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Nonlinear Spinor Fields in Bianchi type-VI spacetime

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Within the scope of Bianchi type-VI cosmological model we study the role of spinor field in the evolution of the Universe. It is found that due to the spinor affine connections the energy-momentum tensor of the spinor field possesses non-diagonal components. The non-triviality of non-diagonal components of the energy-momentum tensor imposes some severe restrictions either on the spinor field or on the metric functions or on both of them. But unlike in cases of Bianchi type-I or VI_0, in case of Bianchi type-VI model it does not lead to the elimination of spinor field nonlinearity or mass term in the spinor field Lagrangian. It is also found that depending on the sign of self-coupling constant the model can give rise to late time acceleration or generate oscillatory mode of evolution.

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I. INTRODUCTION

With the more and more observational data available from far sky, the need for a change in the standard cosmological paradigm becomes inevitable. Prior to 1998 when we had no idea about the accelerating mode of expansion the available observational data were well fit in the standard Einstein model. But the discovery and further reconfirmation of the existence of the late time accelerated mode of expansion [1, 2] have opened a new window for change. Along with that comes out a number of alternative models of the evolution of the Universe.

The most popular among the models are those which consider the old Einstein theory with a new "matter" as a source field. The models with Λ-term [3–5], quintessence [6–11], Chaplygin gas [12–19] etc. are among the most studied ones, though some other models of dark energy are also proposed. After some remarkable works by different authors [20–34], showing the important role of spinor field in the evolution of the Universe, it has been extensively used to model the dark energy. This success is directly related to its ability to answer some fundamental questions of modern cosmology: (i) Problem of initial singularity and its possible elimination [22–26, 35–38]; (ii) problem of isotropization [24, 25, 27, 36, 39] and (iii) late time acceleration of the Universe [28–30, 32, 35–38, 40, 41]. Moreover recently it was found that the spinor field can also describe the different characteristics of matter from ekpyrotic matter to pahntom matter, as well as Chaplygin gas [42–46].

It should be noticed that in earlier works only the diagonal components of the energy-momentum tensor of the spinor field were taken into account. But recently it was shown that due to its specific behavior in curve spacetime the spinor field can significantly change not only the geometry of spacetime but itself as well. The existence of nontrivial non-diagonal components of the energy-momentum tensor plays a vital role in this matter. In [47, 48] it was shown that depending on the type restriction imposed on the non-diagonal components of the energy-momentum tensor, the initially Bianchi type-I evolve into a LRS Bianchi type-I spacetime or FRW one from the very beginning, whereas the model may describe a general Bianchi type-I spacetime but in that case the spinor field becomes massless and linear. The same thing happens for a Bianchi type-VI_0 spacetime, i.e., the geometry of Bianchi type-VI_0 spacetime does not allow the existence of a massive and/or nonlinear spinor field [49].

Anisotropic Bianchi type VI cosmological models were studied by many authors [50–54]. In this report we study the role of spinor field in the evolution of a Bianchi type VI anisotropic cosmological model.

II. BASIC EQUATION

Let us consider the case when the anisotropic space-time is filled with nonlinear spinor field. The corresponding action can be given by

$$\mathscr{S}(g;\psi,\bar{\psi}) = \int L\sqrt{-g}d\Omega \tag{2.1}$$

with

$$L = L_{\rm g} + L_{\rm sp}. \tag{2.2}$$

Here $L_{\rm g}$ corresponds to the gravitational field

$$L_{\rm g} = \frac{R}{2\kappa}, \tag{2.3}$$

where R is the scalar curvature, $\kappa = 8\pi G$, with G being Einstein's gravitational constant and $L_{\rm sp}$ is the spinor field Lagrangian.

A. Gravitational field

The gravitational field in our case is given by a Bianchi type-VI anisotropic space time:

$$ds^2 = dt^2 - a_1^2 e^{-2mx_3} dx_1^2 - a_2^2 e^{2nx_3} dx_2^2 - a_3^2 dx_3^2, \tag{2.4}$$

with a_1, a_2 and a_3 being the functions of time only and m and n are some arbitrary constants.

The nontrivial Christoffel symbols for (2.4) are

$$\begin{aligned}
&\Gamma^1_{01} = \frac{\dot{a}_1}{a_1}, \quad \Gamma^2_{02} = \frac{\dot{a}_2}{a_2}, \quad \Gamma^3_{03} = \frac{\dot{a}_3}{a_3}, \\
&\Gamma^0_{11} = a_1 \dot{a}_1 e^{-2mx_3}, \quad \Gamma^0_{22} = a_2 \dot{a}_2 e^{2nx_3}, \quad \Gamma^0_{33} = a_3 \dot{a}_3, \\
&\Gamma^1_{31} = -m, \quad \Gamma^2_{32} = n, \quad \Gamma^3_{11} = \frac{m a_1^2}{a_3^2} e^{-2mx_3}, \quad \Gamma^3_{22} = -\frac{n a_2^2}{a_3^2} e^{2nx_3}.
\end{aligned} \tag{2.5}$$

The nonzero components of the Einstein tensor corresponding to the metric (2.4) are

$$G^1_1 = -\frac{\ddot{a}_2}{a_2} - \frac{\ddot{a}_3}{a_3} - \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} + \frac{n^2}{a_3^2}, \tag{2.6a}$$

$$G^2_2 = -\frac{\ddot{a}_3}{a_3} - \frac{\ddot{a}_1}{a_1} - \frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} + \frac{m^2}{a_3^2}, \tag{2.6b}$$

$$G^3_3 = -\frac{\ddot{a}_1}{a_1} - \frac{\ddot{a}_2}{a_2} - \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} - \frac{mn}{a_3^2}, \tag{2.6c}$$

$$G^0_0 = -\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} + \frac{m^2 - mn + n^2}{a_3^2}, \tag{2.6d}$$

$$G^0_3 = (m-n)\frac{\dot{a}_3}{a_3} - m\frac{\dot{a}_1}{a_1} + n\frac{\dot{a}_2}{a_2}. \tag{2.6e}$$

