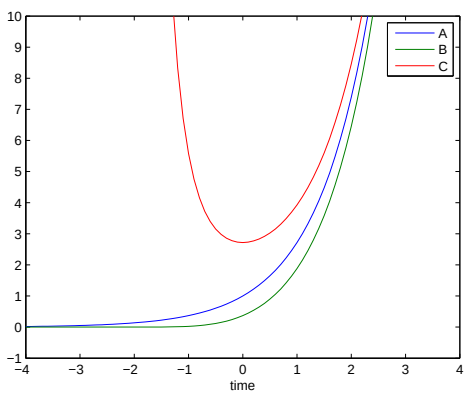

FIG. 23. Variation of scale factors A, B and C with time for the Model 2.

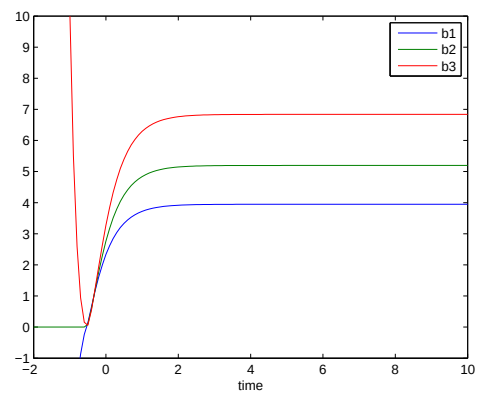

FIG. 24. Variation of Π with time for the subcase Bulk Viscosity Energy-Density Law in Model 2. $b1$, $b2$ and $b3$ represent the variation for $\omega = 1.25$, $\omega = 1.5$ and $\omega = 1.75$ respectively.

The bulk viscosity coefficient in three different theories can be given as
Truncated theory:

$$\zeta = \frac{-\Pi_0[\rho_3 - \rho_6 \exp(-2h_0 t)]^{\omega+1}}{\zeta_{58} - \zeta_{59} \exp(-2h_0 t) + \zeta_{60} \exp(-2h_0 t)[\rho_3 - \rho_6 \exp(-2h_0 t)]^{\omega-1}} \tag{93}$$

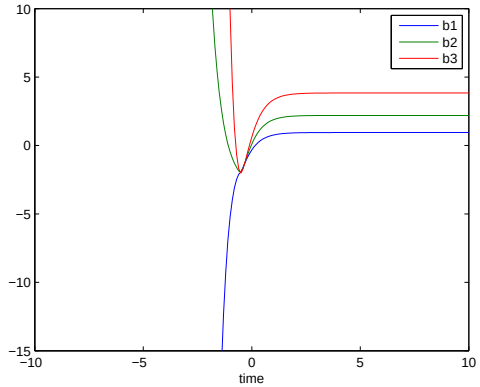

FIG. 25. Variation of creation pressure Pc for $\gamma = 0$ with time for the subcase Bulk Viscosity Energy-Density Law in Model 2. $b1$, $b2$ and $b3$ represent the variation for $\omega = 1.25$, $\omega = 1.5$ and $\omega = 1.75$ respectively.

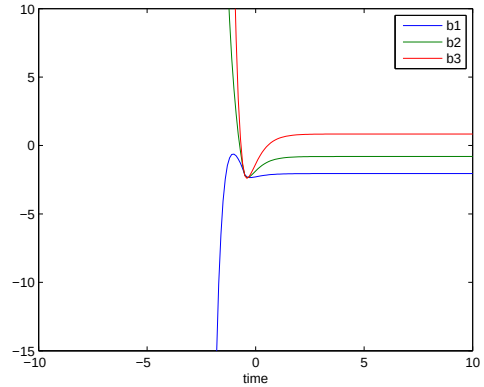

FIG. 26. Variation of creation pressure Pc for $\gamma = 1$ with time for the subcase Bulk Viscosity Energy-Density Law in Model 2. $b1$, $b2$ and $b3$ represent the variation for $\omega = 1.25$, $\omega = 1.5$ and $\omega = 1.75$ respectively.

