Spinor field with polynomial nonlinearity in LRS Bianchi type-I space-time

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Within the scope of the locally rotationally symmetric (LRS) Bianchi type-I cosmological model the role of spinor field on the evolution of the Universe is investigated. In doing so, we have considered a polynomial type of nonlinearity. It is found that, depending on the sign of the self-coupling constant, the model allows either an accelerated mode of expansion or an oscillatory mode of evolution. While the non-diagonal components of the energy-momentum tensor of the spinor field in the case of a full Bianchi type-I model lead to the vanishing mass and nonlinear term in the spinor field Lagrangian, in the case of an LRS Bianchi type-I model neither the mass term nor the nonlinear term of the spinor field vanish.

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I. INTRODUCTION

Recently, after some remarkable works by different authors [1–15], showing the important role that spinor fields play on the evolution of the Universe, the situation began to change. This change of attitude is directly related to some fundamental questions of modern cosmology: (i) problem of initial singularity; (ii) problem of isotropization and (iii) late time acceleration of the Universe.

Given the role that the spinor field can play in the evolution of the Universe, the question that naturally emerges is, if the spinor field can redraw the picture of evolution caused by perfect fluid and dark energy, is it possible to simulate perfect fluid and dark energy by means of a spinor field? An affirmative answer to this question was given in a number of papers [16–20]. In those papers, a spinor description of matter, such as a perfect fluid and dark energy, was given and the evolution of the Universe, given by different Bianchi models, was thoroughly studied. In almost all the papers the spinor field was considered to be a time-dependent function and its energy-momentum tensor was given by the diagonal elements only.

Some latest studies show that because of the specific connection with the gravitational field the energy-momentum tensor of the spinor field possesses non-trivial non-diagonal components as well, and these non-zero non-diagonal components of the energy-momentum tensor play decisive a role in the character of the geometry of space-time as well as on the components of the spinor field [21–24].

In this paper we study the evolution of the Universe filled with spinor field within the scope of a locally rotationally symmetric (LRS) Bianchi type-I (BI) cosmological model. It should be noted that a general BI model in the presence of a nonlinear spinor field duly evolves into a LRS BI model [22]. In this paper we thoroughly study the role of a spinor field in the evolution of the Universe given by a LRS BI model. Here we also consider a more general type of nonlinearity.

II. BASIC EQUATIONS

In this paper we plan to study the evolution of the Universe given by a LRS BI anisotropic cosmological model filled with nonlinear spinor field.

The LRS BI model is the ordinary BI model with two of the three metric functions being equal to each other and can be given by

$$ds^2 = dt^2 - a_1^2\left[dx^2 + dy^2\right] - a_3^2 dz^2, \tag{2.1}$$

with a_1 and a_3 being functions of time only.

The nontrivial components of the Einstein tensor corresponding to metric (2.1) are

$$G_1^1 = G_2^2 = -\left(\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1}\right), \tag{2.2a}$$

$$G_3^3 = -\left(2\frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1^2}{a_1^2}\right), \tag{2.2b}$$

$$G_0^0 = -\left(\frac{\dot{a}_1^2}{a_1^2} + 2\frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1}\right). \tag{2.2c}$$

Keeping in mind the symmetry between ψ and $\bar{\psi}$ we choose the symmetrized Lagrangian [25] for the spinor field as [5]:

$$L = \frac{\imath}{2}\left[\bar{\psi}\gamma^{\mu}\nabla_{\mu}\psi - \nabla_{\mu}\bar{\psi}\gamma^{\mu}\psi\right] - m_{\rm sp}\bar{\psi}\psi - F, \tag{2.3}$$

where the nonlinear term F describes the self-interaction of a spinor field and can be presented as some arbitrary function of invariants generated from the real bilinear forms of a spinor field. We consider $F = F(K)$, with K taking one of the following expressions $\{I, J, I+J, I-J\}$. It can be shown that such a choice describes the nonlinearity in its most general form.

