Mon. Not. R. Astron. Soc. **000**, 000–000 (0000) Printed 14 April 2016 (MN LaTeX style file v1.4)

arXiv:1307.4748v2 [astro-ph.CO] 4 Nov 2013

Effective equation of state for running vacuum: "mirage" quintessence and phantom dark energy

Spyros Basilakos[1] and Joan Solà[2]

[1] *Academy of Athens, Research Center for Astronomy & Applied Mathematics, Soranou Efessiou 4, 11-527, Athens, Greece*

[2] *High Energy Physics Group, Dept. ECM, and Institut de Ciències del Cosmos (ICC), Universitat de Barcelona, Av. Diagonal 647 E-08028 Barcelona, Catalonia, Spain*

E-mails: svasil@academyofathens.gr, sola@ecm.ub.edu,

14 April 2016

ABSTRACT

Past analyses of the equation of state (EoS) of the Dark Energy (DE) were not incompatible with a phantom phase near our time. This has been the case in the years of WMAP observations, in combination with the remaining cosmological observables. Such situation did not completely disappear from the data collected from the *Planck* satellite mission. In it the EoS analysis may still be interpreted as suggesting $\omega_{\rm D} \lesssim -1$, and so a mildly evolving DE cannot be discarded. In our opinion the usual ansatzs made on the structure of the EoS for dynamical DE models (e.g. quintessence and the like) are too simplified. In this work we examine in detail some of these issues and suggest that a general class of models with a dynamical vacuum energy density could explain the persistent phantom anomaly, despite there is no trace of real phantom behavior in them. The spurious or "mirage" effect is caused by an attempt to describe them as if the DE would be caused by fundamental phantom scalar fields. Remarkably, the effective DE behavior can also appear as quintessence in transit to phantom, or vice versa.

Keywords: cosmology: theory, large-scale structure of the universe

1 INTRODUCTION

Dark energy is one of the most mysterious components of our universe (Huterer & Turner, 1999). Despite very little is known on its ultimate nature, compelling evidence exists from independent data sets derived from the luminosity-redshift relation of distant supernovae, the anisotropies of the CMB, the BAO scale produced in the last scattering surface by the competition between the pressure of the coupled baryon-photon fluid and gravity, and the matter power spectrum obtained from the large scale structures (LSS) of the Universe. All of them speak up convincingly for the existence of such mysterious and overwhelmingly abundant entity (cf. Riess et al. 2004; Knop et al. 2004; Spergel et al. 2007; Blaket et al. 2011; Amanullah et al. 2010; Komatsu et al. 2011; and references therein). The recently released analysis of the *Planck* satellite data on the anisotropies of the CMB reinforce the previous observations and confirm that the DE component is an indispensable ingredient of the cosmological puzzle (Ade et al. 2013). Let us however note that CMB data alone are not critically sensitive to DE unless we combine them with other data from large scale structure or cosmic distance measurements. The existence of DE, first directly observed by supernova measurements (Riess et al, 1998; Perlmutter et al, 1999), is required (Spergel et al., 2003) by the combination of CMB power spectrum measurements and any one of the following low redshift observations: measurements of the Hubble constant, measurements of the galaxy power spectrum, galaxy cluster abundances, or supernova measurements of the redshift-distance relation. So the main constraints on expansion history of the universe and reconstructed equation of state of DE is not coming from just CMB data. This is due to the "geometric degeneracy" which prevents both the curvature and expansion rate from being determined simultaneously by the CMB alone (Bond, Efstathiou & Tegmark, 1997; Zaldarriaga, Spergel, & Seljak, 1997). However, inclusion of CMB lensing power spectrum data, which probe structure formation and geometry long after decoupling breaks the CMB geometric degeneracy (B. D. Sherwin et al, 2011; Hazra, Shafieloo & Souradeep, 2013).

Popular proposals for the DE are, among others, quintessence and phantom energy in its various forms, modified gravity etc. (cf. Peebles & Ratra 2003; Padmanabhan 2003; Lima 2004). Interestingly enough, the *Planck* observations (Ade et al. 2013) suggest an effective equation of state (EoS) for the DE centered in the phantom domain $\omega_{\rm D} \lesssim -1$, specifically in the range

$\omega_{\rm D} = -1.13^{+0.13}_{-0.10}$. This situation is not new, as it had also been reckoned by the long series of WMAP observations (Spergel et al. 2007; Komatsu et al. 2011) and indeed it has triggered many works in the literature (see e.g. Alam, Sahni & Starobinsky 2004; Jassal, Bagla & Padmanabhan 2005 and 2010; Feng, Wang & Zhang 2005; Caldwell & Doran 2005; Solà & Štefančić 2005 and 2006).

The observed persistence of this result in the hot data from the *Planck* mission (Ade et al. 2013) is still not compelling evidence of a phantom phase, but depending on the combination of this data with other sources yields a result that is in tension with the $\omega_{\rm D} = -1$ standard expectation at more than 2σ within the phantom domain. This could be interpreted again as a symptom that the DE might have a mild dynamical behavior, as we do not expect a constant EoS result $\omega_{\rm D} < -1$, which would be difficult to expalin. Such dynamics should be very desirable since the idea of a rigid cosmological constant (CC) or vacuum energy is very difficult to reconcile with a possible solution of the cosmological constant problems plaguing theoretical cosmology (Weinberg 1989; Padmanabhan 2003). A constant CC term throughout the entire history of the universe presents strong conceptual difficulties from the point of view of fundamental physics. The apparent lack of evidence of the dynamical component of the DE in the observations could be attributed, in our view, to the fact that it has been tested using exceedingly simple parametrizations which might be blind to this kind of effects. We cannot exclude that the effects are too small, of course, but what is most encouraging is that from the theoretical point of view we expect a dynamical behavior of the vacuum energy in quantum field theory (QFT) in curved spacetime whose evolution should be accessible to future phenomenological observations (for reviews see e.g. Solà 2011, 2013). We will show some examples in this paper.

There are a number of attempts in the literature proposing a dynamical vacuum energy in various ways (cf. e.g. Ozer & Taha 1986; Peccei, Solà & Wetterich 1987; Freese et al. 1987; Peebles & Ratra 1988; Wetterich 1988; Carvalho, Lima & Waga 1992, see also the review by Overduin & Cooperstock, 1998 and references therein for the old literature). However, a deeper theoretical approach can be obtained from the point of view of the so-called running vacuum energy in QFT, where the CC is expected to show some dynamical evolution with the expansion history of the universe (Shapiro & Solà 2002, 2003, 2004; Babić *et al.* 2002; Solà 2007; Shapiro & Solà 2009). This idea has been tested successfully in recent studies using the available data on expansion and structure formation (cf. Basilakos, Plionis & Solà 2009; Grande et al. 2011; Basilakos, Polarski & Solà 2012). Such framework could also help to shed some light on the cosmic coincidence problem (Grande, Solà & Štefančić, 2006, 2007; Grande, Pelinson & Solà, 2008). Furthermore, very recently a class of models of this sort has been employed to describe the full cosmological history from inflation to the present time (cf. Lima, Basilakos & Solà 2012; Perico et al. 2013, Basilakos, Lima & Solà 2013).

In this paper we wish to dwell on a wide class of dynamical models of the vacuum energy inspired in QFT in curved spacetime in which the effective EoS appears indeed as evolving with the expansion of the universe, i.e. it is seen as a slow function of the redshift, $\omega_{\rm D} = \omega_{\rm D}(z)$. We will show that $\omega_{\rm D}(z)$ can approach very close to -1 near our time (even take this exact value at present), both from above and from below. In the first case the dynamical vacuum model mimics quintessence, and in the second phantom energy. The theoretical vacuum model, however, does not hinge on the existence of fundamental quintessence or phantom fields. It only assumes the QFT running of ρ_Λ and G caused by the matter effects in QFT in curved spacetime. Such "mirage" behavior of the EoS is caused by our modeling of the dynamical vacuum energy as if it were a dynamical scalar field. This kind of feature could give some clue as to the interpretation of the phantom phase that persistently appeared in the analysis of the WMAP and stays in the recent *Planck* data.

The structure of the paper is as follows. In section 2 we discuss the cosmologies with running ρ_Λ and/or G. In section 3, we introduce the general form of effective equation of state for the dark energy in these cosmologies. In section 4 we compute in detail the effective equation of state for specific models of this kind. The corresponding numerical analysis is presented in section 5. The final discussion and main conclusions are summarized in section 6.

2 EXPANSION DYNAMICS WITH RUNNING PARAMETERS

When modeling the expanding universe as a perfect fluid with matter-radiation density ρ_m, and corresponding pressure $p_m = \omega_m \rho_m$[⋆], the energy momentum tensor of matter reads $T_{\mu\nu} = -p_m\, g_{\mu\nu} + (\rho_m + p_m) U_\mu U_\nu$. It is well known that the cosmological constant term $\Lambda\, g_{\mu\nu}$ on the *l.h.s.* of Einstein's equations can be absorbed on the *r.h.s.* with a modified energy-momentum tensor $\tilde{T}_{\mu\nu} \equiv T_{\mu\nu} + g_{\mu\nu}\, \rho_\Lambda$, in which $\rho_\Lambda = \Lambda/(8\pi G)$ is the vacuum energy density associated to the presence of Λ, and the corresponding pressure is $p_\Lambda = -\rho_\Lambda$. The field equations then read formally the same way:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, \tilde{T}_{\mu\nu}\,, \tag{1}$$

in which the full energy momentum tensor including the effect of the vacuum energy density takes also the same form: $\tilde{T}_{\mu\nu} = -p_{\rm tot}\, g_{\mu\nu} + (\rho_{\rm tot} + p_{\rm tot}) U_\mu U_\nu$, with $\rho_{\rm tot} = \rho_m + \rho_\Lambda$ and $p_{\rm tot} = p_m + p_\Lambda = p_m - \rho_\Lambda$, or explicitly,

$$\tilde{T}_{\mu\nu} = (\rho_\Lambda - p_m)\, g_{\mu\nu} + (\rho_m + p_m) U_\mu U_\nu\,. \tag{2}$$

⋆ Here $\omega_m = 0$ for non-relativistic matter, and $\omega_m = 1/3$ for the radiation component (relativistic matter). The latter is negligible deep in the matter dominated era. We will not consider transient situations that interpolate between these two clearly differentiated epochs.