B. Spinor field

For a spinor field ψ, the symmetry between ψ and $\bar{\psi}$ appears to demand that one should choose the symmetrized Lagrangian [55]. Keeping this in mind we choose the spinor field Lagrangian as [24]:

$$L_{\rm sp} = \frac{\imath}{2}\left[\bar{\psi}\gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi}\gamma^\mu \psi\right] - m_{\rm sp}\bar{\psi}\psi - F, \tag{2.7}$$

where the nonlinear term F describes the self-interaction of a spinor field and can be presented as some arbitrary functions of invariants generated from the real bilinear forms of a spinor field. Since ψ and $\psi^\star$ (complex conjugate of ψ) have four component each, one can construct $4\times 4 = 16$ independent bilinear combinations. They are

$$S = \bar{\psi}\psi \qquad \text{(scalar)}, \tag{2.8a}$$

$$P = \imath\bar{\psi}\gamma^5\psi \qquad \text{(pseudoscalar)}, \tag{2.8b}$$

$$v^\mu = (\bar{\psi}\gamma^\mu\psi) \qquad \text{(vector)}, \tag{2.8c}$$

$$A^\mu = (\bar{\psi}\gamma^5\gamma^\mu\psi) \qquad \text{(pseudovector)}, \tag{2.8d}$$

$$Q^{\mu\nu} = (\bar{\psi}\sigma^{\mu\nu}\psi) \qquad \text{(antisymmetric tensor)}, \tag{2.8e}$$

where $\sigma^{\mu\nu} = (\imath/2)[\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu]$. Invariants, corresponding to the bilinear forms, are

$$I = S^2, \tag{2.9a}$$
$$J = P^2, \tag{2.9b}$$
$$I_v = v_\mu v^\mu = (\bar\psi\gamma^\mu\psi)\, g_{\mu\nu}(\bar\psi\gamma^\nu\psi), \tag{2.9c}$$
$$I_A = A_\mu A^\mu = (\bar\psi\gamma^5\gamma^\mu\psi)\, g_{\mu\nu}(\bar\psi\gamma^5\gamma^\nu\psi), \tag{2.9d}$$
$$I_Q = Q_{\mu\nu} Q^{\mu\nu} = (\bar\psi\sigma^{\mu\nu}\psi)\, g_{\mu\alpha} g_{\nu\beta}(\bar\psi\sigma^{\alpha\beta}\psi). \tag{2.9e}$$

According to the Fierz identity, among the five invariants only I and J are independent as all others can be expressed by them: $I_v = -I_A = I + J$ and $I_Q = I - J$. Therefore, we choose the nonlinear term F to be the function of I and J only, i.e., $F = F(I,J)$, thus claiming that it describes the nonlinearity in its most general form. Indeed, without losing generality we can choose $F = F(K)$, with $K = \{I, J, I+J, I-J\}$. Here ∇_μ is the covariant derivative of spinor field:

$$\nabla_\mu\psi = \frac{\partial\psi}{\partial x^\mu} - \Gamma_\mu\psi, \quad \nabla_\mu\bar\psi = \frac{\partial\bar\psi}{\partial x^\mu} + \bar\psi\Gamma_\mu, \tag{2.10}$$

with Γ_μ being the spinor affine connection. In (2.7) γ's are the Dirac matrices in curve space-time and obey the following algebra

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \tag{2.11}$$

and are connected with the flat space-time Dirac matrices $\bar\gamma$ in the following way

$$g_{\mu\nu}(x) = e^a_\mu(x) e^b_\nu(x)\eta_{ab}, \quad \gamma_\mu(x) = e^a_\mu(x)\bar\gamma_a \tag{2.12}$$

where e^a_μ is a set of tetrad 4-vectors.

For the metric (2.4) we choose the tetrad as follows:

$$e^{(0)}_0 = 1, \quad e^{(1)}_1 = a_1 e^{-mx_3}, \quad e^{(2)}_2 = a_2 e^{nx_3}, \quad e^{(3)}_3 = a_3. \tag{2.13}$$

The Dirac matrices $\gamma^\mu(x)$ of Bianchi type-VI space-time are connected with those of Minkowski one as follows:

$$\gamma^0 = \bar\gamma^0, \quad \gamma^1 = \frac{e^{mx_3}}{a_1}\bar\gamma^1, \quad \gamma^2 = \frac{e^{-nx_3}}{a_2}\bar\gamma^2, \quad \gamma^3 = \frac{1}{a_3}\bar\gamma^3$$
$$\gamma^5 = -\imath\sqrt{-g}\gamma^0\gamma^1\gamma^2\gamma^3 = -\imath\bar\gamma^0\bar\gamma^1\bar\gamma^2\bar\gamma^3 = \bar\gamma^5$$

with

$$\bar\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \bar\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = \bar\gamma^5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix},$$

where σ_i are the Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -\imath \\ \imath & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Note that the $\bar\gamma$ and the σ matrices obey the following properties:

$$\bar\gamma^i\bar\gamma^j + \bar\gamma^j\bar\gamma^i = 2\eta^{ij}, \quad i,j = 0,1,2,3$$
$$\bar\gamma^i\bar\gamma^5 + \bar\gamma^5\bar\gamma^i = 0, \quad (\bar\gamma^5)^2 = I, \quad i = 0,1,2,3$$
$$\sigma^j\sigma^k = \delta_{jk} + i\varepsilon_{jkl}\sigma^l, \quad j,k,l = 1,2,3$$

where $\eta_{ij} = \{1, -1, -1, -1\}$ is the diagonal matrix, δ_{jk} is the Kronekar symbol and ε_{jkl} is the totally antisymmetric matrix with $\varepsilon_{123} = +1$.

The spinor affine connection matrices $\Gamma_\mu(x)$ are uniquely determined up to an additive multiple of the unit matrix by the equation

$$\frac{\partial \gamma_\nu}{\partial x^\mu} - \Gamma^\rho_{\nu\mu}\gamma_\rho - \Gamma_\mu \gamma_\nu + \gamma_\nu \Gamma_\mu = 0, \tag{2.14}$$

with the solution

$$\Gamma_\mu = \frac{1}{4}\bar{\gamma}_a \gamma^\nu \partial_\mu e^{(a)}_\nu - \frac{1}{4}\gamma_\rho \gamma^\nu \Gamma^\rho_{\mu\nu}. \tag{2.15}$$

From the Bianchi type-VI metric (2.15) one finds the following expressions for spinor affine connections:

$$\Gamma_0 = 0, \tag{2.16a}$$

$$\Gamma_1 = \frac{1}{2}\left(\dot{a}_1 \bar{\gamma}^1 \bar{\gamma}^0 - m\frac{a_1}{a_3}\bar{\gamma}^1\bar{\gamma}^3\right) e^{-mx_3}, \tag{2.16b}$$

$$\Gamma_2 = \frac{1}{2}\left(\dot{a}_2 \bar{\gamma}^2 \bar{\gamma}^0 + n\frac{a_2}{a_3}\bar{\gamma}^2\bar{\gamma}^3\right) e^{nx_3}, \tag{2.16c}$$

$$\Gamma_3 = \frac{\dot{a}_3}{2}\bar{\gamma}^3\bar{\gamma}^0. \tag{2.16d}$$

C. Field equations

Variation of (2.1) with respect to the metric function $g_{\mu\nu}$ gives the Einstein field equation

$$G^\nu_\mu = R^\nu_\mu - \frac{1}{2}\delta^\nu_\mu R = -\kappa T^\nu_\mu, \tag{2.17}$$

where R^ν_μ and R are the Ricci tensor and Ricci scalar, respectively. Here T^ν_μ is the energy momentum tensor of the spinor field.