Here ∇_{μ} is the covariant derivative of spinor field

$$\nabla_{\mu}\psi = \frac{\partial\psi}{\partial x^{\mu}} - \Gamma_{\mu}\psi, \quad \nabla_{\mu}\bar{\psi} = \frac{\partial\bar{\psi}}{\partial x^{\mu}} + \bar{\psi}\Gamma_{\mu}, \tag{2.4}$$

with Γ_{μ} being the spinor affine connection. In (2.3) γ's are the Dirac matrices in curve space-time and obey the following algebra:

$$\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2g^{\mu\nu} \tag{2.5}$$

and are connected with the flat space-time Dirac matrices $\bar{\gamma}$ in the following way

$$g_{\mu\nu}(x) = e_{\mu}^{a}(x)e_{\nu}^{b}(x)\eta_{ab}, \quad \gamma_{\mu}(x) = e_{\mu}^{a}(x)\bar{\gamma}_a, \tag{2.6}$$

where $\eta_{ab} = \mathrm{diag} - (1,-1,-1,-1)$ and e_{μ}^{a} is a set of tetrad 4-vectors. The spinor affine connection matrices $\Gamma_{\mu}(x)$ are uniquely determined up to an additive multiple of the unit matrix by

$$\nabla_{\mu}\gamma_{\nu} = \frac{\partial\gamma_{\nu}}{\partial x^{\mu}} - \Gamma_{\nu\mu}^{\rho}\gamma_{\rho} - \Gamma_{\mu}\gamma_{\nu} + \gamma_{\nu}\Gamma_{\mu} = 0, \tag{2.7}$$

with the solution

$$\Gamma_{\mu} = \frac{1}{4}\bar{\gamma}_a\gamma^{\nu}\partial_{\mu}e_{\nu}^{(a)} - \frac{1}{4}\gamma_{\rho}\gamma^{\nu}\Gamma_{\mu\nu}^{\rho}, \tag{2.8}$$

The spin affine connection corresponding to LRS BI metric (2.1) can be written explicitly as

$$\Gamma_0 = 0, \quad \Gamma_1 = \frac{\dot a_1}{2}\bar\gamma^1\bar\gamma^0, \quad \Gamma_2 = \frac{\dot a_1}{2}\bar\gamma^2\bar\gamma^0, \quad \Gamma_3 = \frac{\dot a_3}{2}\bar\gamma^3\bar\gamma^0, \tag{2.9}$$

Varying (2.3) with respect to $\bar\psi(\psi)$ one finds the spinor field equations:

$$\imath\gamma^\mu\nabla_\mu\psi - m_{\rm sp}\psi - 2F_K(SK_I + \imath PK_J\gamma^5)\psi = 0, \tag{2.10a}$$

$$\imath\nabla_\mu\bar\psi\gamma^\mu + m_{\rm sp}\bar\psi + 2F_K\bar\psi(SK_I + \imath PK_J\gamma^5) = 0. \tag{2.10b}$$

Here we denote $F_K = dF/dK$, $K_I = dK/dI$, and $K_J = dK/dJ$.

The energy-momentum tensor of the spinor field is given by

$$T_\mu^\rho = \frac{i}{4}g^{\rho\nu}\left(\bar\psi\gamma_\mu\nabla_\nu\psi + \bar\psi\gamma_\nu\nabla_\mu\psi - \nabla_\mu\bar\psi\gamma_\nu\psi - \nabla_\nu\bar\psi\gamma_\mu\psi\right) - \delta_\mu^\rho L_{\rm sp} \tag{2.11}$$

where $L_{\rm sp}$ in view of (2.10a) and (2.10b) can be rewritten as

$$\begin{aligned} L_{\rm sp} &= \frac{\imath}{2}\left[\bar\psi\gamma^\mu\nabla_\mu\psi - \nabla_\mu\bar\psi\gamma^\mu\psi\right] - m_{\rm sp}\bar\psi\psi - F(K) \\ &= \frac{\imath}{2}\bar\psi[\gamma^\mu\nabla_\mu\psi - m_{\rm sp}\psi] - \frac{\imath}{2}[\nabla_\mu\bar\psi\gamma^\mu + m_{\rm sp}\bar\psi]\psi - F(K), \\ &= 2F_K(IK_I + JK_J) - F = 2KF_K - F(K). \end{aligned} \tag{2.12}$$