In the flat FLRW metric, the two independent gravitational field equations are the following:

$$8\pi G(\rho_m + \rho_\Lambda) = 3H^2 \,, \tag{3}$$

$$8\pi G(\omega_m \rho_m - \rho_\Lambda) = -2\dot{H} - 3H^2 \,, \tag{4}$$

where the overdot denotes derivative with respect to cosmic time t.

Although a rigid CC term is the simplest possibility, it is remarkable that the Cosmological Principle embodied in the FLRW metric still permits the vacuum energy to be a function of the cosmic time, $\rho_\Lambda = \rho_\Lambda(t)$ or of any collection of homogeneous and isotropic dynamical variables $\chi_i = \chi_i(t)$, i.e. $\rho_\Lambda = \rho_\Lambda(\chi_i(t))$. The same applies to the gravitational coupling, which can also be a function of these dynamical variables: $G = G(\chi_i(t))$. In the following, although we will usually write $\rho_\Lambda = \rho_\Lambda(t)$ and $G = G(t)$ when these parameters evolve with time, we shall indeed understand that they may depend on one or more fundamental dynamical variable (for specific models, cf. Sect. 4).

Let us consider now the different conservation laws that emerge from a FLRW-like cosmology with running parameters (cf. Solà 2011, 2013 and references therein). Despite the possible time evolution of the vacuum energy, the corresponding EoS can still maintain the usual form $p_\Lambda(t) = -\rho_\Lambda(t)$. The Bianchi identities, in their turn, imply that the covariant derivative of the full *r.h.s.* of Einstein's equations (1) is zero, namely $\nabla^\mu \left(G\tilde{T}_{\mu\nu} \right) = 0$, where G is also involved since we admit it could be variable. With the help of the FLRW metric, it is easy to see that the generalized conservation law can be cast as

$$\frac{d}{dt}\left[G(\rho_m + \rho_\Lambda)\right] + 3\,G\,H\,(1+\omega_m)\rho_m = 0\,. \tag{5}$$

The concordance model, or ΛCDM model (Peebles 1984), appears as a particular case of that relation in which both ρ_Λ and G are constant. In this case, the conservation law boils down to the standard one, $\dot{\rho}_m + 3\,H\,(1+\omega_m)\rho_m = 0$, whose solution in terms of the scale factor is well-known:

$$\rho_m(a) = \rho_m^0\, a^{-3(1+\omega_m)}\,. \tag{6}$$

However, the identity (5) allows for interesting possible generalizations of the local conservation laws, that lead to the following three types of generalized models:

Type i) Λ_tCDM model: $\Lambda = \Lambda(t)$ variable, and G =const. In this case, Eq. (5) gives

$$\dot{\rho}_m + 3(1+\omega_m)H\rho_m = -\dot{\rho_\Lambda}\,. \tag{7}$$

The assumption $\dot{\rho_\Lambda} \neq 0$ necessarily requires some energy exchange between matter and vacuum, e.g. through vacuum decay into matter, or vice versa.

Type ii) $\Lambda_t G_t$CDM model: $\Lambda = \Lambda(t)$ variable, and $G = G(t)$ also variable, assuming matter conservation (6). In the present instance the generalized conservation law amounts to

$$(\rho_m + \rho_\Lambda)\dot{G} + G\dot{\rho_\Lambda} = 0\,. \tag{8}$$

Here the time evolution of the vacuum energy density is possible at the expense of a running gravitational coupling. This is what permits local covariant matter conservation in this case.

Type iii) ΛG_tCDM model: Λ =const. and $G = G(t)$ variable. The generalized conservation law (5) renders

$$\dot{G}(\rho_m + \rho_\Lambda) + G[\dot{\rho}_m + 3H(1+\omega_m)\rho_m] = 0\,. \tag{9}$$

Here matter is not conserved, and the gravitational coupling is again running. Despite the vacuum energy here is constant, this situation can also produce an effective or "mirage" dynamical dark energy effect, as we shall see and analyze in detail. For this reason we encompass this model also as a dynamical DE model.

The independent variables of our cosmological analysis can be taken as ρ_m, ρ_Λ and H. Needless to say none of the previous models can be explicitly solved until (based on some theoretical motivation) a dynamical law is given for one of the remaining cosmological variables, e.g. the function $\rho_\Lambda = \rho_\Lambda(\chi_i(t))$ – in the case of the Λ_tCDM and $\Lambda_t G_t$CDM models – or, in the alternative ΛG_tCDM model, a matter density ρ_m that deviates from the standard conservation law (6). Assuming that ω_m is known, the remaining two variables in the triad $(\rho_m, \rho_\Lambda, H)$ can then be solved using either the two Friedmann's equations (3)-(4), or one of them together with the corresponding generalized conservation law (7), (8) or (9). With this information, and following the aforementioned procedure, the corresponding Hubble function and energy densities can be obtained for any of the models i), ii) and iii) mentioned above (cf. Sect. 4 for the details).

We emphasize that the above three generalized models can stay sufficiently close to the standard ΛCDM model provided the possible time variation of ρ_Λ and/or G is sufficiently mild, and of course within bounds. Finally, let us mention that the vacuum can run at fixed G with local covariant conservation of matter. In this situation, however, the DE must be composite, i.e. there must be another DE component apart from Λ– see e.g. the so-called ΛXCDM framework (Grande, Solà & Štefančić 2006).

Because of the proximity of the above running models with the concordance model, it is natural to ask what is the effective equation of state of the dark energy for them, as perceived from an analysis in which the cosmological parameters Λ and G are assumed strictly constant, in particular $\Lambda = 0$, and where the DE is attributed to some smoothly evolving, self-conserved, dynamical entity, such as for example a dynamical scalar field. In the next section we provide an answer to this question.

3 EFFECTIVE EQUATION OF STATE FOR GENERAL RUNNING MODELS

Let us consider a generalized vacuum framework in flat space with normalized Hubble function given as follows:

$$E^2(z) \equiv \frac{H^2(z)}{H_0^2} = \frac{8\pi\,G(z)}{3H_0^2}\left[\rho_m(z) + \rho_\Lambda(z)\right]\,, \tag{10}$$

in which we have explicitly accounted for a possible cosmic evolution of both G and ρ_Λ as a function of the

redshift z. It is convenient to parametrize the above expression in the following way:

$$E^2(z) = \Omega_m^0\, f_m(z;r_i)(1+z)^{\alpha_m} + \Omega_\Lambda^0\, f_\Lambda(z;r_i)\,, \qquad (11)$$

with $\alpha_m \equiv 3\,(1+\omega_m)$, and we have defined the current cosmological parameters $\Omega_m^0 = \rho_m^0/\rho_c^0$ and $\Omega_\Lambda^0 = \rho_\Lambda^0/\rho_c^0$ (superscript 0 meaning at redshift $z=0$, i.e. now). Any of the model types introduced in the previous section can eventually be brought to the form (11). The two functions $f_m(z;r_i)$ and $f_\Lambda(z;r_i)$ of the redshift are assumed to be known in the given vacuum model (or at least calculable), and may depend on some free parameters $r_i = r_1, r_2, ..$ (Solà & Štefančić 2005 and 2006). Their explicit form will depend on the possible evolution of the vacuum energy density ρ_Λ and/or of the gravitational coupling G as functions of z, as well as on a possible anomalous law for the matter energy density (specific examples will be given in the next section). For the standard ΛCDM, these functions take, of course, the trivial values: $f_m = f_\Lambda = 1$. Most important, whatever it be their form in a given generalized vacuum model, they must satisfy $f_m(0;r_i) = f_\Lambda(0;r_i) = 1$ identically, in order to comply with the cosmic sum rule $\Omega_m^0 + \Omega_\Lambda^0 = 1$.

Let us now compute the effective $\omega_{\rm D}$ for the generalized vacuum model introduced above. We proceed as though we would not know that the original Hubble function is the one given by Eq. (10) and we assume that it evolves according to the typical expansion rate of the universe with the DE furnished by a scalar field or some self-conserved entity with negative pressure:

$$H_{\rm D}^2(z) = H_0^2\, \left[\tilde{\Omega}_m^0\,(1+z)^{\alpha_m} + \tilde{\Omega}_{\rm D}^0\,\zeta(z)\right]\,, \qquad (12)$$

where

$$\zeta(z) \equiv \exp\left\{3\int_0^z dz'\,\frac{1+\omega_{\rm D}(z')}{1+z'}\right\}\,. \qquad (13)$$

In Eq.(12) we may assume $\tilde{\Omega}_m^0 = 0.314 \pm 0.02$ provided by the recent *Planck* results (Ade et al. 2013). The dark energy density in this picture is given by $\rho_{\rm D}(z) = \rho_{\rm D}^0\,\zeta(z)$. If $p_{\rm D}$ denotes the negative pressure associated to the DE, we must assume (as usual) that $\omega_{\rm D} = \rho_{\rm D}/p_{\rm D} < -1/3$, in order to grant an accelerated expansion. The subindex "D" in the Hubble function (12) serves to denote the "DE picture" description, in contrast to the original dynamical vacuum energy model, or "CC picture" (10). The tildes in the cosmological parameters in Eq. (12) indicate their values in the new picture. Notice that we also have the corresponding cosmic sum rule $\tilde{\Omega}_m^0 + \tilde{\Omega}_{\rm D}^0 = 1$ in the DE picture, since $\zeta(0) = 1$. However, the parameters in the two pictures need not be identical; in particular, the value of

$$\Delta\Omega_m^0 \equiv \Omega_m^0 - \tilde{\Omega}_m^0\,, \qquad (14)$$

even if it is naturally expected small ($|\Delta\Omega_m^0|/\Omega_m^0 \ll 1$), can play a role in our analysis.

The next point is to implement an important matching condition between the two expansion histories, namely we require the equality of the expansion rates of the original dynamical CC picture (10) and that of the DE picture (12): $H(z) = H_{\rm D}(z)$ ($\forall z$). First of all we note from equations (12) and (13) that $\omega_{\rm D}(z) = -1 + (1/3)\,[(1+z)/\zeta]\,d\zeta/dz$. From here a straightforward calculation leads us to a first operative formula to compute the effective EoS:

$$\omega_{\rm D}(z) = -1 + \frac{1}{3}\,\alpha_m\,(1+z)^{\alpha_m}\;\epsilon(z)\,, \qquad (15)$$

where

$$\epsilon(z) = \frac{\left[\alpha_m\,(1+z)^{\alpha_m-1}\right]^{-1}\,dE(z)/dz - \tilde{\Omega}_m^0}{E^2(z) - \tilde{\Omega}_m^0\,(1+z)^{\alpha_m}}\,. \qquad (16)$$

It is important to remark that $E(z)$ in the EoS formula must be computed from (10), in which the functions $f_m(z;r_i)$ and $f_\Lambda(z;r_i)$ are assumed to be known from the structure of the given dynamical vacuum model.

