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The Validity of the Cosmic String Pattern Search with the Cosmic Microwave Background

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ABSTRACT

We introduce a new technique to detect the discrete temperature steps that cosmic strings might have left in the cosmic microwave background (CMB) anisotropy map. The technique provides a validity test on the pattern search of cosmic strings that could serve as the groundwork for future pattern searches. The detecting power of the technique is only constrained by two unavoidable features of CMB data: (1) the finite pixelization of the sky map and (2) the Gaussian fluctuation from instrumental noise and primordial anisotropy. We set the upper limit on the cosmic string parameter as $G\mu \lesssim 3.7 \times 10^{-6}$ at the 95% confidence level (CL) and find that the amplitude of the temperature step has to be greater than $44\mu K$ in order to be detected for the *Wilkinson Microwave Anisotropy Probe (WMAP)* 3 year data.

Subject headings: CMB anisotropy, cosmic strings

1. Introduction and Modeling

Cosmic string is one of the relic structures that are predicted to be produced in the course of symmetry breaking in the hot, early universe, whose discovery will probably be an important landmark for the high energy physics. The quest for cosmic strings has been conducted in two ways: statistical method and direct search for individual cosmic string. On the statistical side, many studies agreed that the contribution from cosmic strings to statistical observables such as the power spectrum is at most 10% (Pogosian et al. 2003; Wu 2005; Fraisse 2005;

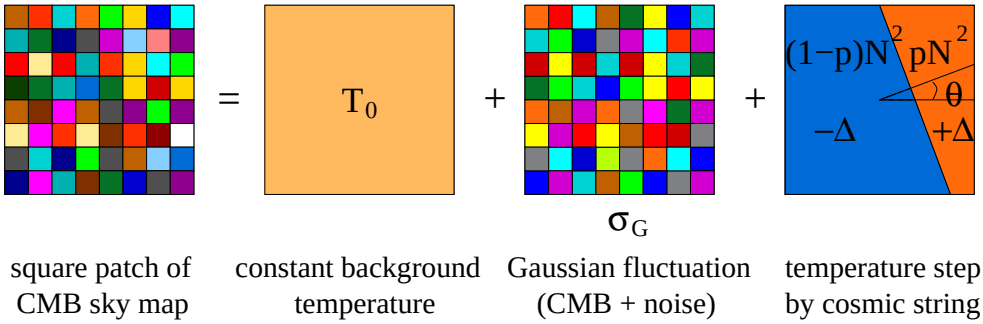

Fig. 1.— Decomposition of a CMB anisotropy sky map patch. A square patch is decomposed into the uniform background temperature (T_0) + Gaussian fluctuation (σ_G)+ discrete temperature step ($\pm\Delta$). The number of pixels in a patch is N^2.

Davis & Kibble 2005; Wyman et al. 2005), reconfirming that cosmic strings played a minor role, if any, in making the universe. Other workers have set upper limits on the cosmic string parameter $G\mu$ (Hindmarsh 1994; Perivolaropoulos & Simatos 1998; Jeong & Smoot 2005; Lo & Wright 2005; Seljak & Slosar 2006; Pogosian et al. 2006; Fraisse 2006). In this Letter, we introduce a technique for estimating how strong a signal from cosmic strings has to be in order to be identified unequivocally. The detecting power of this technique is calibrated by applying it to simulated anisotropy maps based on reasonable modeling. In the last part of this Letter, we apply this technique to the *WMAP* 3 year W-band data set[1] and analyze the implication of the results. A cosmic string can leave discrete temperature steps in a CMB anisotropy map due to the Kaiser-Stebbins effect (Kaiser & Stebbins 1984), with the height of a step δT given by

$$\delta T = 8\pi G\mu\gamma_s\beta_s T\hat{n}\cdot(\hat{v}\times\hat{s}) \tag{1}$$

where $T \simeq 2.725K$ is the universal background temperature of CMB. However, those step patterns are probably obscured by instrumental noises and other physical structures of anisotropies. We consider a square patch of the CMB anisotropy sky map that extends to a size of the horizon at the time of recombination so that the conic spacetime formed by a cosmic string can be in effect within the region. Then another square patch of the CMB anisotropy sky map that contains a segment of moving cosmic string can be decomposed into three parts, as illustrated in Figure 1: (1) the uniform background temperature (T_0), (2) the Gaussian fluctuation (variance σ_G^2), and (3) the discrete temperature step ($\pm\Delta$) from a moving cosmic string. The uniform background temperature T_0 comes from superhorizon-scale primordial fluctuations. Since we pick a horizon-sized region, the super-horizon fluctuations will appear to be a constant temperature shift for the whole patch. Smaller scale (sub-horizon) fluctuations and instrumental noise add up incoherently to form a Gaussian fluctuation with the variance $\sigma_G^2 = \sigma_{CMB}^2 + \sigma_{noise}^2$, where σ_{CMB}^2 and σ_{noise}^2 are