Further inserting (2.4) into (2.11) the energy-momentum tensor of the spinor field can be written as

$$\begin{aligned} T_\mu{}^\rho &= \frac{\imath}{4}g^{\rho\nu}\left(\bar\psi\gamma_\mu\partial_\nu\psi + \bar\psi\gamma_\nu\partial_\mu\psi - \partial_\mu\bar\psi\gamma_\nu\psi - \partial_\nu\bar\psi\gamma_\mu\psi\right) \\ &- \frac{\imath}{4}g^{\rho\nu}\bar\psi\left(\gamma_\mu\Gamma_\nu + \Gamma_\nu\gamma_\mu + \gamma_\nu\Gamma_\mu + \Gamma_\mu\gamma_\nu\right)\psi - \delta_\mu^\rho\left(2KF_K - F(K)\right). \end{aligned} \tag{2.13}$$

Finally, exploiting the explicit form of spin connection (2.9) after some manipulations one finds the following non-trivial components of the energy-momentum tensor of the spinor field

$$T_0^0 = m_{\rm sp}S + F(K), \tag{2.14a}$$

$$T_1^1 = T_2^2 = T_3^3 = F(K) - 2KF_K, \tag{2.14b}$$

$$T_3^1 = \frac{\imath}{4}\frac{a_3}{a_1}\left(\frac{\dot a_3}{a_3} - \frac{\dot a_1}{a_1}\right)\bar\psi\bar\gamma^3\bar\gamma^1\bar\gamma^0\psi = \frac{1}{4}\frac{a_3}{a_1}\left(\frac{\dot a_3}{a_3} - \frac{\dot a_1}{a_1}\right)A^2, \tag{2.14c}$$

$$T_3^2 = \frac{\imath}{4}\frac{a_3}{a_1}\left(\frac{\dot a_1}{a_1} - \frac{\dot a_3}{a_3}\right)\bar\psi\bar\gamma^2\bar\gamma^3\bar\gamma^0\psi = \frac{1}{4}\frac{a_3}{a_1}\left(\frac{\dot a_1}{a_1} - \frac{\dot a_3}{a_3}\right)A^1. \tag{2.14d}$$

So the complete set of Einstein equations for a BI metric should be

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} = \kappa(F(K) - 2KF_K), \tag{2.15a}$$

$$2\frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1^2}{a_1^2} = \kappa(F(K) - 2KF_K), \tag{2.15b}$$

$$\frac{\dot{a}_1^2}{a_1^2} + 2\frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} = \kappa(m_{\rm sp}S + F(K)), \tag{2.15c}$$

$$0 = \left(\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1}\right) A^2,, \tag{2.15d}$$

$$0 = \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_3}{a_3}\right) A^1. \tag{2.15e}$$

Before solving the Einstein equations let us first write the equations for the bilinear spinor forms. Recalling that there are 16 bilinear spinor forms, namely, $S = \bar{\psi}\psi, \quad P = \imath\bar{\psi}\gamma^5\psi, \quad v^\mu = \bar{\psi}\gamma^\mu\psi, \quad A^\mu = \bar{\psi}\gamma^5\gamma^\mu\psi$, and $Q^{\mu\nu} = \bar{\psi}\sigma^{\mu\nu}\psi$ are the scalar, pseudoscalar, vector, pseudovector and antisymmetric tensor, respectively, for the LRS BI metric one finds the following system of equations:

$$\dot{S}_0 + \mathscr{G}A_0^0 = 0, \tag{2.16a}$$

$$\dot{P}_0 - \Phi A_0^0 = 0, \tag{2.16b}$$

$$\dot{A}_0^0 + \Phi P_0 - \mathscr{G}S_0 = 0, \tag{2.16c}$$

$$\dot{A}_0^3 = 0, \tag{2.16d}$$

$$\dot{v}_0^0 = 0, \tag{2.16e}$$

$$\dot{v}_0^3 + \Phi Q_0^{30} + \mathscr{G}Q_0^{21} = 0, \tag{2.16f}$$

$$\dot{Q}_0^{30} - \Phi v_0^3 = 0, \tag{2.16g}$$

$$\dot{Q}_0^{21} - \mathscr{G}v_0^3 = 0, \tag{2.16h}$$

where we denote $S_0 = SV, P_0 = PV, A_0^\mu = A_\mu V, v_0^\mu = v^\mu V, Q_0^{\mu\nu} = Q^{\mu\nu}V$ and $\Phi = m_{\rm sp} + \mathscr{D}$. We also denote $\mathscr{D} = 2SF_K K_I$ and $\mathscr{G} = 2PF_K K_J$, with $F_K = dF/dK$, $K_I = dK/dI$, and $K_J = dK/dJ$.

Here we also introduce the volume scale

$$V = a_1^2 a_3. \tag{2.17}$$

III. SOLUTION TO THE FIELD EQUATIONS

From (2.16a) - (2.16h) one finds the following relations:

$$(S_0)^2 + (P_0)^2 + (A_0^0)^2 = C_1 = \text{Const}, \tag{3.1a}$$

$$A_0^3 = C_2 = \text{Const}, \tag{3.1b}$$

$$v_0^0 = C_3 = \text{Const}, \tag{3.1c}$$

$$(Q_0^{30})^2 + (Q_0^{21})^2 + (v_0^3)^2 = C_4 = \text{Const}. \tag{3.1d}$$

Let us now go back to the Einstein equations. The off-diagonal components of Einstein equations (2.15d) and (2.15e) impose the following restrictions either on the components of the spinor field or on the metric functions:

$$A^2 = 0, \quad A^1 = 0, \tag{3.2a}$$

$$\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1} = 0. \tag{3.2b}$$

The restriction (3.2b) leads to $a_3 = q_0 a_1$ with q_0 being some constant. In this case the system can be described by a Friedmann-Robertson-Walker (FRW) model from the very beginning. Here we do not consider this case, which we will address in a later work, within the scope of a FRW model.

We consider the case when the restriction is imposed on the components of the spinor field in detail. Subtraction of (2.15b) from (2.15a) gives

$$\frac{\ddot{a}_3}{a_3} - \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1}{a_1}\left(\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1}\right) = 0, \tag{3.3}$$

that leads to [5]

$$a_1 = D_1 V^{1/3} \exp\left(X_1 \int \frac{dt}{V}\right), \quad a_3 = (1/D_1^2) V^{1/3} \exp\left(-2X_1 \int \frac{dt}{V}\right). \tag{3.4}$$

with D_1 and X_1 being the integration constants. Thus we see that the metric functions can be expressed in terms of V.

The solutions to spinor field equation (2.10a) in this case can be presented as [5]

$$\psi_{1,2}(t) = \frac{C_{1,2}}{\sqrt{V}} \exp\left(-i \int \Phi dt\right), \quad \psi_{3,4}(t) = \frac{C_{3,4}}{\sqrt{V}} \exp\left(i \int \Phi dt\right), \tag{3.5}$$

with C_1, C_2, C_3, and C_4 being the integration constants and related to V_0 as

$$C_1^* C_1 + C_2^* C_2 - C_3^* C_3 - C_4^* C_4 = V_0.$$

Here we assumed that $K = I$, i.e., $F = F(I)$. The reason for this choice is discussed later.