It is immediately checked that for the special case of the ΛCDM model ($f_m = f_\Lambda = 1$) and for $\Omega_m^0 = \tilde{\Omega}_m^0$, one has $\epsilon(z) = 0$ and Eq. (15) reduces to $\omega_{\rm D} = -1$, as it should. Therefore, any departure from this result will be a clear sign that the background cosmology cannot be one with Λ =const. In particular, if the resulting effective EoS evolves with the expansion, $\omega_{\rm D} = \omega_{\rm D}(z)$, it will be a sign of a dynamical vacuum. As we realize now, however – and this is an important remark underlying all this work – the traces of DE dynamics need not be necessarily attributed to a scalar field with some specific potential. We will identify later on distinctive features that can emerge between the dynamics triggered by a running vacuum model and that of a purely scalar field model. But before unraveling these differences, let us come back to the above effective EoS formula and show that it can be brought into a reduced form that is much more convenient for a physical interpretation.

It turns out that thanks to the constraint imposed by the general Bianchi identity (5) the EoS formula can be expressed in a more compact fashion, directly in terms of the functions $f_m(z)$ and $f_\Lambda(z)$ of the generalized vacuum model. To see this, let us first trade the cosmic time variable for the redshift variable in Eq. (5), which is easy done using $d/dt = -(1+z)\,H(z)\,d/dz$, and we find

$$(1+z)\frac{d}{dz}\,\left[G(z)(\rho_m(z)+\rho_\Lambda(z))\right] = \alpha_m\,G(z)\,\rho_m(z)\,. \qquad (17)$$

From this equation and Eq. (10) we arrive at the following expression:

$$\frac{d}{dz}\,\left[\Omega_m^0\,f_m(z;r_i)(1+z)^{\alpha_m} + \Omega_\Lambda^0\,f_\Lambda(z;r_i)\right] = \alpha_m\,\Omega_m^0\,f_m(z;r_i)(1+z)^{\alpha_m-1} \qquad (18)$$

Working out this expression, we find:

$$\Omega_m^0\,f_m'(z;r_i)(1+z)^{\alpha_m} + \Omega_\Lambda^0\,f_\Lambda'(z;r_i) = 0\,, \qquad (19)$$

where the primes denote derivatives with respect to the redshift variable. The differential relation (19) between the functions $f_m(z)$ and $f_\Lambda(z)$ is a reflection of the Bianchi identity, and it plays a key role to simplify the structure of the effective equation of state (15). Indeed, computing dE/dz from (10), and using Eq. (19), the distinctive dynamical part of the effective EoS (15) can be cast as

$$\epsilon(z) = \frac{\Omega_m^0 f_m(z;r_i) - \tilde{\Omega}_m^0}{E^2(z) - \tilde{\Omega}_m^0\,(1+z)^{\alpha_m}} \qquad (20)$$

$$= \frac{\Omega_m^0 f_m(z;r_i) - \tilde{\Omega}_m^0}{\left(\Omega_m^0\, f_m(z;r_i) - \tilde{\Omega}_m^0\right)\,(1+z)^{\alpha_m} + \Omega_\Lambda^0\, f_\Lambda(z;r_i)}$$

In this simpler form it becomes transparent that $\omega_{\rm D}(z)$ reduces to $\omega_{\rm D} = -1$ for the ΛCDM case ($f_m = f_\Lambda = 1$) provided the parameter difference (14) is zero.

Equation (20) leads to an interesting observation. As the function f_m must satisfy $f_m(0;r_i) = 1$, and the parameter difference (14) between the two pictures should be small, it follows that for any generalized CC model in which $f_m(z;r_i)$ is a monotonous function of z there will be a point $z = z^*$ near our present ($z = 0$) where $\epsilon(z^*) = 0$, equivalently $\omega_{\rm D}(z^*) = -1$, and hence the effective EoS function (15) will change from $\omega_{\rm D}(z) > -1$ ("effective quintessence") to $\omega_{\rm D}(z) < -1$ ("effective phantom") around that point, or vice versa. For instance, assume that $\Delta\Omega_m^0 < 0$ and that the function $f_m(z;r_i)$ is monotonously increasing with z (therefore decreasing with the expansion). It means that well in our past $\epsilon(z) > 0$, equivalently, $\omega_{\rm D}(z) > -1$, so the running vacuum model behaves there as quintessence. Let us now approach the present time. The condition $f_m(z;r_i) \to 1$ for $z \to 0$ is satisfied in the manner $f_m(z;r_i) \gtrsim 1$, and hence there is a point z^* in our recent past where the effective EoS entered the phantom regime: $\epsilon(0 \lesssim z \lesssim z^*) < 0$. Here we assume that the denominator of (20) stays positive near our present, thanks to the second term in it, which tends to $\Omega_\Lambda^0 f_\Lambda(0;r_i) = \Omega_\Lambda^0 > 0$ – in effect larger than the first term, which is of order $\Delta\Omega_m^0$ near $z = 0$. Similarly, if $\Delta\Omega_m^0 > 0$, the phantom regime can only be attained after the present time, namely when the condition $f_m(z;r_i) \lesssim 1$ is fulfilled (to the necessary degree) at some $z \lesssim 0$ (in our future). Should, instead, the function $f_m(z;r_i)$ be monotonically decreasing with z (i.e. increasing with the expansion), then for $\Delta\Omega_m^0 < 0$ there could be a crossover from phantom into quintessence in the future, whereas for $\Delta\Omega_m^0 > 0$ the same kind of crossover could occur in our recent past. This is true only if the denominator of (20) stays positive. If it changes to negative sign in the past but stays positive near our present, then for $\Delta\Omega_m^0 < 0$ we can actually have the opposite situation, i.e. a transition from quintessence to phantom near our time. This situation will actually occur in one of our specific examples (cf. Sect. 5 for details).

Needless to say, without giving further details of the parameter values and of the structure of the generalized vacuum model (10) – above all the explicit form of the functions $f_m(z;r_i)$ and $f_\Lambda(z;r_i)$ – it is not possible to firmly conclude if the aforementioned crossing will be in our recent past or in our immediate future. In the next section we shall illustrate various possibilities by studying a rather general class of running vacuum models, in which the functions f_m and f_Λ are given, or can be computed.

4 SPECIFIC RUNNING VACUUM ENERGY MODELS

We have seen that in the wide class of generalized vacuum models (10) we should expect the existence of at least a redshift point $z = z^*$ representing a crossover of the CC divide by the effective EoS. However, the crossing is not realized by the presence of fundamental phantom fields in combination with quintessence fields, as amply discussed within very different points of view in the past (confer e.g. Feng, Wang & Zhang 2005, Vikman 2005; Caldwell & Doran 2005, see also the review by Copeland, Sami & Tsujikawa 2006 and references therein). The effective DE behavior of the EoS is caused, in our case, by the (mild) running of the vacuum energy density ρ_Λ and/or the participation of a (slightly) anomalous conservation law for matter, sometimes even in combination with the (slow) running of the gravitational coupling G. Such possibility should be welcome since it shows that the crossover of the CC divide $\omega_{\rm D} = -1$ need not generally be associated to any QFT anomaly, but to the effective description of what could be a perfectly normal QFT behavior. To illustrate this fact in concrete terms, in the present section we shall compute the effective EoS in the context of the three model types introduced in Sect 2, all of them being particular cases of the general structure (10).

4.1 Running vacuum Λ_tCDM models

Let us start with the type i) model of Sect. 2, i.e. the class Λ_tCDM. We exemplify it with the running vacuum models whose energy density evolves with the expansion rate in the following way:

$$\rho_\Lambda = n_0 + n_2 H^2 + n_{\dot{H}}\, \dot{H}\,. \tag{21}$$

Notice that the constant additive term n_0 should be the dominant one, in order that the vacuum energy remains approximately constant near our present and also during relatively long cosmological intervals of time. In other words, we expect $n_0 \sim \rho_\Lambda^0$. We will be more precise later on. However, the additional H-dependent terms endow the vacuum energy with a dynamical behavior. From the two Friedmann's equations (3) and (4) it is easy to show that

$$\frac{H^2}{\dot{H}} = -\frac{2}{\alpha_m}\,(1+r) = -\frac{2}{3}\,\frac{1+r}{1+\omega_m}\,, \tag{22}$$

where $r = \rho_\Lambda/\rho_m$ is the ratio between vacuum energy density and matter energy density. This ratio is of $\mathcal{O}(1)$ at present ($r \sim 7/3$), whereas in the past $r \to 0$. Therefore, for the relevant epoch H^2 and $\dot{H}$ are dynamical terms of the same order of magnitude (H_0^2 is roughly twice $|\dot{H}_0|$). It is only in the remote future that $r \to \infty$ – except for the ΛXCDM model, where r stays always bounded (Grande, Solà & Štefančić 2006). As the dynamical behavior should obviously be mild, we expect that these terms play a sub-leading role, but at the same time we consider the possibility that they can have some measurable effect. The coefficients of H^2 and $\dot{H}$ can be parameterized more conveniently in the form

$$n_2 = \frac{3\nu}{8\pi}\, M_P^2 \qquad n_{\dot{H}} = \frac{\alpha}{4\pi}\, M_P^2\,, \tag{23}$$

M_P being the Planck mass, and where we have traded the dimensionful parameters n_2 and $n_{\dot{H}}$ by the dimensionless ones ν and α. The theoretical motivation for the structure (21)-(23) is based on the renormalization group running. This is also the reason why we have omitted the linear terms in H (Shapiro & Solà 2002; Solà 2007; Shapiro & Solà 2009). Should the dimensionless parameters ν and α be of order one, the vacuum dynamics would be too pronounced and we would have detected it, so we expect that

$$|\nu| \ll 1\,, \quad |\alpha| \ll 1\,. \tag{24}$$

This theoretical expectation is substantiated by particular QFT calculations (Solà 2007), where ν is found to satisfy $|\nu| = \mathcal{O}(10^{-3})$ at most. On the other hand, we can also use Eq. (21) as a phenomenological ansatz and compare the model with the data so as to extract the maximum allowed values for these parameters. This has been done in (Basilakos, Polarski & Solà, 2012), extending the analysis of (Basilakos et al. 2009; Grande et al. 2010, 2011; Fabris, Shapiro & Solà, 2007). These works use the data on type Ia supernovae (SNIa), the Baryonic Acoustic Oscillations (BAOs), the shift parameter of the Cosmic Microwave Background (CMB) and the power matter spectrum. The results suggest that both $|\nu|$ and $|\alpha|$ can be at most of order 10^{-3}, in very good agreement with the expectation (24). A bound of this order, even if it is relatively tight, could still enable detection of a mildly time evolving vacuum energy. It is also interesting that, even with such small values of the parameters, these dynamical vacuum models could be able to explain the hints presumably detected in experiments devoted to find evidence of the possible variation of the so-called fundamental "constants" of nature (cf. Fritzsch & Solà 2012).