[1]See http://lambda.gsfc.nasa.gov/product/map/

variances for fluctuations of primordial origin and instrumental noise, respectively. We introduce five parameters that characterize a square patch of a string-embedded sky map, T_0, σ_G, Δ, p (blueshifted pixels/total pixels), and θ (orientation of step). We concoct a simulated patch of the CMB anisotropy map by assigning arbitrary legitimate values to the parameters and adding the three components illustrated in Figure 1. To recover these parameters from the CMB anisotropy map where the step pattern is intermixed, we employ five observables (p, Δ, θ, σ_G, T_0) that can be expressed in terms of

$$\text{mean} \quad : \quad \tau = N^{-2} \sum_i T_i \tag{2}$$

$$\text{dipole moment} \quad : \quad \mathbf{d} = N^{-3} \sum_i T_i \mathbf{r}_i \tag{3}$$

$$\lambda - \text{inertia} \quad : \quad \lambda = 4N^{-3} \sum_i T_i \left|\mathbf{d} \cdot \mathbf{r}_i\right| / \left|\mathbf{d}\right| \tag{4}$$

$$\text{variance} \quad : \quad \sigma^2 = N^{-2} \sum_i \left(T_i - \tau\right)^2 \tag{5}$$

where T_i represents the signal at the ith pixel and $\mathbf{r}_i$ is the two-dimensional gridded position vector for the ith pixel defined in a square patch. In the absence of the background fluctuation ($\sigma_G = 0$), the following analytic expressions for the characteristic parameters return the exact input values:

$$p = \frac{2d - \frac{1}{2}\left(|\lambda - \tau| + \lambda - \tau\right)}{4d - |\lambda - \tau|} \tag{6}$$

$$\Delta = \frac{\left(|\lambda - \tau| - 4d\right)^2}{2\left(2d - |\lambda - \tau|\right)} \tag{7}$$

$$\theta = \tan^{-1}\left(d_y/d_x\right) \tag{8}$$

$$\sigma_G = \left(\sigma^2 - 4\Delta^2 p\left(1-p\right)\right)^{1/2} \tag{9}$$

$$T_0 = \tau - \Delta\left(2p - 1\right). \tag{10}$$

With the background fluctuation turned on, an observed characteristic parameter $\mathcal{Q}_{i,obs}$ is expressed as an unbiased estimate $\mathcal{Q}_i$ plus a Gaussian error $\sigma_{\mathcal{Q}_i}$ ($\mathcal{Q}_i = p$, Δ, θ, σ_G, T_0),

$$\mathcal{Q}_{i,obs} = \mathcal{Q}_i \pm \sigma_{\mathcal{Q}_i}. \tag{11}$$

The errors $(\sigma_p, \sigma_\Delta, \sigma_\theta, \sigma_{\sigma_G}, \sigma_{T_0})$ present in the equations (11) are measures of how precisely the information on a temperature step screened by background fluctuation is recovered. Figure 2 shows the behaviors of errors as functions of Δ for three different pixel numbers for a patch. The standard deviation of the background fluctuation chosen here is the mean standard deviation of patches with angular radius $\theta_R = 1.8°$ (the angular size of the horizon at the recombination for the ΛCDM model) for the WMAP 3 year W-band data set with the Kp2 mask applied. The most pronounced feature of the graphs is that the error bars are wildly undulating for small Δ compared to σ_G, with attenuating envelopes as Δ increases. The algorithm works poorly for low Δ compared to σ_G, such as the $\pm 90°$ error on the orientation estimate. Even the breakdowns of the algorithm do happen

Parameter	Input	Output (1 σ)
Patch with 8×8 pixels		
p	0.5	0.5 ± 0.1
Δ (μK)	80.0	83.4 ± 16.6
θ (deg)	0.0	-0.1 ± 12.3
$\sigma_G(\mu$K)	116.0	113.0 ± 12.0
$T_0(\mu$K)	10.0	9.7 ± 18.9
Patch with 14×14 pixels		
p	0.5	0.5 ± 0.1
Δ (μK)	45.0	47.1 ± 9.5
θ (deg)	0.0	-0.2 ± 12.6
$\sigma_G(\mu$K)	116.0	115.0 ± 6.2
$T_0(\mu$K)	10.0	9.6 ± 10.9
Patch with 28×28 pixels		
p	0.5	0.5 ± 0.1
Δ (μK)	25.0	25.8 ± 4.7
θ (deg)	0.0	0.2 ± 11.2
$\sigma_G(\mu$K)	116.0	115.7 ± 3.0
$T_0(\mu$K)	10.0	9.9 ± 5.3