Thus we see that the metric functions, the components of spinor field as well as the invariants constructed from metric functions and spinor fields are some inverse functions of V of some degree. Hence any space-time point where $V = 0$ is a singular point. So it is important to study the behavior of V, which we do in the next section.

IV. RESULTS AND DISCUSSION

In this section we discuss the results obtained in the previous section. In doing so, we pay special attention to the volume scale, V.

Let us first see whether the model becomes asymptotically isotropic. It can be shown that for an expanding Universe, when $V \to \infty$ as $t \to \infty$, the isotropization process of the Universe takes place. To prove that we exploit the isotropization condition proposed in [26]

$$\left.\frac{a_i}{a}\right|_{t\to\infty} \to \text{const.} \tag{4.1}$$

Then by rescaling some of the coordinates, we can make $a_i/a \to 1$, and the metric will become manifestly isotropic at large t.

Taking into account that $a = V^{1/3}$ from (3.4) we find

$$\frac{a_1}{a} = D_1 \exp\left(X_1 \int \frac{dt}{V}\right) \to D_1, \quad \frac{a_3}{a} = (1/D_1^2)\exp\left(-2X_1 \int \frac{dt}{V}\right) \to 1/D_1^2, \tag{4.2}$$

as $V \to \infty$. Recall that the isotropic FRW model has the same scale factor in all three directions (i.e., $a_1(t) = a_2(t) = a_3(t) = a(t)$). So for the LRS BI universe to evolve into a FRW one we should have $D_1 = 1$. Moreover, the isotropic nature of the present Universe leads to the fact that $|X_1| << 1$, so that $\int [V(t)]^{-1} dt \to 0$ for $t < \infty$ (for $V(t) = t^n$ with $n > 1$ the integral tends to zero as $t \to \infty$).

Our next step will be to define V. Combining the diagonal Einstein equations (2.15a) – (2.15c) in a certain way for V we find [5]

$$\ddot{V} = \frac{3\kappa}{2}\big(m_{\rm sp} S + 2(F(K) - K F_K)\big) V. \tag{4.3}$$

Now in order to solve (4.3) we have to know the relation between the spinor and the gravitational fields. Using the equations (2.16a) and (2.16b) it can be show that

$$K = \frac{V_0^2}{V^2}. \tag{4.4}$$

Relation (4.4) holds only for massless spinor field if K takes one of the expressions $\{J, I + J, I - J\}$, while for $K = I$ it holds both for massless and massive spinor fields. In the case of $K = I + J$ one can write $S = \sin(V_0/V)$ and $P = \cos(V_0/V)$, whereas for $K = I - J$ one can write $S = \cosh(V_0/V)$ and $P = \sinh(V_0/V)$. In what follows, we will consider the case for $K = I$, as in this case further setting spinor mass $m_{\rm sp} = 0$ we can revive the results for other cases. Assuming

$$F = \sum_k \lambda_k I^{n_k} = \sum_k \lambda_k S^{2n_k} \tag{4.5}$$

on account of $S = V_0/V$ we find

$$\ddot{V} = \frac{3\kappa}{2}\left[m_{\rm sp} V_0 + 2\sum_k \lambda_k (1-n_k) V_0^{2n_k} V^{1-2n_k}\right], \tag{4.6}$$

with the solution in quadrature

$$\int \frac{dV}{\sqrt{3\kappa\left[m_{\rm sp} V_0 V + \sum_k \lambda_k V_0^{2n_k} V^{2(1-n_k)}\right] + \bar{C}}} = t + t_0, \tag{4.7}$$

with $\bar{C}$ and t_0 being some arbitrary constants.