The model can be explicitly solved following the procedure outlined in Sect. 2. Essentially, one has to solve Eq. (7) using the explicit form (21) and the reparameterization (23). The computation is straightforward, and the emerging matter and vacuum energy densities read as follows:

$$\rho_m(z) = \rho_m^0\ (1+z)^{3\xi} \tag{25}$$

and

$$\rho_\Lambda(z) = \rho_\Lambda^0 + \rho_m^0\ (\xi^{-1}-1)\left[(1+z)^{3\xi}-1\right]\,, \tag{26}$$

with

$$\xi = \frac{1-\nu}{1-\alpha}\,. \tag{27}$$

In the above formula, ρ_m^0 is the cold matter density at present. We have omitted the radiation part, ρ_r^0, because it is not relevant for the study of the EoS behavior near our time. Its inclusion is nevertheless essential to fit the CMB data. We omit these details, but for the sake of completeness we quote the normalized Hubble rate when both the relativistic and nonrelativistic matter components are included:

$$E^2(z) = \frac{\Omega_m^0}{\xi}\ (1+z)^{3\xi} + \frac{\Omega_r^0}{\xi'}\ (1+z)^{4\xi'} + \frac{\Omega_\Lambda^0 - \Delta\nu}{1-\nu}\,, \tag{28}$$

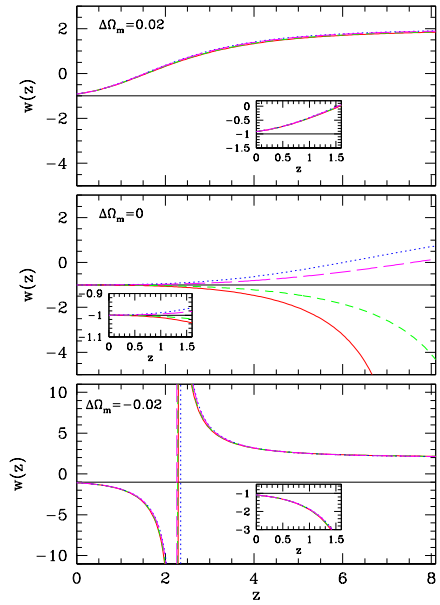

Figure 1. *Outer Plots:* The Λ_tCDM evolution of the effective EoS parameter for various $\Delta\Omega_m^0$ values. We assume $\alpha = 3\nu/4$ (see Sect. 5 for details). To produce the lines we used $\nu = -0.002$ (blue/dotted), $\nu = -0.001$ (magenta/long dashed), $\nu = 0.001$ (green/dashed) and $\nu = 0.002$ (red/solid). *Inner Plots:* detail of the evolution of $w_D(z)$ in the shorter range $0 \leqslant z \lesssim 1.5$ relevant for SNIa observations.

where $\Delta\nu = \nu - \bar{\nu}$, with $\bar{\nu} = \alpha\,\Omega_m^0 + (4/3)\,\alpha\,\Omega_r^0$. Here

$$\xi' = \frac{1-\nu}{1-\frac{4}{3}\alpha}\,. \tag{29}$$

The current normalized contributions from both relativistic and nonrelativistic matter, Ω_m^0 and Ω_r^0, and vacuum energy Ω_Λ^0, satisfy $\Omega_m^0 + \Omega_r^0 + \Omega_\Lambda^0 = 1$. The term Ω_r^0 can be omitted from that sum rule when the radiation component is neglected. But the full expression (28) is indispensable in order to determine the model parameter values from the CMB data following the procedure of Basilakos et al. (2012). In this sense, the fit is actually sensitive to two parameters (ξ, ξ'), or alternatively (ν, α). The first set allows a more compact notation, but the second set is more convenient for the numerical analysis because any deviation of ν and/or α from zero indicates a departure from the ΛCDM.

We can indeed check that all the above expressions retrieve their standard forms when ν and α are both vanishing (equivalently, $\xi = \xi' = 1$)[†]; in particular, ρ_Λ in (26) then becomes strictly constant ($\rho_\Lambda = \rho_\Lambda^0$) and the cold matter evolution law (25) recovers its conventional form $\rho_m(z) = \rho_m^0\,(1+z)^3$, i.e. as in (6) for $\omega_m = 0$.

Inserting equations (25) and (26) in Eq. (10), and comparing with Eq. (11) in the nonrelativistic matter epoch ($\omega_m = 0$), we can explicitly identify the functions

[†] Notice that in the absence of the condition $\xi' = 1$ the two parameters ν and α could be large and almost equal, and this option would still be compatible with $\xi = 1$. However, the simultaneous constraint $\xi' = 1$ is what enforces that ν and α must be both small in the physical parameter space.

$f_m(z;r_i)$ and $f_\Lambda(z;r_i)$ for the dynamical vacuum model (21). They read

$$f_m(z) = (1+z)^{3(\xi-1)} = (1+z)^{3\frac{\alpha-\nu}{1-\alpha}} \quad (30)$$

$$f_\Lambda(z) = 1+\frac{\Omega_m^0}{\Omega_\Lambda^0}\left(\xi^{-1}-1\right)\left[(1+z)^{3\xi}-1\right] = 1+\frac{\Omega_m^0}{\Omega_\Lambda^0}\frac{\nu-\alpha}{1-\nu}\left[(1+z)^{3\frac{1-\nu}{1-\alpha}}-1\right]. \quad (31)$$

As expected, they satisfy $f_m(0) = 1 = f_\Lambda(0)$ at $z = 0$. Notice that that these functions are of the form $f_m(z;\alpha,\nu)$ and $f_\Lambda(z;\alpha,\nu)$, i.e. they depend on two independent parameters α and ν. It is interesting to note that despite they are expected to be small in absolute value, the sign difference $\alpha-\nu>0$ or $\alpha-\nu<0$ determines whether $f_m(z)$ increases or decreases with z, and this feature may determine the effective quintessence or phantom behavior of the model (cf. Sect. 3). From Eq. (19) it is obvious that when $f_m(z;r_i)$ increases, $f_\Lambda(z;r_i)$ must decrease, and vice versa. We can check that this is indeed the case for the specific functions of the model under consideration. Finally, substituting the expressions (30) and (31) in Eq. (20) and rearranging, we find

$$\epsilon(z) = \frac{\xi\left[\Omega_m^0(1+z)^{3(\xi-1)}-\tilde{\Omega}_m^0\right]}{\Omega_m^0\left[(1+z)^{3\xi}-1\right]-\xi\left[\tilde{\Omega}_m^0(1+z)^3-1\right]}, \quad (32)$$

where we recall that ξ can be written in terms of (ν,α) through Eq. (27). We note immediately that $\epsilon(z)=0$ $(\forall z)$ if $\nu=\alpha=0$ and if the parameter difference (14) vanishes, as expected. Moreover, if $\Delta\Omega_m^0=0$, we have $\epsilon(0)=0$, equivalently $\omega_{\rm D}(0)=-1$, for any value of ν and α. This does not apply, of course, for $z\neq 0$. It is instructive to expand $\epsilon(z)$ linearly in the small parameters ν and α, assuming that z is not very large (i.e. for points relatively close to our current universe) and with the natural assumption $|\Delta\Omega_m^0/\Omega_m^0|\ll 1$. After substituting the result in Eq. (15), we arrive at the following approximate effective EoS

$$\omega_{\rm D}(z) \simeq -1+\frac{\Omega_m^0}{\Omega_\Lambda^0}(1+z)^3\left[\frac{\Delta\Omega_m^0}{\Omega_m^0}+3(\alpha-\nu)\ln(1+z)\right]. \quad (33)$$

This equation, even though only approximate, reveals the essential qualitative facts of the effective EoS for the present model. For example, we confirm that we can recover the ΛCDM limit, $\omega_{\rm D}(z)=-1$, for $\nu=\alpha=0$ and $\Delta\Omega_m^0=0$, as it should be; and that $\omega_{\rm D}(0)=-1$ irrespective of ν and α. Moreover, for $\Delta\Omega_m^0=0$ and nonvanishing α and ν, the effective EoS mimics quintessence ($\omega_{\rm D}\gtrsim -1$) for $\alpha-\nu>0$, and phantom DE ($\omega_{\rm D}\lesssim -1$) for $\alpha-\nu<0$. If, however, there is a significant relative deviation between the parameters Ω_m^0 and $\tilde{\Omega}_m^0$, the situation could change, depending on the size and sign of the term $\Delta\Omega_m^0/\Omega_m^0$. It is easy to check that with the current values of the cosmological parameters and their precisions we can easily get $\omega_{\rm D}=-1\pm 0.1$ for supernovae data at $z\simeq 1$ (cf. Fig. 1 and Sect.5 for the detailed numerical analysis). Therefore, both the quintessence and phantom regime can be accounted for in an effective way by the dynamical class of Λ_tCDM models, without invoking fundamental scalar or phantom fields as responsible for the DE. Interestingly, the effective EoS of the dynamical vacuum model does not, though, adapt to the simple parameterizations of the DE in vogue in the literature, which cannot describe this kind of scenarios. We shall come back to this issue in Sect.4.4. Let us also note that the kind of effective EoS plots obtained here (and the impact on them from the assumptions of matter density) are also observed in generic approaches where one reconstructs the equation of state of dark energy (see e.g. Sahni, Shafieloo, & Starobinsky, 2008).