Varying (2.7) with respect to $\bar{\psi}(\psi)$ one finds the spinor field equations:

$$\imath\gamma^\mu \nabla_\mu \psi - m_{\rm sp}\psi - \mathscr{D}\psi - \imath\mathscr{G}\gamma^5\psi = 0, \tag{2.18a}$$

$$\imath\nabla_\mu \bar{\psi}\gamma^\mu + m_{\rm sp}\bar{\psi} + \mathscr{D}\bar{\psi} + \imath\mathscr{G}\bar{\psi}\gamma^5 = 0, \tag{2.18b}$$

where we denote $\mathscr{D} = 2SF_K K_I$ and $\mathscr{G} = 2PF_K K_J$, with $F_K = dF/dK$, $K_I = dK/dI$ and $K_J = dK/dJ$. In view of (2.18) can be rewritten as

$$\begin{aligned} L_{\rm sp} &= \frac{\imath}{2}\left[\bar{\psi}\gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi}\gamma^\mu \psi\right] - m_{\rm sp}\bar{\psi}\psi - F(I,J) \\ &= \frac{\imath}{2}\bar{\psi}[\gamma^\mu \nabla_\mu \psi - m_{\rm sp}\psi] - \frac{\imath}{2}[\nabla_\mu \bar{\psi}\gamma^\mu + m_{\rm sp}\bar{\psi}]\psi - F(I,J), \\ &= 2(IF_I + JF_J) - F = 2KF_K - F(K), \end{aligned} \tag{2.19}$$

where $K = \{I, J, I+J, I-J\}$.

D. Energy momentum tensor of the spinor field

The energy-momentum tensor of the spinor field is given by

$$T_{\mu}^{\rho} = \frac{i}{4} g^{\rho\nu} \left(\bar{\psi}\gamma_{\mu}\nabla_{\nu}\psi + \bar{\psi}\gamma_{\nu}\nabla_{\mu}\psi - \nabla_{\mu}\bar{\psi}\gamma_{\nu}\psi - \nabla_{\nu}\bar{\psi}\gamma_{\mu}\psi \right) - \delta_{\mu}^{\rho} L_{\mathrm{sp}}. \tag{2.20}$$

Then in view of (2.10) and (2.19) the energy-momentum tensor of the spinor field can be written as

$$\begin{aligned} T_{\mu}{}^{\rho} &= \frac{i}{4} g^{\rho\nu} \left(\bar{\psi}\gamma_{\mu}\partial_{\nu}\psi + \bar{\psi}\gamma_{\nu}\partial_{\mu}\psi - \partial_{\mu}\bar{\psi}\gamma_{\nu}\psi - \partial_{\nu}\bar{\psi}\gamma_{\mu}\psi \right) \\ &- \frac{i}{4} g^{\rho\nu} \bar{\psi} \left(\gamma_{\mu}\Gamma_{\nu} + \Gamma_{\nu}\gamma_{\mu} + \gamma_{\nu}\Gamma_{\mu} + \Gamma_{\mu}\gamma_{\nu} \right) \psi - \delta_{\mu}^{\rho} \left(2KF_{K} - F(K) \right). \end{aligned} \tag{2.21}$$

As is seen from (2.21), is case if for a given metric Γ_{μ}'s are different, there arise nontrivial non-diagonal components of the energy momentum tensor.

We consider the case when the spinor field depends on t only, i.e. $\psi = \psi(t)$. Then inserting (2.10) into (2.21) one finds

$$T_{0}^{0} = \frac{i}{2} g^{00} \left(\bar{\psi}\gamma_{0}\dot{\psi} - \dot{\bar{\psi}}\gamma_{0}\psi \right) - L_{\mathrm{sp}}, \tag{2.22a}$$

$$T_{1}^{1} = -\frac{i}{2} g^{11} \bar{\psi} \left(\gamma_{1}\Gamma_{1} + \Gamma_{1}\gamma_{1} \right) \psi - L_{\mathrm{sp}}, \tag{2.22b}$$

$$T_{2}^{2} = -\frac{i}{2} g^{22} \bar{\psi} \left(\gamma_{2}\Gamma_{2} + \Gamma_{2}\gamma_{2} \right) \psi - L_{\mathrm{sp}}, \tag{2.22c}$$

$$T_{3}^{3} = -\frac{i}{2} g^{33} \bar{\psi} \left(\gamma_{3}\Gamma_{3} + \Gamma_{3}\gamma_{3} \right) \psi - L_{\mathrm{sp}}, \tag{2.22d}$$

$$T_{1}^{0} = \frac{i}{4} g^{00} \left(\bar{\psi}\gamma_{1}\dot{\psi} - \dot{\bar{\psi}}\gamma_{1}\psi \right) - \frac{i}{4} g^{00} \bar{\psi} \left(\gamma_{0}\Gamma_{1} + \Gamma_{1}\gamma_{0} \right) \psi, \tag{2.22e}$$

$$T_{2}^{0} = \frac{i}{4} g^{00} \left(\bar{\psi}\gamma_{2}\dot{\psi} - \dot{\bar{\psi}}\gamma_{2}\psi \right) - \frac{i}{4} g^{00} \bar{\psi} \left(\gamma_{0}\Gamma_{2} + \Gamma_{2}\gamma_{0} \right) \psi, \tag{2.22f}$$

$$T_{3}^{0} = \frac{i}{4} g^{00} \left(\bar{\psi}\gamma_{3}\dot{\psi} - \dot{\bar{\psi}}\gamma_{3}\psi \right) - \frac{i}{4} g^{00} \bar{\psi} \left(\gamma_{0}\Gamma_{3} + \Gamma_{3}\gamma_{0} \right) \psi, \tag{2.22g}$$

$$T_{2}^{1} = -\frac{i}{4} g^{11} \bar{\psi} \left(\gamma_{2}\Gamma_{1} + \Gamma_{1}\gamma_{2} + \gamma_{1}\Gamma_{2} + \Gamma_{2}\gamma_{1} \right) \psi, \tag{2.22h}$$

$$T_{3}^{2} = -\frac{i}{4} g^{22} \bar{\psi} \left(\gamma_{3}\Gamma_{2} + \Gamma_{2}\gamma_{3} + \gamma_{2}\Gamma_{3} + \Gamma_{3}\gamma_{2} \right) \psi, \tag{2.22i}$$

$$T_{3}^{1} = -\frac{i}{4} g^{11} \bar{\psi} \left(\gamma_{3}\Gamma_{1} + \Gamma_{1}\gamma_{3} + \gamma_{1}\Gamma_{3} + \Gamma_{3}\gamma_{1} \right) \psi. \tag{2.22j}$$