Thus we see that the metric functions, the components of the spinor field, as well as the invariants constructed from metric functions and spinor fields are some inverse functions of V of some degree. Hence any space-time point where $V = 0$ is a singular point. So we consider that the initial value of $V(0)$ is small but non-zero. As a result for the nonlinear term to prevail in (4.6) we should have $n_k = n_1 : 1 - 2n_1 < 0$ (i.e., $n_1 > 1/2$) whereas for an expanding Universe when $V \to \infty$ as $t \to \infty$ one should have $n_k = n_2 : 1 - 2n_2 > 0$ (i.e., $n_2 < 1/2$). As is seen from (4.6), $n_k = n_0 : n_0 = 1/2$ leads to a term that can be added to the mass term.

In this case we obtain

$$\begin{aligned} \ddot{V} &= \Phi_1(V), \\ \Phi_1(V) &= \frac{3\kappa}{2}\left[\left(m_{\rm sp} + \lambda_0\right) V_0 + 2\lambda_1(1-n_1) V_0^{2n_1} V^{1-2n_1} + 2\lambda_2(1-n_2) V_0^{2n_2} V^{1-2n_2}\right]. \end{aligned} \tag{4.8}$$

Equation (4.8) allows the first integral

$$\dot{V} = \Phi_2(V), \tag{4.9}$$

$$\Phi_2(V) = \sqrt{3\kappa\left[\left(m_{\rm sp}+\lambda_0\right)V_0 V + \lambda_1 V_0^{2n_1} V^{2(1-n_1)} + \lambda_2 V_0^{2n_2} V^{2(1-n_2)} + \bar{C}\right]}. \tag{4.10}$$

The solution to (4.8) can be written in quadrature as follows:

$$\int \frac{dV}{\Phi_2(V)} = t + t_0. \tag{4.11}$$

To solve (4.8) we should choose the problem parameters V_0, $m_{\rm sp}$, κ, $\bar{C}$, λ_k, as well as the initial value of $V(0)$ in such a way that does not lead to

$$\left(m_{\rm sp}+\lambda_0\right)V_0 V + \lambda_1 V_0^{2n_1} V^{2(1-n_1)} + \lambda_2 V_0^{2n_2} V^{2(1-n_2)} + \bar{C} < 0.$$

For simplicity let us set $V_0 = 1$, $m_{\rm sp} = 1$, $C_0 = 10$, and $\kappa = 1$. In line with our discussion earlier we consider $n_0 = 1/2$, $n_1 = 2$, and $n_2 = 0$. In our case we set $V(0) = 0.5$. We set $\lambda_0 = 1$, whereas $\lambda_1 = \pm 1$ and $\lambda_2 = \pm 1$ were taken in different combinations. It was found that depending on the sign of λ_2 the model gives principally different types of solutions, namely, in the case of positive λ_2 we have an accelerated mode of expansion of the Universe, while for negative λ_2 we have oscillatory solution.

Defining the deceleration parameter

$$q = -\frac{V\ddot{V}}{\dot{V}^2} = -\frac{V\Phi_1(V)}{\Phi_2^2(V)}, \tag{4.12}$$

from (4.8) and (4.9) we have

$$q = -\frac{\frac{3\kappa}{2}\left[\left(m_{\rm sp}+\lambda_0\right)V_0 V + 2\lambda_1(1-n_1)V_0^{2n_1} V^{2(1-n_1)} + 2\lambda_2(1-n_2)V_0^{2n_2} V^{2(1-n_2)}\right]}{3\kappa\left[\left(m_{\rm sp}+\lambda_0\right)V_0 V + \lambda_1 V_0^{2n_1} V^{2(1-n_1)} + \lambda_2 V_0^{2n_2} V^{2(1-n_2)} + \bar{C}\right]}. \tag{4.13}$$

Taking into account that for an expanding Universe at large t the term V^{1-2n_2} prevails, for deceleration parameter we find

$$\lim_{V\to\infty} q \longrightarrow -(1-n_2) < 0, \quad \text{since} \quad n_2 < 1/2. \tag{4.14}$$

Thus we see that spinor field nonlinearity generates late time acceleration of the Universe.