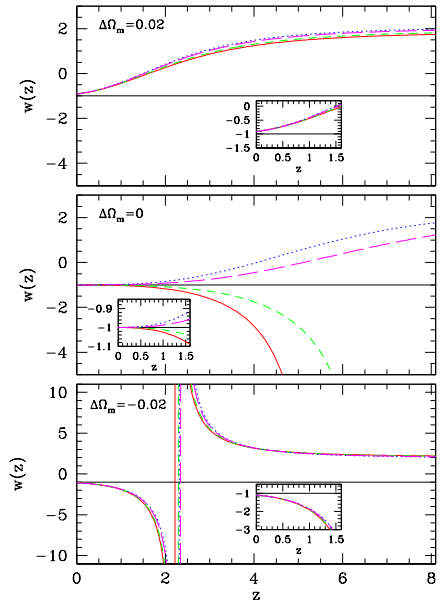

Figure 2. The $\Lambda_t G_t$CDM evolution of the effective EoS parameter. For more details concerning the curves see caption of Fig. 1

4.2 $\Lambda_t G_t$CDM models: running Λ and G

We discuss now, more briefly, the type ii) model introduced in Sect. 2, i.e. the $\Lambda_t G_t$CDM model. In this case, matter is covariantly conserved and the Bianchi identity can be fulfilled through a dynamical interplay between a running vacuum energy and a running gravitational coupling, see Eq. (8). Unfortunately, in general in this class of models it is not possible to provide a simple analytical solution of the cosmological equations, especially if we start again with a vacuum dynamical law of the full form as indicated in Eq. (21), i.e. containing both H^2 and $\dot{H}$ terms. We have, however, seen from the previous section that the two dynamical terms, H^2 and $\dot{H}$, play a similar role in the effective EoS. We will assume that this is also the case here. Moreover, as we discussed in Eq. (22), H^2 and $\dot{H}$ are of the same order of magnitude at present and in the past. Therefore it should suffice to focus here on the model type (21) under the assumption that $n_{\dot{H}}=0$ – equivalently, $\alpha=0$ (as in Shapiro, Solà & Štefančić, 2005; Grande et al. 2010; Grande et al. 2011).

Then we are left with a single parameter, n_2 (or ν) and this simplifies the analysis and the confrontation of the model with the data. Such assumption enables also to obtain a relatively simple analytical expression for the effective EoS of this model, as we shall see below.

For the $\Lambda_t G_t$CDM model, the functions f_m and f_Λ involved in the expansion rate (11) can be written as follows:

$$f_m(z) = \frac{G(z)}{G_0} \equiv g(z)\,, \qquad f_\Lambda(z) = f_m(z)\,\frac{\Omega_\Lambda(z)}{\Omega_\Lambda^0}\,, \quad (34)$$

where $G_0 \equiv 1/M_P^2$ is the current value of the gravitational coupling, and $\Omega_\Lambda(z) = \rho_\Lambda(z)/\rho_c^0$ is the vacuum energy density normalized to the current critical density. Clearly, $f_m(0) = 1 = f_\Lambda(0)$ is fulfilled as a basic normalization condition that these functions were supposed to satisfy. The function $g(z)$ is the dimensionless gravitational coupling at any cosmic epoch, normalized to the current Newton's constant. Solving Eq. (8) in combination with (3) it can be exactly determined as a function of the normalized Hubble rate $E(z) = H(z)/H_0$:

$$g(E) = \frac{1}{1+\nu\,\ln E^2}\,. \quad (35)$$

The logarithmic behavior clearly shows that G varies very slowly with the cosmic evolution. If we focus on the nonrelativistic epoch, where the EoS is measured, the corresponding solution in terms of the redshift can be obtained in the form of an implicit equation:

$$\frac{1}{g(z)} - 1 + \nu\,\ln\left[\frac{1}{g(z)} - \nu\right] = \nu\,\ln\left[\Omega_m(z) + \Omega_\Lambda^0 - \nu\right]\,. \quad (36)$$

Similarly an expression for $f_\Lambda(z)$, which hinges on the previous one, ensues:

$$f_\Lambda(z) = \frac{g(z)}{1-\nu\,g(z)}\left\{1 + \frac{\nu}{\Omega_\Lambda^0}\,\left[\Omega_m(z)\,g(z) - 1\right]\right\}\,. \quad (37)$$

Here $\Omega_m(z) = \rho_m(z)/\rho_c^0 = \Omega_m^0\,(1+z)^3$ is the matter energy density normalized to the current critical density. Notice from (36) and (37) that we get $g = 1$ and $f = 1$ identically for $\nu = 0$. Moreover, for any ν we can easily check the correct normalizations $f_m(0) = g(0) = 1$ and $f_\Lambda(0) = 1$ in these explicit formulas, after recalling that $\Omega_m^0 + \Omega_\Lambda^0 = 1$.

Finally, we can insert the above expressions in the general effective EoS equation (15),(20) so as to obtain the desired result for the $\Lambda_t G_t$CDM model:

$$\epsilon(z) = \frac{(1-\nu g(z))\left(\Omega_m^0 g(z) - \tilde{\Omega}_m^0\right)}{\left[\Omega_m^0 g(z) - \tilde{\Omega}_m^0 + \nu g(z)\tilde{\Omega}_m^0\right](1+z)^3 + g(z)\,(\Omega_\Lambda^0 - \nu)}\,. \quad (38)$$

Note that in order to evaluate this expression exactly one has to obtain first $g(z)$ by numerically solving the implicit equation (36). It may, however, be illuminating to expand equations (36) and (38) linearly in the small parameter ν, assuming that z is not very large (i.e., once more for points relatively close to our current universe) and for $|\Delta\Omega_m^0/\Omega_m^0| \ll 1$. The final result for the effective EoS, in this approximation, is the following:

$$\omega_{\rm D}(z) \simeq -1 + \frac{\Omega_m^0}{\Omega_\Lambda^0}(1+z)^3\left[\frac{\Delta\Omega_m^0}{\Omega_m^0} - \nu\,\ln\left[\Omega_m(z) + \Omega_\Lambda^0\right]\right]\,. \quad (39)$$

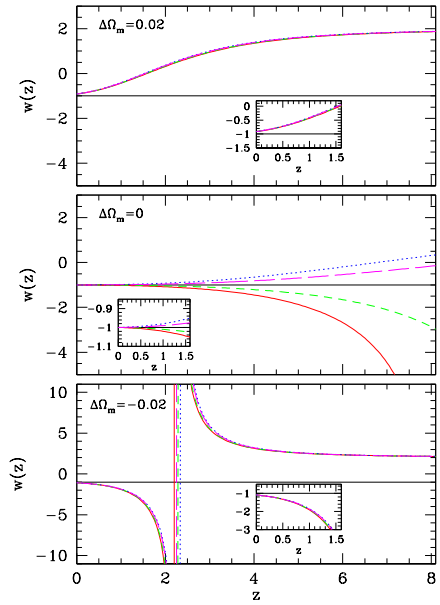

Figure 3. The ΛG_tCDM evolution of the effective EoS parameter. Remaining details as in Fig. 1.

Once more, using the present data, we can easily recreate scenarios leading to $\omega_{\rm D} = -1 \pm 0.1$, thus mimicking both the quintessence and phantom regimes in a purely artificial way, namely without calling upon fundamental quintessence or phantom fields (cf. Fig. 2 and Sect.5 for the detailed numerical analysis).

4.3 Class ΛG_tCDM: running G with anomalous matter conservation law

Our final example is the type iii) class of models of Sect. 2, or ΛG_tCDM models. Here the original vacuum term is not evolving with the expansion, but the effective description of this scenario leads to a case of virtually dynamical DE. The effect is caused by the fact that matter is not conserved in this kind of model, and this can be covariantly consistent provided there is a small running of the gravitational coupling. As a result, two anomalous functions $f_m \neq 1$ and $f_\Lambda \neq 1$ will be generated in Eq. (10), implying a nontrivial evolution of the effective DE. To illustrate the present class, we take a model of this sort that was discussed in (Fritzsch & Solà, 2012) within the framework of dynamical vacuum energy, and in (Guberina, Horvat & Nikolic, 2006) in the holographic context. The model assumes an anomalous evolution law for matter, of the form

$$\rho_m(z) = \rho_m^0\,\,(1+z)^{3(1-\eta)}\,, \quad (40)$$

which is similar to Eq.(25), but now with no accompanying variation of the vacuum energy.

The coefficient η that appears in Eq.(25) has a different interpretation to that of ν for the Λ_tCDM and $\Lambda_t G_t$CDM models of sections 4.1 and 4.2 since the CC remains constant here. Even so, for convenience we adopt the same notation $\nu \equiv \eta$. The anomalous law (40)

for matter conservation was first considered in (Shapiro & Solà, 2003) and then also in (Wang & Meng, 2005) and (Alcaniz & Lima, 2005).

Next we compute the effective EoS of this model. The corresponding function f_m reads as follows:

$$f_m(z) = f_\Lambda(z)\,(1+z)^{-3\nu}\,, \tag{41}$$

where $f_\Lambda(z)$ in this formula is given by

$$f_\Lambda(z) = \frac{G(z)}{G_0} = \left[\Omega_m^0\,(1+z)^{3(1-\nu)} + \Omega_\Lambda^0\right]^{\nu/(1-\nu)}\,. \tag{42}$$

The explicit form for $G(z)$ in the last expression follows from solving the differential equation (9) in the matter epoch. With the help of (20) and the previous expressions the effective EoS of this model becomes completely determined. Its dynamical part reads

$$\epsilon(z) = \frac{\Omega_m^0 f_\Lambda(z)\,(1+z)^{-3\nu} - \tilde{\Omega}_m^0}{\left[\Omega_m^0 f_\Lambda(z)\,(1+z)^{-3\nu} - \tilde{\Omega}_m^0\right]\,(1+z)^3 + \Omega_\Lambda^0\, f_\Lambda(z)}\,. \tag{43}$$

Expanding this formula linearly in the small parameter ν for not very high z and $|\Delta\Omega_m^0/\Omega_m^0| \ll 1$, the effective EoS can be approximated as follows:

$$\omega_{\rm D}(z) = -1 + \frac{\Omega_m^0}{\Omega_\Lambda^0}\,(1+z)^3\left\{\frac{\Delta\Omega_m^0}{\Omega_m^0} + \nu\,\ln\left[\Omega_m^0 + \frac{\Omega_\Lambda^0}{(1+z)^3}\right]\right\}\,. \tag{44}$$

For a numerical example, see Fig. 3. In Sect.5 we present the details of the numerical analysis.

4.4 Some usual parameterizations of the effective EoS

We may now briefly compare the various forms for the effective EoS obtained in the previous sections with the most usual ones employed in the literature and in the practical analysis of the cosmological data, say from WMAP (Komatsu et al. 2011) and *Planck* (Ade et al. 2013).