Further inserting (2.16) into (2.22) after a little manipulations for the components of the energy-

momentum tensor one finds:

$$T_0^0 = m_{\rm sp} S + F(K), \tag{2.23a}$$

$$T_1^1 = T_2^2 = T_3^3 = F(K) - 2KF_K, \tag{2.23b}$$

$$T_1^0 = -\frac{\imath n e^{-mx_3}}{4}\frac{a_1}{a_3}\,\bar\psi\bar\gamma^3\bar\gamma^1\bar\gamma^0\psi = -\frac{n e^{-mx_3}}{4}\frac{a_1}{a_3}A^2, \tag{2.23c}$$

$$T_2^0 = -\frac{\imath m e^{nx_3}}{4}\frac{a_2}{a_3}\,\bar\psi\bar\gamma^2\bar\gamma^3\bar\gamma^0\psi = -\frac{m e^{nx_3}}{4}\frac{a_2}{a_3}A^1, \tag{2.23d}$$

$$T_3^0 = 0, \tag{2.23e}$$

$$\begin{aligned} T_2^1 &= \frac{\imath e^{(m+n)x_3}}{4}\frac{a_2}{a_1}\left[\left(\frac{\dot a_1}{a_1} - \frac{\dot a_2}{a_2}\right)\bar\psi\bar\gamma^1\bar\gamma^2\bar\gamma^0\psi - \frac{m+n}{a_3}\bar\psi\bar\gamma^1\bar\gamma^2\bar\gamma^3\psi\right] \\ &= \frac{e^{(m+n)x_3}}{4}\frac{a_2}{a_1}\left[\left(\frac{\dot a_1}{a_1} - \frac{\dot a_2}{a_2}\right)A^3 - \frac{m+n}{a_3}A^0\right], \end{aligned} \tag{2.23f}$$

$$T_3^1 = \frac{\imath e^{mx_3}}{4}\frac{a_3}{a_1}\left(\frac{\dot a_3}{a_3} - \frac{\dot a_1}{a_1}\right)\bar\psi\bar\gamma^3\bar\gamma^1\bar\gamma^0\psi = \frac{e^{mx_3}}{4}\frac{a_3}{a_1}\left(\frac{\dot a_3}{a_3} - \frac{\dot a_1}{a_1}\right)A^2 \tag{2.23g}$$

$$T_3^2 = \frac{\imath e^{-nx_3}}{4}\frac{a_3}{a_2}\left(\frac{\dot a_2}{a_2} - \frac{\dot a_3}{a_3}\right)\bar\psi\bar\gamma^2\bar\gamma^3\bar\gamma^0\psi = \frac{e^{-nx_3}}{4}\frac{a_3}{a_2}\left(\frac{\dot a_2}{a_2} - \frac{\dot a_3}{a_3}\right)A^1. \tag{2.23h}$$

As one sees from (2.22) and (2.23) the non-triviality of non-diagonal components of the energy momentum tensors is directly connected with the affine spinor connections Γ_i's.

From (2.18) one can write the equations for bilinear spinor forms (2.8):

$$\dot S_0 + \mathscr{G} A_0^0 = 0, \tag{2.24a}$$

$$\dot P_0 - \Phi A_0^0 = 0, \tag{2.24b}$$

$$\dot A_0^0 - \frac{m-n}{a_3}A_0^3 + \Phi P_0 - \mathscr{G} S_0 = 0, \tag{2.24c}$$

$$\dot A_0^3 - \frac{m-n}{a_3}A_0^0 = 0, \tag{2.24d}$$

$$\dot v_0^0 - \frac{m-n}{a_3}v_0^3 = 0, \tag{2.24e}$$

$$\dot v_0^3 - \frac{m-n}{a_3}v_0^0 + \Phi Q_0^{30} + \mathscr{G} Q_0^{21} = 0, \tag{2.24f}$$

$$\dot Q_0^{30} - \Phi v_0^3 = 0, \tag{2.24g}$$

$$\dot Q_0^{21} - \mathscr{G} v_0^3 = 0, \tag{2.24h}$$

where we denote $S_0 = SV$, $P_0 = PV$, $A_0^\mu = A_\mu V$, $v_0^\mu = v^\mu V$, $Q_0^{\mu\nu} = Q^{\mu\nu}V$ and $\Phi = m_{\rm sp} + \mathscr{D}$. Here we also introduce the volume scale

$$V = a_1 a_2 a_3. \tag{2.25}$$

Combining these equations together and taking the first integral one gets

$$(S_0)^2 + (P_0)^2 + (A_0^0)^2 - (A_0^3)^2 = C_1 = \text{Const}, \tag{2.26a}$$

$$(Q_0^{30})^2 + (Q_0^{21})^2 + (v_0^3)^2 - (v_0^0)^2 = C_2 = \text{Const} \tag{2.26b}$$