where $\zeta_{58} = 3h_0\rho_3$, $\zeta_{59} = 3h_0\rho_6$ and $\zeta_{60} = 2h_0\Pi_0\omega\rho_6$.
Full Causal theory:

$$\zeta = \frac{-2\Pi_0[\rho_3 - \rho_6 \exp(-2h_0 t)]^{\omega+2}}{\begin{array}{c}\zeta_{61}\exp(-2h_0 t)[\rho_3 - \rho_6\exp(-2h_0 t)]^{\omega} + \zeta_{62}[\rho_3 - \rho_6\exp(-2h_0 t)]^{\omega+1} \\ +6h_0[\rho_3 - \rho_6\exp(-2h_0 t)]^2 - \zeta_{63}\exp(-2h_0 t)\end{array}} \tag{94}$$

where $\zeta_{61} = 4h_0\rho_6\omega\Pi_0$, $\zeta_{62} = 3\Pi_0 h_0$ and $\zeta_{63} = \frac{2(1+2\gamma)h_0\rho_6}{(1+\gamma)}$.
Eckart theory:

$$\zeta = -\frac{\Pi_0}{3h_0}[\rho_3 - \rho_6\exp(-2h_0 t)]^{\omega}. \tag{95}$$

2. *Uniform Particle Number Density* ($\dot{\eta} = 0$)

The bulk viscous stress and creation pressure can be obtained as follows:

$$\Pi = -\frac{2}{3}\rho_6\exp(-2h_0 t) \tag{96}$$

and

$$p_c = -(1+\gamma)[\rho_3 - \rho_6\exp(-2h_0 t)]. \tag{97}$$

The behavior of Π and Pc for Uniform Particle Number Density in Model2 are presented graphically in figures 28 and 29.
The bulk viscosity coefficient in different theories can be written as
Truncated theory:

$$\zeta = \frac{\zeta_{64}\exp(-2h_0 t)[\rho_3 - \rho_6\exp(-2h_0 t)]}{\zeta_{65} - \zeta_{66}\exp(-2h_0 t)} \tag{98}$$

where $\zeta_{64} = \frac{2}{3}\rho_6$, $\zeta_{65} = 3h_0\rho_3$ and $\zeta_{66} = \frac{5}{3}h_0\rho_6$.

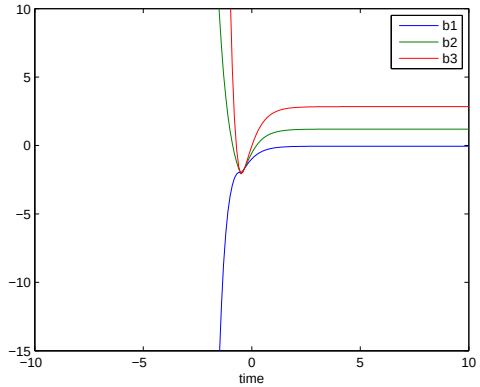

FIG. 27. Variation of creation pressure Pc for $\gamma = 1/3$ with time for the subcase Bulk Viscosity Energy-Density Law in Model 2. $b1$, $b2$ and $b3$ represent the variation for $\omega = 1.25$, $\omega = 1.5$ and $\omega = 1.75$ respectively.

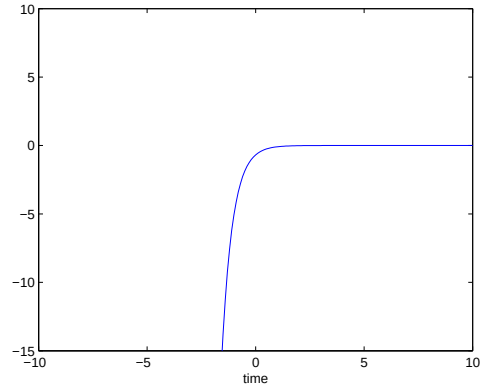

FIG. 28. Variation of Π with time for the subcase Uniform Particle Number Density in Model 2.

Full Causal theory:

$$\zeta = \frac{\zeta_{67}\exp(-2h_0t)[\rho_3 - \rho_6\exp(-2h_0t)]^2}{\zeta_{68}\exp(-2h_0t) + \zeta_{69}\exp(-4h_0t)} \tag{99}$$

where $\zeta_{67} = \frac{2}{3}\rho_6$, $\zeta_{68} = -\frac{17}{3}h_0\rho_3\rho_6$ and $\zeta_{69} = \left[\frac{2(1+2\gamma)}{3(1+\gamma)} + \frac{8}{3}\right]h_0\rho_6^2$.
Eckart theory:

$$\zeta = \zeta_{70}\exp(-2h_0t). \tag{100}$$

Here $\zeta_{70} = \frac{2\rho_6}{9h_0}$.