Popular parameterizations have been proposed and tested in the past in the literature. They intended to be simple and as useful as possible for a large class of models, but they have their own limitations. As an example, consider the very simple form

$$\omega_{\rm D}(z) = \omega_0 + \omega_1 z\,. \tag{45}$$

This form is conceivable only for recent data since its linear behavior with the redshift cannot be extended to the early universe. A parametrization that overcomes this difficulty is the CPL parametrization (Chevallier & Polarski 2001; Linder 2003):

$$\omega_{\rm D}(a) = \omega_0 + \omega_0'(1-a)\,, \tag{46}$$

where a is the scale factor. Finally, let us quote (see Jassal et al. 2005; 2010)

$$\omega_{\rm D}(z) = \omega_0 + \omega_0'\,\frac{z}{(1+z)^p}\,, \quad (p = 1, 2, ...)\,, \tag{47}$$

where $\omega_0' = (d\omega_{\rm D}/dz)_{z=0}$ is assumed to be significantly smaller than ω_0 (in absolute value), i.e. $|\omega_0'/\omega_0| \ll 1$, as ω_0' controls the dynamical part of the DE. One naturally expects that ω_0 is close to -1, i.e. $\omega_0 = -1+\delta$, with $|\delta| \ll 1$. It is clear that (47) is a generalization of (46) since for $p = 1$ they coincide. Notice that the asymptotic behavior of $\omega_{\rm D}(z)$ in this kind of parametrizations is smooth: for $p = 1$ one has $\omega_{\rm D}(+\infty) = \omega_0 + \omega_0'$, and for $p \geq 2$, $\omega_{\rm D}(+\infty) = \omega_0$. Let us consider the corresponding Hubble rate. For example, for $p = 1$ one obtains from equations (12)-(13) the following result:

$$\begin{aligned} E^2(z) &= \tilde{\Omega}_m^0\,(1+z)^3 \\ &+ \tilde{\Omega}_{\rm D}^0\,(1+z)^{3(1+\omega_0+\omega_0')}\,\exp\left[-3\,\omega_0'\,\frac{z}{1+z}\right]\,. \end{aligned} \tag{48}$$

Even though the parameter family of EoS (47) is quite general and useful for many models, the kind of running vacuum models under consideration cannot be described by it. The expansion history traced by the Hubble function (48) is still too simple to cover the behavior of the dynamical vacuum models discussed in the previous sections. Both at small and at high redshift redshift the DE part of (48) basically behaves as a power $\sim (1+z)^{3(1+\omega_0+\omega_0')}$, with just a small correction from the exponential since $|\omega_0'/\omega_0| \ll 1$. In other words, it roughly takes on the form $\sim (1+z)^{3(\delta+\omega_0')}$. This expression can effectively behave as a CC term (as suggested by observations), provided $|\delta|$ and $|\omega_0'|$ are small, as indeed assumed. This is at least the logic followed by the usual parameterizations of the DE. For example, for $p = 1$ Eq. (47) exhibits the approximate behavior

$$\omega_{\rm D}(z) \simeq -1 + \delta + \omega_0'\,\frac{z}{1+z}\,. \tag{49}$$

The latter describes quintessence for $\delta + \omega_0'\,z/(1+z) > 0$ and phantom energy for $\delta + \omega_0'\,z/(1+z) < 0$. While this is of course only a parametrization, at the fundamental level one usually attributes this behavior to some dynamical scalar field, which is supposed to underlie the EOS (47). In fact, it is well-known (cf. Peebles & Ratra 2003) that a dynamical scalar field yields a contribution to the expansion rate of the form given in Eq. (12). In point of fact, within the quintessence-like scenarios one assumes that the DE behavior is fully generated in that way (i.e. neglecting the existence of the CC term *ab initio*).

However, as we have seen in the various examples provided in the previous section, we can actually reproduce all kinds of quintessence or phantom-like behaviors from a dynamical model in which the vacuum density and/or the gravitational constant are running. In such framework, which is conceptually quite different from the one usually adopted in the literature, the fundamental scalar fields are not primarily responsible for the DE, and in particular there is no need of fundamental phantom fields (i.e. fields with negative kinetic term). For example, the dynamical vacuum models that could be ultimately responsible for the generic behavior (21) might emerge from the expected quantum effects of the effective action of QFT in curved spacetime (Solà, 2013). Specifically, in an expanding universe these quantum effects should endow the vacuum energy density of a time evolution inherited from additional (even) powers of the Hubble rate (Shapiro & Solà 2002 and 2003; Solà

2007; Shapiro & Solà 2009). In this kind of framework the scalar (and fermion) fields can just enter through their virtual quantum effects that renormalize the vacuum energy density, all of them with standard kinetic terms.

Furthermore, it is pretty obvious that the EoS behaviors inferred for the various dynamical vacuum models considered in the previous sections, cannot be accommodated into the relatively simple form (49) or to any of the initial expressions (45), (46) or (47). This is particularly transparent when we compare the approximate forms (33), (39) and (44) with (46) or (49). But these are nevertheless the ones used e.g. for the WMAP and *Planck* analysis of the cosmological parameters.

5 NUMERICAL ANALYSIS OF THE EFFECTIVE EQUATION OF STATE

In the following we show the evolution of the effective EoS parameter for the three vacuum models considered in the text, Λ_tCDM (Fig. 1), $\Lambda_t G_t$CDM (Fig. 2) and ΛG_tCDM (Fig. 3). Note that we sample $\Delta\Omega_m^0 \in [-0.02, 0.02]$ in steps of 0.02 that corresponds to the 1σ *Planck* error (Ade et al. 2013). We remind the reader that for the DE models we use $\tilde{\Omega}_m^0 = 0.314$ (Ade et al. 2013) which implies that the corresponding Ω_m^0 of the vacuum models lies in the interval $[0.294, 0.334]$. Owing to the fact that $|\nu|$ is found to be rather small when the vacuum models are confronted to the cosmological data, $|\nu| \leq \mathcal{O}(10^{-3})$ (Basilakos et al. 2009; Grande et al. 2011; Basilakos et al. 2012), we decide to use four different values of ν around $\nu = 0$ – the latter providing the ΛCDM result. As an example, the lines in the figures correspond to $\nu = -0.002$ (blue/dotted), $\nu = -0.001$ (magenta/ long dashed), $\nu = 0.001$ (green/dashed) and $\nu = 0.002$ (red/solid).

Generally, as it can be seen for $\Delta\Omega_m^0 \neq 0$ (see upper and lower panels in Figs. 1, 2 and 3) the effective EoS parameter $w_D(z)$ shows almost the same behavior for all three vacuum models. This is not the case for the Λ_tCDM model with $\Delta\Omega_m^0 = 0$. In particular the main comparison results can be summarized in the following statements (for nomenclature of models, see sections 2 and 4):

• **Λ_tCDM model** As we said, this model has, in principle, two independent parameters, ν and α. However, one can actually show that BBN bounds imply $|\nu - 4\alpha/3| < \mathcal{O}\left(10^{-3}\right)$ (Solà, 2012). The simplest possibility is to take $\alpha = 3\nu/4$, and this is what we will do for most of the numerical analysis, except for an special case considered at the end of this section. Notice that $\alpha = 3\nu/4$ is tantamount to take a strictly standard behavior for the radiation density, $\rho_r \sim a^{-4}$ (as assumed in Basilakos, Polarski & Solà, 2012).

We find the following situations (see Fig. 1):

a) For $\Delta\Omega_m^0 = 0.02$, the effective EoS parameter remains always $w_D(z) > -1$ as well as it tends to $w_D(0) \simeq -0.9$ at the present time (see the detail in the magnified part of Fig. 1 in the interval $0 \leqslant z \lesssim 1.5$). It is therefore a quintessence-like behavior in that region. We also find that for $z \geq 1.5$ the EoS parameter is positive, which means that the cosmic expansion in this case is more rapidly decelerated than in the usual ΛCDM cosmological model. As a result the acceleration process is a bit retarded as compared to the standard model. Notice that in the far future the EoS parameter tends to -1, as seen from Eq. (15) and (32);

b) In the special situation where $\Delta\Omega_m^0$ is strictly equal to 0, the effective EoS remains always either phantom (for $\nu > 0$) or quintessence (for $\nu < 0$) for $z \geq 0$. In all these cases the current value is exactly $w_D(0) = -1$. It interesting to mention that for $\nu > 0$ we find a transition from phantom into quintessence in the far future ($z \simeq -0.5$);

c) In the case $\Delta\Omega_m^0 = -0.02$, the evolution of the EoS parameter is in the phantom regime with $w_D(0) \simeq -1.09$, i.e. slightly below -1 for $\nu < 0$. Before reaching $z = 0.5$ it already takes the value $w_D \lesssim -1.2$. These are precisely the kind of phantom behaviors we usually encounter in the analyses of WMAP (Spergel et al. 2007; Komatsu et al. 2011) and *Planck* (Ade et al. 2013) data. On the other hand, close to $z \simeq 2$ one can see a kind of divergent feature due to the fact that the denominator of Eq.(32) vanishes at this point. We would like to stress that there is no real singularity at this point because the assumed fundamental RG model behaves smoothly for all values of redshifts, Eq.(10), i.e. all densities are perfectly finite at this point. It is only the effective EoS description of the original RG model that points to this fake singularity, which is nothing but an artifact of the parametrization of the running vacuum model as if it were a DE model with expansion rate (12) [‡]. From the observational point of view, it is interesting to mention that if we would identify a sort of anomaly like the above when comparing with the the future $w(z)$ data form the the next generation of surveys (based on *Euclid* satellite, Cimatti *et al.*, Laureijs *et al.* 2011), then we could suspect that there is no fundamental dynamical field behind the EoS but something else, in particular the RG model under consideration. Finally, at very large redshifts $z \gg 10$, we find $w_D(z) \to 0$. Once more we find $w_D(z \to -1) \to -1$. Next we describe briefly the results found for the other two models, the details are in Figs. 2 and 3.

• **$\Lambda_t G_t$CDM model** (cf. Fig. 2): in this case we observe that the behavior of the $\Lambda_t G_t$CDM effective EoS parameter is similar to that of Λ_tCDM [for comparison see cases (a), (b) and (c) in Fig.1]. Once more, for $\Delta\Omega_m^0 = 0$, the model behaves as phantom (for $\nu < 0$), and as quintessence (for $\nu > 0$). The evolution into these regimes with increasing z is actually faster than in the Λ_tCDM case studied before.