Now let us consider the Einstein field equations. In view of (2.6) and (2.23) we find the following system of Einstein Equations

$$\frac{\ddot{a}_2}{a_2}+\frac{\ddot{a}_3}{a_3}+\frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3}-\frac{n^2}{a_3^2}=\kappa\left(F(K)-2KF_K\right), \tag{2.27a}$$

$$\frac{\ddot{a}_3}{a_3}+\frac{\ddot{a}_1}{a_1}+\frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1}-\frac{m^2}{a_3^2}=\kappa\left(F(K)-2KF_K\right), \tag{2.27b}$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2}+\frac{mn}{a_3^2}=\kappa\left(F(K)-2KF_K\right), \tag{2.27c}$$

$$\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3}+\frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1}-\frac{m^2-mn+n^2}{a_3^2}=\kappa\left(m_{\rm sp}S+F(K)\right), \tag{2.27d}$$

$$(m-n)\frac{\dot{a}_3}{a_3}-m\frac{\dot{a}_1}{a_1}+n\frac{\dot{a}_2}{a_2}=0, \tag{2.27e}$$

$$0=\frac{n\,e^{-mx_3}}{4}\frac{a_1}{a_3}A^2, \tag{2.27f}$$

$$0=\frac{m\,e^{nx_3}}{4}\frac{a_2}{a_3}A^1, \tag{2.27g}$$

$$0=\frac{e^{(m+n)x_3}}{4}\frac{a_2}{a_1}\left[\left(\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}\right)A^3-\frac{m+n}{a_3}A^0\right], \tag{2.27h}$$

$$0=\frac{e^{mx_3}}{4}\frac{a_3}{a_1}\left(\frac{\dot{a}_3}{a_3}-\frac{\dot{a}_1}{a_1}\right)A^2, \tag{2.27i}$$

$$0=\frac{e^{-nx_3}}{4}\frac{a_3}{a_2}\left(\frac{\dot{a}_2}{a_2}-\frac{\dot{a}_3}{a_3}\right)A^1. \tag{2.27j}$$

From (2.27f) and (2.27g) one dully finds

$$A^2=0, \quad \text{and} \quad A^1=0. \tag{2.28}$$

In view of (2.28) the relations (2.27i) and (2.27j) fulfill even without imposing restrictions on the metric functions. From (2.27h) one finds the following relations between A^0 and A^3:

$$\left(\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}\right)A^3=\frac{m+n}{a_3}A^0. \tag{2.29}$$

Inserting (2.29) into (2.24d) one finds

$$\frac{m+n}{m-n}\frac{\dot{A}_0^3}{A_0^3}=\left(\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}\right), \tag{2.30}$$

with the solution

$$\left(A_0^3\right)^{\frac{m+n}{m-n}}=X_{03}\left(\frac{a_1}{a_2}\right), \quad X_{03}=\text{const}. \tag{2.31}$$

On the other hand from (2.27e) one finds the following relation between the metric functions

$$a_3=X_0\left(\frac{a_1^m}{a_2^n}\right)^{1/(m-n)}. \tag{2.32}$$

Thus the non-diagonal components of Einstein equations not only connected the different metric functions as was found in [26], but also imposes some restrictions on the components of the spinor field.

To find the metric functions explicitly we have to address the diagonal components of Einstein system. Explicit presence of a_3 force us to impose some additional conditions. In an early work [26] we propose two different situations, namely, set $a_3 = \sqrt{V}$ and $a_3 = V$ which allows us to obtain exact solutions for the metric functions.

In a recent paper we imposed the proportionality condition, widely used in literature. Demanding that the expansion is proportion to a component of the shear tensor, namely

$$\vartheta = N_3 \sigma_3^3. \tag{2.33}$$

The motivation behind assuming this condition is explained with reference to Thorne [56]. The observations of the velocity-red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 per cent [57, 58]. To put more precisely, red-shift studies place the limit

$$\frac{\sigma}{H} \leq 0.3, \tag{2.34}$$

on the ratio of shear σ to Hubble constant H in the neighborhood of our Galaxy today. Collins et al. [59] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies the condition $\frac{\sigma}{\theta}$ is constant. Under this proportionality condition it was also found that the energy-momentum distribution of the model is strictly isotropic, which is absolutely true for our case.

Let us now find expansion and shear for BVI metric. The expansion is given by

$$\vartheta = u^{\mu}_{;\mu} = u^{\mu}_{\mu} + \Gamma^{\mu}_{\mu\alpha} u^{\alpha}, \tag{2.35}$$

and the shear is given by

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu}, \tag{2.36}$$

with

$$\sigma_{\mu\nu} = \frac{1}{2}\left[u_{\mu;\alpha}P^{\alpha}_{\nu} + u_{\nu;\alpha}P^{\alpha}_{\mu}\right] - \frac{1}{3}\vartheta P_{\mu\nu}, \tag{2.37}$$

where the projection vector P:

$$P^2 = P, \quad P_{\mu\nu} = g_{\mu\nu} - u_{\mu}u_{\nu}, \quad P^{\mu}_{\nu} = \delta^{\mu}_{\nu} - u^{\mu}u_{\nu}. \tag{2.38}$$

In comoving system we have $u^{\mu} = (1,0,0,0)$. In this case one finds

$$\vartheta = \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} = \frac{\dot{V}}{V}, \tag{2.39}$$

and

$$\sigma^1_1 = -\frac{1}{3}\left(-2\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) = \frac{\dot{a}_1}{a_1} - \frac{1}{3}\vartheta, \tag{2.40a}$$

$$\sigma^2_2 = -\frac{1}{3}\left(-2\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} + \frac{\dot{a}_1}{a_1}\right) = \frac{\dot{a}_2}{a_2} - \frac{1}{3}\vartheta, \tag{2.40b}$$

$$\sigma^3_3 = -\frac{1}{3}\left(-2\frac{\dot{a}_3}{a_3} + \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2}\right) = \frac{\dot{a}_3}{a_3} - \frac{1}{3}\vartheta. \tag{2.40c}$$

One then finds

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{3}\left(\frac{\dot{a}_i}{a_i}\right)^2 - \frac{1}{3}\vartheta^2\right] = \frac{1}{2}\left[\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\vartheta^2\right]. \tag{2.41}$$

Inserting (2.32) into (2.39), (2.40) and (2.41) we find

$$\vartheta = \frac{2m-n}{m-n}\frac{\dot{a}_1}{a_1} + \frac{m-2n}{m-n}\frac{\dot{a}_2}{a_2}, \tag{2.42}$$

and

$$\sigma_1^1 = \frac{m-2n}{3(m-n)}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right), \tag{2.43a}$$

$$\sigma_2^2 = \frac{n-2m}{3(m-n)}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right), \tag{2.43b}$$

$$\sigma_3^3 = \frac{m+n}{3(m-n)}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right). \tag{2.43c}$$