3. Ideal Gas

The value of the particle number density and Bulk viscous stress in this case can be obtained as

$$\eta = \eta_0\exp(-3h_0t) \tag{101}$$

and

$$\Pi = -\Pi_1 + \Pi_2\exp(-2h_0t) \tag{102}$$

where $\Pi_1 = (1+\gamma)\rho_3$, $\Pi_2 = \frac{(1+3\gamma)}{3}\rho_6$. The behaviour of Π in Ideal Gas in this model can be observed in figure 30.
The coefficient of Bulk viscosity in all three different theories can be calculated as
Truncated theory:

$$\zeta = \frac{[\Pi_1 - \Pi_2\exp(-2h_0t)][\rho_3 - \rho_6\exp(-2h_0t)]}{\zeta_{71} - \zeta_{72}\exp(-2h_0t)} \tag{103}$$

where $\zeta_{71} = 3h_0\rho_3$, $\zeta_{72} = (3\rho_6 + 2\Pi_2)h_0$.
Full Causal theory:

$$\zeta = \frac{[\Pi_1 - \Pi_2\exp(-2h_0t)][\rho_3 - \rho_6\exp(-2h_0t)]^2}{\zeta_{73} + \zeta_{74}\exp(-2h_0t) - \zeta_{75}\exp(-4h_0t)} \tag{104}$$

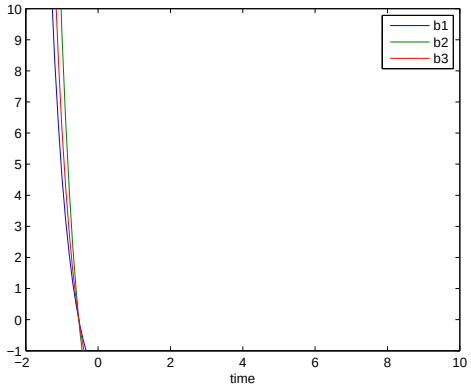

FIG. 29. Variation of creation pressure Pc with time for the subcase Uniform Particle Number Density in Model 2. $b1$, $b2$ and $b3$ represent the variation for $\gamma = 0$, $\gamma = 1$ and $\gamma = 1/3$ respectively.

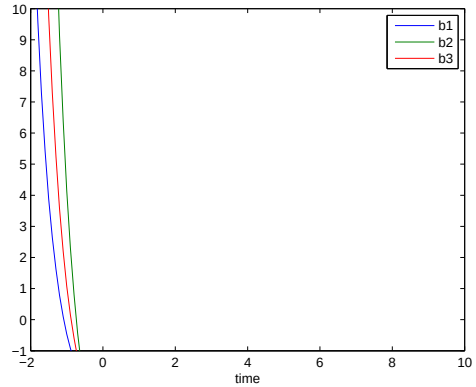

FIG. 30. Variation of Π with time for the subcase Ideal Gas in Model 2. $b1$, $b2$ and $b3$ represent variation for $\gamma = 0$, $\gamma = 1$ and $\gamma = 1/3$ respectively.

where $\zeta_{73} = \rho_3 \left(3h_0\rho_3 - \frac{3}{2}h_0\Pi_1\right)$,
$\zeta_{74} = \left[\rho_3 \left(\frac{3}{2}h_0\Pi_2 - 2h_0\Pi_2 - 3h_0\rho_6\right) - \rho_6 \left(3h_0\rho_3 - \frac{3}{2}h_0\Pi_1\right) + \frac{(1+2\gamma)}{(1+\gamma)}h_0\rho_6\Pi_1\right]$
and $\zeta_{75} = \left[\rho_6 \left(\frac{3}{2}h_0\Pi_2 - 2h_0\Pi_2 - 3h_0\rho_6\right) + \frac{(1+2\gamma)}{(1+\gamma)}h_0\rho_6\Pi_2\right]$.
Eckart theory:

$$\zeta = \frac{[\Pi_1 - \Pi_2 \exp(-2h_0 t)]}{3h_0}. \tag{105}$$

4. *Creation with Second Order Correction in H*

Also in this case, the creation pressure and bulk viscous stress can be given as

$$p_c = -\frac{(1+\gamma)dh_0}{3}[\rho_3 - \rho_6 \exp(-2h_0 t)] \tag{106}$$

$$\Pi = \Pi_3 - \Pi_4 \exp(-2h_0 t) \tag{107}$$

where $\Pi_3 = \frac{d(1+\gamma)h_0\rho_3}{3} - 3(1+\gamma)\rho_3 h_0$, $\Pi_4 = \frac{d(1+\gamma)h_0\rho_6}{3} - 3(1+\gamma)\rho_6 h_0$. The nature of the graphs in this case for Π and Pc can be observed in the figures 31 and 32.
The bulk viscosity coefficient in all three cases can be obtained
Truncated theory:

$$\zeta = \frac{-[\Pi_3 - \Pi_4 \exp(-2h_0 t)][\rho_3 - \rho_6 \exp(-2h_0 t)]}{\zeta_{76} \exp(-2h_0 t) + 3h_0[\rho_3 - \rho_6 \exp(-2h_0 t)]} \tag{108}$$

where $\zeta_{76} = 2h_0\rho_6 \left[\frac{d(1+\gamma)h_0}{3} - 3(1+\gamma)h_0 - \frac{2}{3}\right]$.
Full Causal theory:

$$\zeta = \frac{-[\Pi_3 - \Pi_4 \exp(-2h_0 t)][\rho_3 - \rho_6 \exp(-2h_0 t)]^2}{[\rho_3 - \rho_6 \exp(-2h_0 t)][\zeta_{77} + \zeta_{78} \exp(-2h_0 t) - \zeta_{79} \exp(-4h_0 t)]} \tag{109}$$

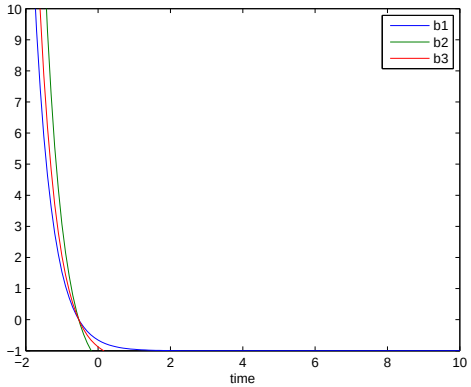

FIG. 31. Variation of creation pressure Pc with time for the subcase Creation with Second Order Correction in H in Model 2. $b1$, $b2$ and $b3$ represent the variation for $\gamma = 0$, $\gamma = 1$ and $\gamma = 1/3$ respectively.

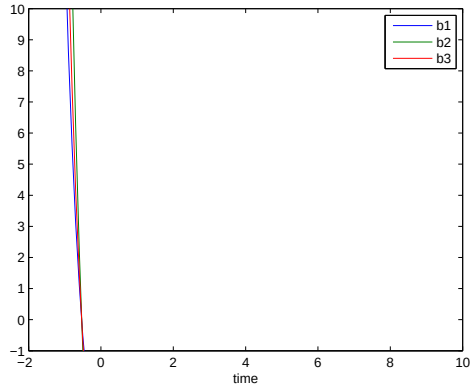

FIG. 32. Variation of Π with time for the subcase Creation with Second Order Correction in H in Model 2. $b1$, $b2$ and $b3$ represent the variation for $\gamma = 0$, $\gamma = 1$ and $\gamma = 1/3$ respectively.

where $\zeta_{77} = \left(3h_0\rho_3 + \frac{3}{2}h_0\Pi_3\right)$, $\zeta_{78} = \left[\zeta_{76} - 3h_0\rho_6 - \frac{3}{2}h_0\Pi_4 + \frac{(1+2\gamma)\rho_6\Pi_3}{2(1+\gamma)}\right]$ and $\zeta_{79} = \frac{(1+2\gamma)\rho_6\Pi_4}{2(1+\gamma)}$.
Eckart theory:

$$\zeta = \frac{-[\Pi_3 - \Pi_4 \exp(-2h_0 t)]}{3h_0}. \tag{110}$$

IV. CONCLUSION

In this paper, we have obtained exact solutions for the classical Einstein's general relativity equation in Bianchi type V space-time with bulk viscosity in the presence of particle creation. Here, we have applied the variation law of Hubble's parameter that yields a constant value of deceleration parameter to find out the solution. Following this, we have found two types of cosmological models for two different values of average scale factor obtained from the assumption. We have found singularity in the first model whereas the second model is free from it. We have then studied the bulk viscosity and particle creation in each model for four different cases. Also we have obtained the bulk viscosity coefficient for Truncated, Full Causal and Eckart theories in all four cases. Different physical and kinematical parameters have been obtained and studied graphically in both the models.

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