• **ΛG_tCDM model** (see Fig. 3): this case behaves qualitatively more similarly to the Λ_tGCDM vacuum model, although the quantitative differences are manifest, especially for $z \geq 1$. Both of these models share

‡ The presence of this kind of fake singularities has also been observed in the effective EoS description of other DE models in the literature, see e.g. Shafieloo *et al.* 2005; Sahni & Shtanov 2005 and 2002, as well as in Solà & Štefančić 2005 and 2006.

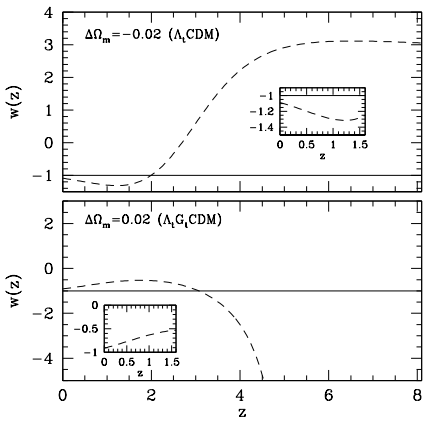

Figure 4. *Upper Panel:*The Λ_tCDM evolution of the effective EoS parameter for $\Delta\Omega_m^0 = \nu = -0.02$. *Lower Panel:* w_D versus z for $\Lambda_t G_t$CDM $\Delta\Omega_m^0 = \nu = 0.02$. *Inner Plots:* The corresponding $w_D(z)$ for $z \lesssim 1.5$.

the non-conservation of matter, which is compensated by the running of G and Λ respectively.

Before closing the numerical analysis of the various models, let us address a couple of special situations. First, assume relatively large values of $|\nu| = \mathcal{O}(10^{-2})$. These are not favored by the cosmological data within the simple class of running models examined here (Basilakos et al. 2009; Grande et al. 2011; Basilakos et al. 2012), but could be accommodated within some generalized versions such as the ΛXCDM models (Grande, Solà & Štefančić, 2006, 2007). Let us analyze this case, as it will serve to illustrate the possibilities at our disposal. We find that the general $w_D(z)$ evolution for the majority of the vacuum models is similar to those of $|\nu| = \mathcal{O}(10^{-3})$ with only two exceptions. Specifically, the first model is the Λ_tCDM with $(\Delta\Omega_m^0, \nu) = (-0.02, -0.02)$ (upper panel of Fig.4). The passing to the $w_D(z) < -1$ phase took place at $z \simeq 2$ where the Λ_tCDM model switched from the quintessence- to the phantom-like EoS. Interestingly, at $z \simeq 0.7$ which is the redshift that the universe enters the accelerated period, the effective EoS takes the value $w_D(0.7) \simeq -1.25$. The current value of the effective EoS parameter behavior is $w_D(0) \simeq -1.09$. The second vacuum model is the $\Lambda_t G_t$CDM with $\Omega_m^0 = \nu = 0.02$ (lower panel of Fig.4). In this case we meet an alternative crossover from phantom into quintessence at $z \simeq 3$, sustained until the present epoch, where $w_D(0) \simeq -0.9$. The effective quintessence phase therefore spans the entire interval where low and high redshift supernovae have been measured.

The second special case we would like to consider is when we keep the Λ_tCDM parameters ν and α within the strictly allowed range for these models, namely $|\nu, \alpha| = \mathcal{O}(10^{-3})$, but we assume that the future accuracy of the measurements has reached $|\Delta\Omega_m^0| = 0.005$ (rather than 0.02 as it is right now). Notice that we can relax here the assumption $\alpha = 3\nu/4$ provided $|\nu|$ and $|\alpha|$ are both small as indicated above. Two specific examples of this situation are depicted in Fig. 5. The interesting feature here is that we have an effective transition from quintessence into phantom regime near our time, namely at around $z = 1.5$ (the larger is $\alpha > 0$ the closer is the transition to $z = 0$), and therefore accessible to future DE surveys such as the WFIRST project (Green *et al.* 2011; D. Spergel *et al.* 2013). A similar dynamical situation occurs for the $\Lambda_t G_t$CDM model, although in this case the transition from quintessence to phantom is significantly earlier in cosmic time (cf. lower panel of Fig.5). On the other hand, if $\Delta\Omega_m^0 = 0.005$ then the corresponding effective EoS parameter of the different vacuum models is approximated by the upper panels of Figs.1-3.

Finally, we would like to stress that in order to investigate whether the expansion of the observed universe follows one of the above possibilities, we need a robust extragalactic equation of state indicator at redshifts $z \geq 1$. Such high quality $w(z)$ data are expected to be available from the future surveys like the aforementioned WFIRST project and the *Euclid* mission (Cimatti *et al.*, Laureijs *et al.* 2011).

6 DISCUSSION AND CONCLUSIONS

In this work we have shown that there is a wide class of models with variable vacuum energy density ρ_Λ and/or gravitational coupling G that can mimic the behavior of quintessence and phantom fields. These dynamical vacuum models generally lead to a non-trivial effective equation of state (EoS) which cannot be described by the usual parameterizations. The usual observational procedures to pin down the empirical form of the EoS for the DE in the current generation of precision cosmology experiments should keep in mind this possibility. While the precise dynamics behind ρ_Λ is not known, various works in the literature suggest that a fundamental ρ_Λ can display a running in QFT in curved spacetime which can be translated into redshift evolution. This evolution endows the effective EoS of a behavior which can help explaining the observation of possible "mirage transitions" from quintessence to phantom DE, or vice versa, without relying at all on the existence of fundamental quintessence or phantom scalars fields.

In this work we presented some theoretical possibilities that can fit the cosmological data pretty well but that are not covered by popular EoS dark energy parametric forms such as the well known CPL one ((Chevallier & Polarski 2001; Linder 2003)). This is important since we do not know what is the actual model of DE and using an inappropriate parametric form of it to fit the data can become misleading. We have also shown that assuming different values of matter density can substantially affect the form of the effective $w(z)$. While the concept of cosmographic degeneracy has been discussed for a long time in the literature, where it is shown how assumptions of matter density or curvature can affect the reconstructed equation of state of dark energy (Clarkson, Cortes, & Bassett, 2007; Sahni, Shafieloo, & Starobinsky, 2008; Shafieloo & Linder, 2011). still it is important to highlight this issue that working with the

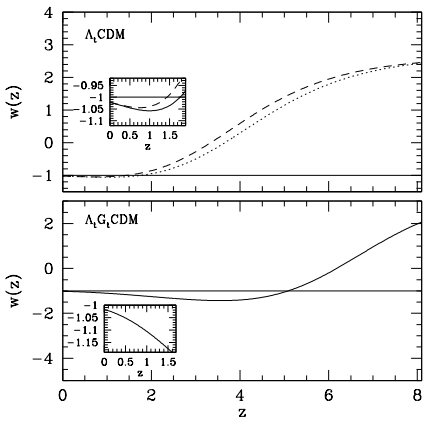

Figure 5. *Upper Panel:* The Λ_tCDM evolution of the effective EoS parameter. We assume an improved future accuracy $\Delta\Omega_m^0 = -0.005$ and we consider two cases: $\alpha = -\nu = 3 \times 10^{-3}$ (dashed line) and $\alpha = -3\nu/4$, with $\nu = -3 \times 10^{-3}$ (dotted line). *Lower Panel:* w_D versus z for the $\Lambda_t G_t$CDM model under the same parameter inputs as with the previous model. In this case we have set $\alpha = 0$ and we consider only the line for $\nu = -3 \times 10^{-3}$. *Inner Plots:* The corresponding $w_D(z)$ for a more focused redshift interval near out time.

EoS of DE can be very much tricky having broad parametric degeneracy and dealing with unknown DE.

We have illustrated these features analyzing various types of dynamical vacuum and/or evolving gravitational coupling G models. These results are timely in view of the recent features on the DE uncovered in the *Planck* mission, which reinforce the possibility (persistently recorded in the data releases by WMAP) of a phantom phase near our time. As it is highly unlikely that this phase, if real at all, remained constant across the cosmic evolution, we should suppose that it is a temporal one, and in this sense it points to a possible dynamical evolution of the DE. However, and most important, even if this phase would fade away from the next generation of high precision cosmological experiments, the analysis presented in this work shows that the usual parameterizations of the DE may be too simpleminded to encompass the more realistic possibilities offered by QFT, and in this sense we believe that it is unjustified to conclude, on the basis of these parameterizations, that there is no evidence of a dynamical vacuum energy. The observation of a fairly stable vacuum at present (i.e. the successfulness of the ΛCDM) should not lead us to conclude that this is the most natural expectation within the context of fundamental physics. As a matter of fact, nothing should be more natural in quantum field/string theory in an expanding universe that a vacuum which is time evolving (hence redshifting) with the cosmic expansion (Solà 2011, 2013). A mild evolution at present would suggest a more rapid evolution in the past, and this can be very useful to trace the entire cosmic history in a framework where inflation, matter and dark energy epochs can be encompassed in a unified framework (see Lima et al. 2013). We suspect that these are the kind of scenarios that should be able to eventually explain the cosmological constant problem in its various faces.

Let us finish by mentioning the fact that the recent observation, at the CERN Large Hadron Collider, of a Higgs-like boson particle (Aad *et. al*, 2012; Chatrchyan *et. al*, 2012) reminded us that the vacuum energy (through the spontaneous symmetry breaking mechanism) can be a fully tangible concept in real phenomenology. Despite the usual difficulties associated with the value of the vacuum energy in QFT, the palpable reality of the Higgs mechanism cannot be easily denied anymore, and this means that we have to keep thinking on how to solve the CC problem associated to the vacuum energy, not just blindly replacing it by other concepts. Let us note that the difficulties with the CC problem are no less severe than those associated to any ersatz entity trying to substitute the CC. What is important is that the CC could be, after all, a cosmic time evolving quantity. This is perfectly allowed by the Cosmological Principle, and it could help in better dealing with the old CC problem and the cosmic coincidence problem. In the meanwhile the effects associated to changes in the vacuum energy can be as real as they are in, say, the Casimir effect, which is sensitive not to the total vacuum energy density but to changes in the vibrational modes of the vacuum configuration. The possible analogy (Solà, 2013) suggests that in the cosmological case the changes in the vacuum energy density are described in terms of mildly evolving functions of the expansion rate, specifically in the form of powers of H^2 and $\dot{H}$.

To summarize, in this work we have emphasized on the following two important issues. First, there are some theoretical models that while they do not have any theoretical ghost, their effective equation of state can have a phantom like behavior and such models can fit the data pretty well comparable with cosmological constant. Second, parametric degeneracies (or, as it called also, cosmographic degeneracies) along with unknown nature of dark energy makes it very tricky to work with parametric forms of the equation of state of dark energy fitting cosmological data.