On account of (2.32), (2.40c), (2.25) from (2.33) one finds

$$a_1 = \left[\frac{X_1^{(m-n)/(m-2n)}}{X_0}V\right]^{\frac{1}{3}+N_3\frac{m-2n}{m+n}}, \tag{2.44a}$$

$$a_2 = X_1\left[\frac{X_1^{(m-n)/(m-2n)}}{X_0}V\right]^{\frac{1}{3}+N_3\frac{n-2m}{m+n}}, \tag{2.44b}$$

$$a_3 = X_0 X_1^{-n/(m-n)}\left[\frac{X_1^{(m-n)/(m-2n)}}{X_0}V\right]^{\frac{1}{3}+N_3}, \tag{2.44c}$$

where X_1 is an integration constant. Further taking into account that $V = a_1a_2a_3$ from (2.44) one finds

$$X_1^{\frac{m-n}{m-2n}+\frac{m-2n}{m-n}} = 1, \tag{2.45}$$

with either $X_1 = 1$ or $\frac{m-n}{m-2n} + \frac{m-2n}{m-n} = 0$. Since $(m-n)^2 + (m-2n)^2 \neq 0$, we conclude $X_1 = 1$. Hence for the metric functions finally we obtain

$$a_1 = \left[\frac{V}{X_0}\right]^{\frac{1}{3}+N_3\frac{m-2n}{m+n}}, \quad a_2 = \left[\frac{V}{X_0}\right]^{\frac{1}{3}+N_3\frac{n-2m}{m+n}}, \quad a_3 = X_0\left[\frac{V}{X_0}\right]^{\frac{1}{3}+N_3}. \tag{2.46}$$

The equation for V can be found from the Einstein Equation (2.6) which for some manipulation looks

$$\ddot{V} = 2(m^2 - mn + n^2)X_0^{2N_3-4/3}V^{1/3-2N_3} + \frac{3\kappa}{2}\left[m_{\rm sp}S + 2\left(F(K) - KF_K\right)\right]V. \tag{2.47}$$

In order to solve (2.47) we have to know the relation between the spinor and the gravitational fields. Let us first find those relations for different K. Let us recall that $K = \{I, J, I+J, I-J\}$, with $\mathscr{D} = 2SF_KK_I$ and $\mathscr{G} = 2PF_KK_J$.

In case of $K = I$, i.e. $\mathscr{G} = 0$ from (2.24a) we duly have

$$\dot{S}_0 = 0, \tag{2.48}$$

with the solution

$$K = I = S^2 = \frac{V_0^2}{V^2}, \quad \Rightarrow \quad S = \frac{V_0}{V}, \quad V_0 = \text{const}. \tag{2.49}$$

In this case spinor field can be either massive or massless.

As far as case with $K = \{J, I+J, I-J\}$ that gives $K_J = \pm 1$ is concerned, it can be solved exactly only for a massless spinor field.

In case of $K = J$, i.e. $\Phi = \mathscr{D} = 0$ from (2.24b) we duly have

$$\dot{P}_0 = 0, \tag{2.50}$$

with the solution

$$K = J = P^2 = \frac{V_0^2}{V^2}, \quad \Rightarrow \quad P = \frac{V_0}{V}, \quad V_0 = \text{const}. \tag{2.51}$$

In case of $K = I + J$ the equations (2.24a) and (2.24b) can be rewritten as

$$\dot{S}_0 + 2PF_K A_0^0 = 0, \tag{2.52a}$$

$$\dot{P}_0 - 2SF_K A_0^0 = 0, \tag{2.52b}$$

which can be rearranged as

$$S_0\dot{S}_0 + P_0\dot{P}_0 = \frac{d}{dt}\left(S_0^2 + P_0^2\right) = \frac{d}{dt}\left(V^2 K\right) = 0, \tag{2.53}$$

with the solution

$$K = \frac{V_0^2}{V^2}, \quad V_0 = \text{const}. \tag{2.54}$$

Note that one can represent S and P as follows:

$$S = \frac{V_0}{V}\sin\theta, \quad P = \frac{V_0}{V}\cos\theta. \tag{2.55}$$

The term θ can be determined from (2.52a) or (2.52b) on account of (2.29), (2.31) and (2.46).

Finally, for $K = I - J$ the equations (2.24a) and (2.24b) can be rewritten as

$$\dot{S}_0 - 2PF_K A_0^0 = 0, \tag{2.56a}$$

$$\dot{P}_0 - 2SF_K A_0^0 = 0, \tag{2.56b}$$

which can be rearranged as

$$S_0\dot{S}_0 - P_0\dot{P}_0 = \frac{d}{dt}\left(S_0^2 - P_0^2\right) = \frac{d}{dt}\left(V^2 K\right) = 0, \tag{2.57}$$

with the solution

$$K = \frac{V_0^2}{V^2}, \quad V_0 = \text{const}. \tag{2.58}$$

As in previous case one can rewrite S and P as follows:

$$S = \frac{V_0}{V}\cosh\theta, \quad P = \frac{V_0}{V}\sinh\theta. \tag{2.59}$$

Like previous case θ can be determined from (2.56a) or (2.56b) on account of (2.29), (2.31) and (2.46).

III. SOLUTION TO THE FIELD EQUATIONS

In this section we solve the field equations. Let us begin with the spinor field equations. In view of (2.10) and (2.16) the spinor field equation (2.18a) takes the form

$$\imath\bar{\gamma}^0\left(\dot{\psi}+\frac{1}{2}\frac{\dot{V}}{V}\psi\right)-m_{\rm sp}\psi-\frac{m-n}{2a_3}\bar{\gamma}^3\psi-\mathscr{D}\psi-\imath\mathscr{G}\bar{\gamma}^5\psi=0, \tag{3.1a}$$

$$\imath\left(\dot{\bar{\psi}}+\frac{1}{2}\frac{\dot{V}}{V}\bar{\psi}\right)\bar{\gamma}^0+m_{\rm sp}\bar{\psi}-\frac{m-n}{2a_3}\bar{\psi}\bar{\gamma}^3+\mathscr{D}\bar{\psi}+\imath\mathscr{G}\bar{\psi}\bar{\gamma}^5=0. \tag{3.1b}$$

As we have already mentioned, ψ is a function of t only. We consider the 4-component spinor field given by

$$\psi=\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix}. \tag{3.2}$$

Denoting $\phi_i=\sqrt{V}\psi_i$ and $\bar{X}_0=(m-n)X_0^{N_3-2/3}$ from (3.1) for the spinor field we find we find

$$\dot{\phi}_1+\imath\Phi\phi_1+\left[\imath\frac{\bar{X}_0}{2V^{1/3+N_3}}+\mathscr{G}\right]\phi_3=0, \tag{3.3a}$$

$$\dot{\phi}_2+\imath\Phi\phi_2-\left[\imath\frac{\bar{X}_0}{2V^{1/3+N_3}}-\mathscr{G}\right]\phi_4=0, \tag{3.3b}$$

$$\dot{\phi}_3-\imath\Phi\phi_3+\left[\imath\frac{\bar{X}_0}{2V^{1/3+N_3}}-\mathscr{G}\right]\phi_1=0, \tag{3.3c}$$

$$\dot{\phi}_4-\imath\Phi\phi_4-\left[\imath\frac{\bar{X}_0}{2V^{1/3+N_3}}+\mathscr{G}\right]\phi_2=0. \tag{3.3d}$$