In Fig. 1 and Fig. 2 we plotted the evolution of volume scale V for a positive and negative self-coupling constant λ_2, respectively. As one sees from Fig. 1, a positive λ_2 gives rise to an

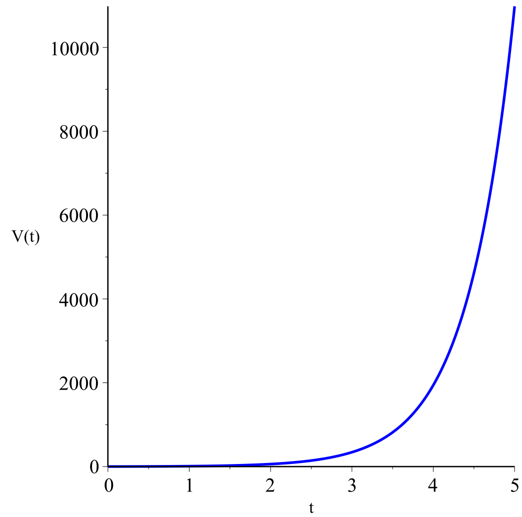

FIG. 1. Evolution of the Universe for a positive λ_2

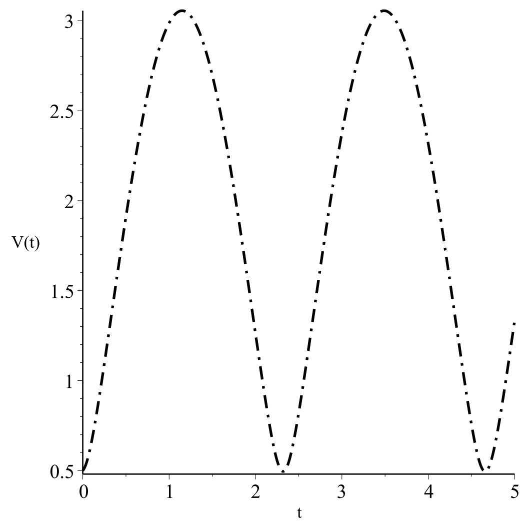

FIG. 2. Evolution of the Universe for a negative λ_2

accelerated mode of expansion, whereas Fig. 2 with negative λ_2 shows the oscillatory mode of expansion. In Fig. 3 the deceleration parameter q is illustrated for a positive λ_2.

Finally we study what happens to shear and anisotropic parameters in this case. In doing so, let us first rewrite the corresponding quantities. The expansion ϑ for LRS BI metric reads

$$\vartheta = 2\frac{\dot a_1}{a_1} + \frac{\dot a_3}{a_3} = \frac{\dot V}{V}, \tag{4.15}$$

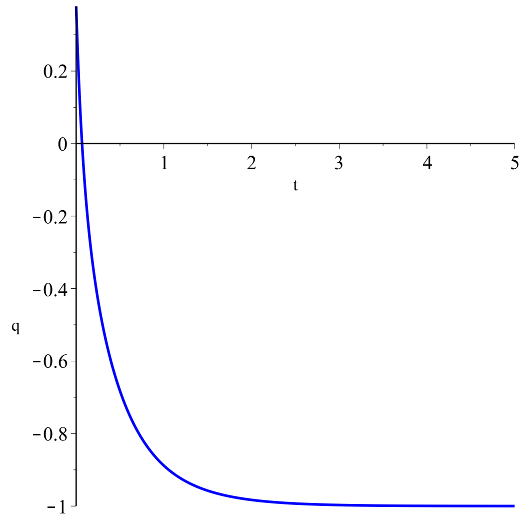

FIG. 3. Plot of deceleration parameter q for a positive λ_2

whereas, from

$$\sigma_1^1 = \sigma_2^2 = \frac{\dot a_1}{a_1} - \frac{1}{3}\vartheta = \frac{1}{3}\left(\frac{\dot a_1}{a_1} - \frac{\dot a_3}{a_3}\right), \tag{4.16a}$$