We have shown that these effects could potentially be detected in future observations through the dynamical features encoded in the effective equation of state of the dark energy. We have also emphasized that we should stay open minded to this possibility. Although the usual parameterizations of the EoS existing in the literature might not be able to capture these effects at present, we expect that future high precision observations and the use of more general parameterizations should help in eventually unraveling the dynamical nature of the cosmic vacuum energy.

ACKNOWLEDGMENTS

We thank V. Sahni for interesting comments on our work and for pointing out some relevant references. SB acknowledges support by the Research Center for Astronomy of the Academy of Athens in the context of the program *"Tracing the Cosmic Acceleration"*. JS has been supported in part by projects FPA2010-20807 and CPAN (Consolider CSD2007-00042). Also by

2009SGR502 Generalitat de Catalunya. SB thanks the Dept. ECM of the Univ. de Barcelona for the hospitality when part of this work was being done.

REFERENCES

Aad G. *et al.* (ATLAS Collab.), Phys. Lett. B, 2012, **716**, 1 [arXiv:1207.7214]

Ade P.A.R. *et al.*, Planck Collab. *Cosmological parameters* [arXiv:1303.5076]

Alam U., Sahni V., & Starobinsky A.A., JCAP, 2004, 0406 008; Alam U., Sahni V., Saini T. D., & Starobinsky A. A., Mon. Not.Roy. Astron. Soc., 2004, 354, 275

Alcaniz J. S., & Lima J. A. S., Phys. Rev. D., 2005, **72**, 063516

Amanullah R. et al., ApJ, 2010, **716**, 712

Babić A., Guberina B., Horvat R. & Štefančić H., Phys. Rev. D, 2002, **65**, 085002.

Basilakos S., Lima, J.A.S & Solà J., Int. J. of Mod. Phys. D (2013).(Work awarded Honorable Mention by the Gravity Research Foundation, 2013) [arXiv:1307.6251]

Basilakos S., Plionis M., & Solà J., Phys. Rev. D., 2009 **80**, 3511 [arXiv:0907.4555]

Basilakos S., Polarski D., & Solà J., Phys. Rev. D., 2012 **86**, 043010

Blake C. et al., Mon. Not.Roy. Astron. Soc., 2011, **418**, 1707

Bond J. R., Efstathiou G., and Tegmark M., 1997, MNRAS, **291**, L33

Caldwell R. R., & Doran M, Phys. Rev. D., 2005, **72**, 043527

Carvalho J. C., Lima J. A. S., & Waga I., Phys. Rev. D., 1992, **46**, 2404

Chatrchyan S. *et al.* (CMS Collab.), Phys. Lett. B, 2012, **716**, 30 [arXiv:1207.7235]

Chevallier M, & Polarski D., Int. J. Mod. Phys. D, 2001, **10**, 213

Cimatti A. *et al.*, *Euclid Assessment Study Report for the ESA Cosmic Visions* [arXiv:0912.0914]

Clarkson C., Cortes M., Bassett B. A., 2007, JCAP, **0708**, 011

Copeland E.J., Sami M. & Tsujikawa S., Int. J. of Mod. Phys, 2006, **15**, 1753

Fabris J. C., Shapiro I. L., Solà J. , JCAP, 2007, 0702, **016** [arXiv:gr-qc/0609017].

Feng B., Wang X. L., & Zhang X. M., Phys. Lett. B, 2005, **607**, 35

Freese K., Adams F.C., Frieman J.A., & Mottola E., Nucl. Phys. 1987, **287**, 797

Fritzsch H., & Solà J., Class. Quant. Grav., 2012, **29**, 215002 [arXiv:1202.5097]

Huterer D. & Turner M.S., Phys. Rev. D, 1999, **60**, 081301 [arXiv:astro-ph/9808133]

Grande J., Solà J., Basilakos S., & Plionis M., JCAP, 2011 **08**, 007 [arXiv:1103.4632]

Grande J., Pelinson A., & Solà J, Phys. Rev. D, 2009, **79**, 043006 [arXiv:0809.3462]; Proc. of the 7th Int. Heidelberg Conf., ed. H.V. Klapdor-Kleingrothaus and I. V. Krivosheina (World Scientific, Singapore, 2010) p.380 [arXiv:0904.3293].

Grande J., Solà J., Fabris J.C., & Shapiro I.L., 2010, Class. Quant. Grav., 2010, **27**, 105004 [arXiv:1001.0259]

Grande J., Solà J., & Štefančić H, JCAP, 2006, **0608**, 011 [arXiv:: gr-qc/0604057];

Grande J., Solà J., & Štefančić, Phys. Lett. B, 2007, **645**, 236 [arXiv:gr-qc/060908]

Hazra D. K., Shafieloo, A. & Souradeep T. , 2013, Phys. Rev. **D 87**, 123528

Green J, *et al.*, *Wide-Field InfraRed Survey Telescope (WFIRST) Interim Report* [arXiv:1108.1374]

Guberina B., Horvat R. & Nikolic H., Phys. Lett. B, 2006, **636**, 80

H.K. Jassal, J.S. Bagla, & T. Padmanabhan, Mon. Not.Roy. Astron. Soc., 2005, **356**, L11; Phys.Rev. D, 2005, **72**, 103503.

H.K. Jassal, J.S. Bagla, T. Padmanabhan, Mon. Not.Roy. Astron. Soc., 2010, **405**, 2639

Knop R., Astrophys. J. , 2004, **598**, 102

Komatsu E. et al., Astrophys. J. Suplem., 2011, **192**, 18

Lima J.A.S., Basilakos S. & Solà J., Mon. Not.Roy. Astron. Soc., 2013, **43**, 923 [arXiv:1209.2802]
5597

Lima J. A. S., Braz. Journ. Phys., 2004, **34**, 194, (astro-ph/0402109)

Linder E., Phys. Rev. Lett., 2003, **90**, 091301

Laureijs R. *et al.*,*Euclid Definition Study Report* [arXiv:1110.3193]

Maia J. M. F., & Lima J. A. S., Phys. Rev. D., 2002, **65**, 083513

Montenegro Jr., & Carneiro S., Class. Quant. Grav., 2007, **24**, 313

Overduin J. M., & Cooperstock, F. I., Phys. Rev. D., 1998 **58**, 043506

Ozer M., & Taha, O., Phys. Lett. B, 1986 171, 363; Nucl. Phys. B, 1987, **287**, 776.

Peccei R., Solà J. & Wetterich C., Phys. Lett. B, 1987, **195** (1987) 183

Padmanabhan T., Phys. Rept., 2003, **380**, 235

Peebles P. J. E. , Astrophys. J, 1984, **284**, 439.

Peebles P. J. E., & Ratra, B., ApJ, 1988, **325**, L17

Perlmutter S. et al., 1999, ApJ, **517**, 565

Peebles P. J., & Ratra B., Rev. Mod. Phys., 2003, **75**, 559

Perico E. L. D., Lima J.A.S., Basilakos S. & Solà J., 2013 Phys. Rev. 2013, **D88**, 063531 [arXiv:1306.0591].

Riess A. G. et al., AJ, 1998, **116**, 1009

Riess A.G. et al., ApJ, 2004, **607**, 665

Sahni V., Shafieloo A., and Starobinsky A.A., 2008, Phys.Rev. **D78**, 103502

Sahni V., & Shtanov Y., Phys. Rev. D, 2005, **71**, 084018 [arXiv:astro-ph/0410221]

Sahni V., & Shtanov Y., JCAP, 2003, **0311**, 014 [arXiv:astro-ph/0202346]

Shafieloo A., Alam, U., Sahni V., & Starobinsky A.A., Mon. Not. Roy. Astron. Soc., 2006, **366**, 1081 [arXiv:astro-ph/0505329]

Shafieloo A. and Linder E., 2011, Phys.Rev. **D84** 063519

Shapiro, I. L., & Solà, J., Phys. Lett. B., 2000, **475**, 236 [hep-ph/9910462]

Shapiro, I. L., & Solà, J., JHEP, 2002 **02**, 006 [arXiv:hep-th/0012227].

I.L. Shapiro, J. Solà, Nucl. Phys. Proc. Suppl., 2004, **127**, 71 [arXiv:hep-ph/0305279];

Shapiro I. L. , Solà J. España-Bonet C., Ruiz-Lapuente P., Phys. Lett. B, 2003, **574** (149) [arXiv:astro-ph/0303306]; JCAP, 2004, **0402** 006 [arXiv:hep-ph/0311171]

Shapiro, I. L., & Solà, J., Phys. Lett. B., 2009, **682**, 105 [arXiv:0910.4925]

Shapiro, I. L., & Solà, J., 2009, Štefančić H., 2005, JCAP **0501** (2005) 012 [arXiv:hep-ph/0410095].

Sherwin B. D. et al, 2011, Phys. Rev. Lett., **107**, 021302

Solà J., 2012 (unpublished).

Solà J., J. of Phys. A., 2008, **41**, 164066 [arXiv:0710.4151].

Solà J., J. Phys. Conf. Ser., 2011, **283**, 012033 [arXiv:1102.1815].

Solà J., *Cosmological constant and vacuum energy: old*

and new ideas, J.Phys.Conf.Ser. (2013), **453**, 012015 [arXiv:1306.1527]
Solà J., & Štefančić H., Phys. Lett. B. 2005 **624**, 147 [arXiv:astro-ph/0505133]
Solà J., & Štefančić H., Mod. Phys. Lett. A., 2006, **21**, 479 [arXiv:astro-ph/0507110]
Spergel D. N. ApJS, 2003, **148**, 175
Spergel D. N. *et al.*, ApJ, 2007, **170**, 377
Steinhardt J. P., 1997, "Critical Problems in Physics", edited by V. L. Fitch et al. D. R. Marlow & M. A. E. Dementi (Princeton Univ. 1997).
Turner M. S., & Riess A. G., ApJ, 2002, **569**, 18
Vikman A., Phys. Rev. D, 2005, **71**, 023515
Wang P, & Meng X.-H., Class.Quant.Grav., 2005, **22**, 283
Weinberg S., Rev. Mod. Phys., 1989, **61**, 1
Xu L., Wang Y., Tong M., & Noh H., Phys. Rev. D., 2011, **84**, 123004
Zaldarriaga M., Spergel D. N., and Seljak U., 1997, ApJ, **488**, 1