Further denoting $\mathscr{Y}=\frac{\bar{X}_0}{2V^{1/3+N_3}}$ we can write the foregoing system of equation in the form:

$$\dot{\phi}=A\phi, \tag{3.4}$$

with $\phi=\mathrm{col}\,(\phi_1,\,\phi_2,\,\phi_3,\,\phi_4)$ and

$$A=\begin{pmatrix}-\imath\Phi & 0 & -\imath\mathscr{Y}-\mathscr{G} & 0\\ 0 & -\imath\Phi & 0 & \imath\mathscr{Y}-\mathscr{G}\\ -\imath\mathscr{Y}+\mathscr{G} & 0 & \imath\Phi & 0\\ 0 & \imath\mathscr{Y}+\mathscr{G} & 0 & \imath\Phi\end{pmatrix}. \tag{3.5}$$

It can be easily found that

$$\det A=\left(\Phi^2+\mathscr{Y}^2+\mathscr{G}^2\right)^2. \tag{3.6}$$

The solution to the equation (3.4) can be written in the form

$$\phi(t)=\mathrm{Texp}\left(-\int_t^{t_1}A_1(\tau)d\tau\right)\phi(t_1), \tag{3.7}$$

where

$$A=\begin{pmatrix} -\imath\mathscr{D} & 0 & -\imath\mathscr{Y}-\mathscr{G} & 0 \\ 0 & -\imath\mathscr{D} & 0 & \imath\mathscr{Y}-\mathscr{G} \\ -\imath\mathscr{Y}+\mathscr{G} & 0 & \imath\mathscr{D} & 0 \\ 0 & \imath\mathscr{Y}+\mathscr{G} & 0 & \imath\mathscr{D} \end{pmatrix}. \tag{3.8}$$

and $\phi(t_1)$ is the solution at $t=t_1$. As we have already shown, $K=V_0^2/V^2$ for $K=\{J, I+J, I-J\}$ with trivial spinor-mass and $K=V_0^2/V^2$ for $K=I$ for any spinor-mass. Since our Universe is expanding, the quantities $\mathscr{D}$, $\mathscr{Y}$ and $\mathscr{G}$ become trivial at large t. Hence in case of $K=I$ with non-trivial spinor-mass one can assume $\phi(t_1)=\mathrm{col}\,(e^{-\imath m_{\rm sp}t_1}, e^{-\imath m_{\rm sp}t_1}, e^{\imath m_{\rm sp}t_1}, e^{\imath m_{\rm sp}t_1})$, whereas for other cases with trivial spinor-mass we have $\phi(t_1)=\mathrm{col}\left(\phi_1^0, \phi_2^0, \phi_3^0, \phi_4^0\right)$ with ϕ_i^0 being some constants. Here we have used the fact that $\Phi=m_{\rm sp}+\mathscr{D}$. The other way to solve the system (3.3) is given in [26].

As far as equation for V, i.e., (2.47) is concerned, we solve it setting $K=I$ as in this case we can use the mass term as well. Assuming $F=\lambda I^N$ we find

$$\ddot{V}=\Phi(V), \quad \Phi(V)=\bar{X}V^{1/3-2N_3}+\frac{3\kappa}{2}\left[m_{\rm sp}V_0+2\lambda(1-N)V_0^{2N}V^{1-2N}\right], \tag{3.9}$$

where $\bar{X}=2\left(m^2-mn+n^2\right)X_0^{(2N_3-4/3)}$. The first integral of (3.9) is

$$\dot{V}=\Phi_1(V), \quad \Phi_1(V)=\sqrt{\bar{X}_1V^{(4/3-2N_3)}+3\kappa\left[m_{\rm sp}V_0V+\lambda V_0^{2N}V^{2(1-N)}\right]+\bar{C}}, \tag{3.10}$$

where we denote $\bar{X}_1=6\bar{X}/(4-6N_3)$ and $\bar{C}$ is the constant of integration. The solution for V can be written in quadrature as

$$\int\frac{dV}{\sqrt{\bar{X}_1V^{(4/3-2N_3)}+3\kappa\left[m_{\rm sp}V_0V+\lambda V_0^{2N}V^{2(1-N)}\right]+\bar{C}}}=t+t_0, \quad t_0=\text{const.} \tag{3.11}$$

In what follows we solve the Eqn. (3.9) numerically. In doing so we determine $\dot{V}(0)$ from (3.10) for the given value of $V(0)$.

To determine the character of the evolution, let us first study the asymptotic behavior of the equation (3.9). It should be recalled that we have $K=V_0^2/V^2$. Since all the physical quantities constructed from the spinor fields as well as the invariants of gravitational fields are inverse function of V of some degree, it can be concluded that at any spacetime point where the volume scale becomes zero, it is a singular point [24]. So we assume at the beginning V was small but non-zero. Then from (3.9) we see that at $t\to 0$ the term $m_{\rm sp}V_0$ prevails if $1-2N>0$, i.e., $N<1/2$ and $1/3-2N_3>0$, i.e. $N_3<1/6$. Otherwise one of the remaining two terms become predominant depending on which one of $1-2N$ and $1/3-2N_3$ is negative and possesses greater absolute value. Recalling that we are considering an expanding Universe, at $t\to\infty$ the volume scale should be quite large. In that case the nonlinear term prevails over the first term if $1-2N>1/3-2N_3$, i.e., $N<1/3+N_3$ with $N<1/2$, the the first term dominates if $N>1/3+N_3$ and $N_3<1/6$.