$$\sigma_3^3 = \frac{\dot a_3}{a_3} - \frac{1}{3}\vartheta = -\frac{2}{3}\left(\frac{\dot a_1}{a_1} - \frac{\dot a_3}{a_3}\right) \tag{4.16b}$$

one finds the expression for shear

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{3}\left(\frac{\dot a_i}{a_i}\right)^2 - \frac{1}{3}\vartheta^2\right] = \frac{1}{3}\left(\frac{\dot a_1}{a_1} - \frac{\dot a_3}{a_3}\right)^2. \tag{4.17}$$

The anisotropic parameter in this case has the form

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i}{H} - 1\right)^2 = \frac{1}{3H^2}\left[2\left(\frac{\dot a_1}{a_1}\right)^2 + \left(\frac{\dot a_3}{a_3}\right)^2\right] - 1, \tag{4.18}$$

where $H = \frac{1}{3}\left(2\frac{\dot a_1}{a_1} + \frac{\dot a_3}{a_3}\right) = \frac{1}{3}\frac{\dot V}{V}$. Further from (3.4) we find that

$$\frac{\dot a_1}{a_1} = \frac{1}{3}\frac{\dot V}{V} + \frac{X_1}{V}, \quad \frac{\dot a_3}{a_3} = \frac{1}{3}\frac{\dot V}{V} - 2\frac{X_1}{V}. \tag{4.19}$$

Now inserting (4.19) into (4.16) – (4.18) we finally find

$$\sigma_1^1 = \sigma_2^2 = \frac{X_1}{V}, \quad \sigma_3^3 = -2\frac{X_1}{V}, \tag{4.20}$$

$$\sigma^2 = 3\frac{X_1^2}{V^2}, \tag{4.21}$$

$$A_m = 18\frac{X_1^2}{\dot{V}^2}. \tag{4.22}$$

As was shown in (4.8) and (4.9) in the case of a positive λ_2 that generates late time acceleration, both V and $\dot{V}$ become large with the expansion of the Universe leading to $\sigma_i^i \to 0$, $\sigma^2 \to 0$ and $A_m \to 0$. This corresponds to our earlier conclusion regarding isotropization. As far as negative λ_2 is concerned, in this case the model gives rise to an oscillatory mode of expansion. In this case we have both local minima and maxima. The maximum (minimum) value of volume scale V depends on the parameters and the initial condition and may be as large as possible. Hence in case of a negative λ_2 though it is possible to attain a solution such that $\sigma_i^i|_{V=V_{\rm max}} \to 0$ and $\sigma^2|_{V=V_{\rm max}} \to 0$, but at the same time we have $A_m|_{V=V_{\rm max}} \to \infty$ because at any space-time point where $V = V_{\rm max(min)}$ we have $\dot{V}|_{V=V_{\rm max(min)}} = 0$. This means that at any space-time point where evolution changes its direction (expansion to contraction and vice versa) the Universe becomes highly anisotropic.

V. CONCLUSION

Within the scope of the LRS BI cosmological model we studied the role of the spinor field in the evolution of the Universe. The reason for considering the LRS BI model lies in the fact that in the case of a full BI model the non-diagonal components of the energy-momentum tensor of the spinor fields imposes severe restrictions on the components of the spinor field, resulting in vanishing scalar $S = \bar{\psi}\psi$ and pseudoscalar $P = \imath\bar{\psi}\gamma^5\psi$ [21, 22]. As a result both the mass term and nonlinear term in the Lagrangian disappear. But, as was shown here, in the case of an LRS BI cosmological model, neither the mass term nor the nonlinear term vanish. Moreover, unlike the Bianchi type-VI model the present model leads to asymptotic isotropization. It is was also found that, depending on the sign of the self-coupling constant, the model allows either the accelerated mode of expansion or the oscillatory mode of evolution.

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