To define whether the model allows decelerated or accelerated mode of expansion we also plot deceleration parameter q defines as

$$q=-\frac{V\ddot{V}}{\dot{V}^2}, \tag{3.12}$$

which in view of (3.9) and (3.10) can be rewritten as

$$q=-\frac{V\Phi(V)}{\Phi_1^2(V)}=-\frac{\bar{X}V^{4/3-2N_3}+\frac{3\kappa}{2}\left[m_{\rm sp}V_0V+2\lambda(1-N)V_0^{2N}V^{2(1-N)}\right]}{\bar{X}_1V^{(4/3-2N_3)}+3\kappa\left[m_{\rm sp}V_0V+\lambda V_0^{2N}V^{2(1-N)}\right]+C}, \tag{3.13}$$

Now let us see what happens to deceleration parameter as $t \to \infty$. As we have already established, for $N < 1/2$ and $N < 1/3 + N_3$ the nonlinear tern prevails and in this case we find

$$q \approx -(1-N) < 0, \tag{3.14}$$

whereas for $N_3 < 1/6$ and $N > 1/3 + N_3$ we have

$$q \approx -\frac{\bar{X}}{X_1} = -(2/3 - N_3) < 0. \tag{3.15}$$

Thus we see that in both cases the Universe expands with acceleration.

It should also be emphasized that for $N > 1/2$ and $N_3 > 1/6$ the mass term prevails asymptotically at $t \to \infty$ and the Universe expands as a quadratic function of time, i.e., $V|_{t\to\infty} \propto t^2$.

Since we are interested in qualitative picture here, so we set the value of problem parameters very simple. Here we set $m = 1, n = 2, X_0 = 1, V_0 = 1, m_{\rm sp} = 1, C_0 = 1, \kappa = 1$. Fixing $N_3 = 1/8$ and $N = 1/4$ we consider two cases with positive and a negative λ, namely $\lambda = 1$ (corresponds to Fig. 1 and Fig. 3) and $\lambda = -1$ (corresponds to Fig. 2 and Fig. 4). The initial value of $V(0)$ is taken to be $V(0) = 0.5$. In Fig. 1 and Fig. 2, we plotted the evolution of the volume scale for a positive and negative self-coupling constant, while we did the same in Fig. 3 and Fig. 4 for deceleration parameter. As it was expected the positive λ in this case gives rise to a Universe that expands with acceleration, while the negative λ generates a cyclic or periodic mode of evolution of the Universe.

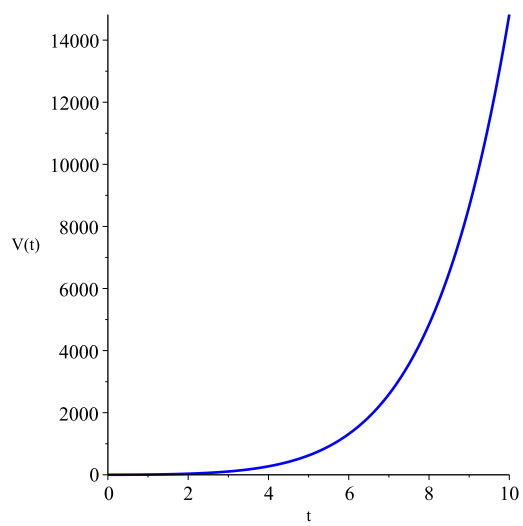

FIG. 1. Evolution of the Universe filled with massive spinor field with a positive self-coupling constant.

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IV. CONCLUSION

Within the scope of Bianchi type-VI spacetime we study the role of spinor field on the evolution of the Universe. It is found that in this case the non-diagonal components of the energy-momentum tensor of spinor field, unlike in the cases Bianchi type I [48] and Bianchi type-VI_0 [49], does not lead to the elimination of spinor field nonlinearity and the mass term in spinor field Lagrangian. Depending of the sign of self-coupling constant the model in this case allows either late time acceleration or oscillatory mode of evolution.

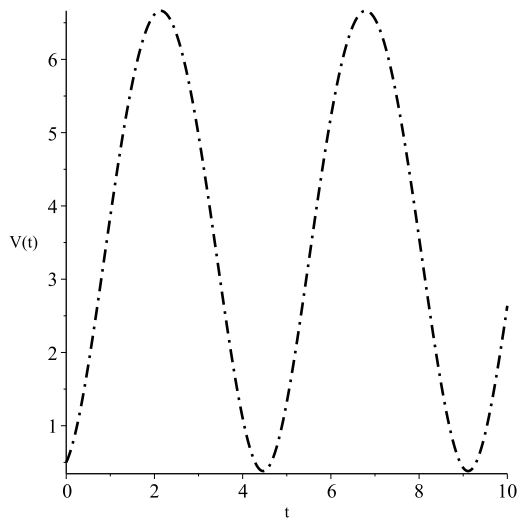

FIG. 2. Evolution of the Universe filled with massive spinor field with a negative self-coupling constant.

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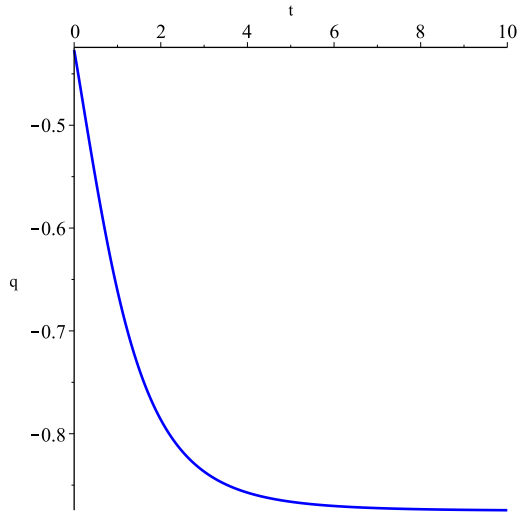

FIG. 3. Plot of deceleration parameter q with a positive self-coupling constant..

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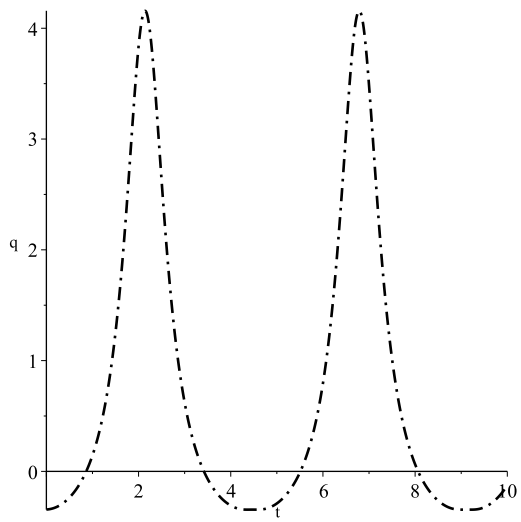

FIG. 4. Plot of deceleration parameter q with a negative self-coupling constant.

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