Table 1: Simulation results at the critical values of Δ above which σ_Δs are small enough and do not decrease any more.

in the shaded region, resulting in erroneous parameter values such as negative Δ or p not in the range between 0 and 1. Simulations with collapsed results are not included in the statistics shown in Figure 2 since those cases are evidently the ones with insufficiently strong signals against background fluctuation. As Δ increases, we begin obtaining the computed parameters that are very close to the true values with acceptable errors (dubbed the "good" results), and it allows us to recover the temperature step parameters faithfully. We also find from Figure 2 that a patch with more pixels works better with a faster attenuation of uncertainties. Table 1 displays the performances of the algorithm for the values of Δ at the borders, above which errors for the temperature step (σ_Δ) do not get any better. We use circular patches for *WMAP* data analysis rather than square patches because of the computational advantage. Algebraic expressions for the observables given in equations (6)-(10) work very well for circular patches with negligible numerical differences for the case of square patches. A square patch with 28×28 pixels at the normal resolution of *WMAP* W-band data covers nearly the same area as the circular region with angular radius $\theta_R = 1.8°$. We performed a further detecting power test with 28×28 pixels patches and found empirically that the relation between the critical value of Δ, Δ_c, above which "good" results start to come out, and

σ_G is

$$\Delta_c \simeq 0.25\sigma_G. \tag{12}$$

The different choices of input values for the parameters p, θ, or T_0 showed no noticeable difference in the evolutions of errors.

2. Application to *WMAP*

An observed CMB anisotropy map is an aggregate of various independent modes of perturbation ranging from tiny sub-horizon scales to super-horizon scales well beyond the correlation length. As illustrated in Figure 1, fluctuations with larger or smaller scales compared to the size of a test patch are neatly prescribed, but the intermediate scales whose wavelengths are comparable to the size of a test patch will appear to be continuous temperature tilts that also give plausible values of the step parameters. One drawback of this algorithm is that it does not distinguish between a discrete temperature step and a smooth temperature slope. However, this shortcoming can be easily fixed: If an apparent temperature step is detected at a spot on the map, we repeat the analysis with a half-sized patch at the same spot. If the structure is a continuous slope, then it would return half the value of Δ_{obs} than the previous result, while for the signal from a discrete step, the returned Δ_{obs} should stay the same within the error. We conducted the step signal search through the *WMAP* 3 year W-band data set and found 193,160 unqualified signals against background fluctuation (the constraint in eq. [12] is not applied), 129,049 qualified steps+tilts (signals that meet the constraint eq. [12]; i.e., $\Delta_{obs} \geq 0.25\sigma_G$), and 12,330 qualified discriminated steps. The Δ_{obs} value of qualified steps is in the range

$$(18.3 \pm 3.0)\mu K < \Delta_{obs} < (115.4 \pm 6.0)\mu K \tag{13}$$

as the curves show in Figure 3. Patches with radius $\theta_R = 1.8^\circ$ in the *WMAP* 3 year W-band have a background fluctuation σ_G less than 176 μK in the Kp2-mask-cleared region. This means, at the worst case, we can identify a step signal as low as 176 μK/4 = 44 μK with moderate errors. Therefore, if there are cosmic string signals with $\Delta \geq 115.4$ μK and if they are located in the available region (out of the Galactic plane or the Kp2 masked region), they would not be missed. Thus, we can set an upper limit on the cosmic string signal

$$\Delta = 4\pi G\mu\gamma_s\beta_s T|\cos\phi| < 127.4\mu K, \quad 95\%\ \mathrm{CL} \tag{14}$$

where ϕ is arbitrary. Thus, with $\langle\gamma_s\beta_s\rangle \simeq 1$ (Vilenkin & Shellard 1994), the upper limit of the cosmic string parameter $G\mu$ can be estimated as

$$G\mu \lesssim 3.7 \times 10^{-6}, \quad 95\%\ \mathrm{CL}. \tag{15}$$

3. CONCLUSIONS

We have presented and tested a new technique to directly discover cosmic strings via the patterns they would produce in a CMB anisotropy map. We found that the minimum magnitude of the step signal that is required to be unequivocally identified is 44 μK for the *WMAP* 3 year W-band data set. This algorithm can be used to crop the reliable step signals from the CMB anisotropy data, and it will serve as the valuable ground work for future pattern searches with more refined data, such as further *WMAP* data releases or *PLANCK* data.

Computer simulations and data analysis with the *WMAP* data set were done using HEALPix[2] (Górski et al. 2005). This work was supported by LBNL and the Department of Physics at the University of California, Berkeley.

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[2]See http://healpix.jpl.nasa.gov

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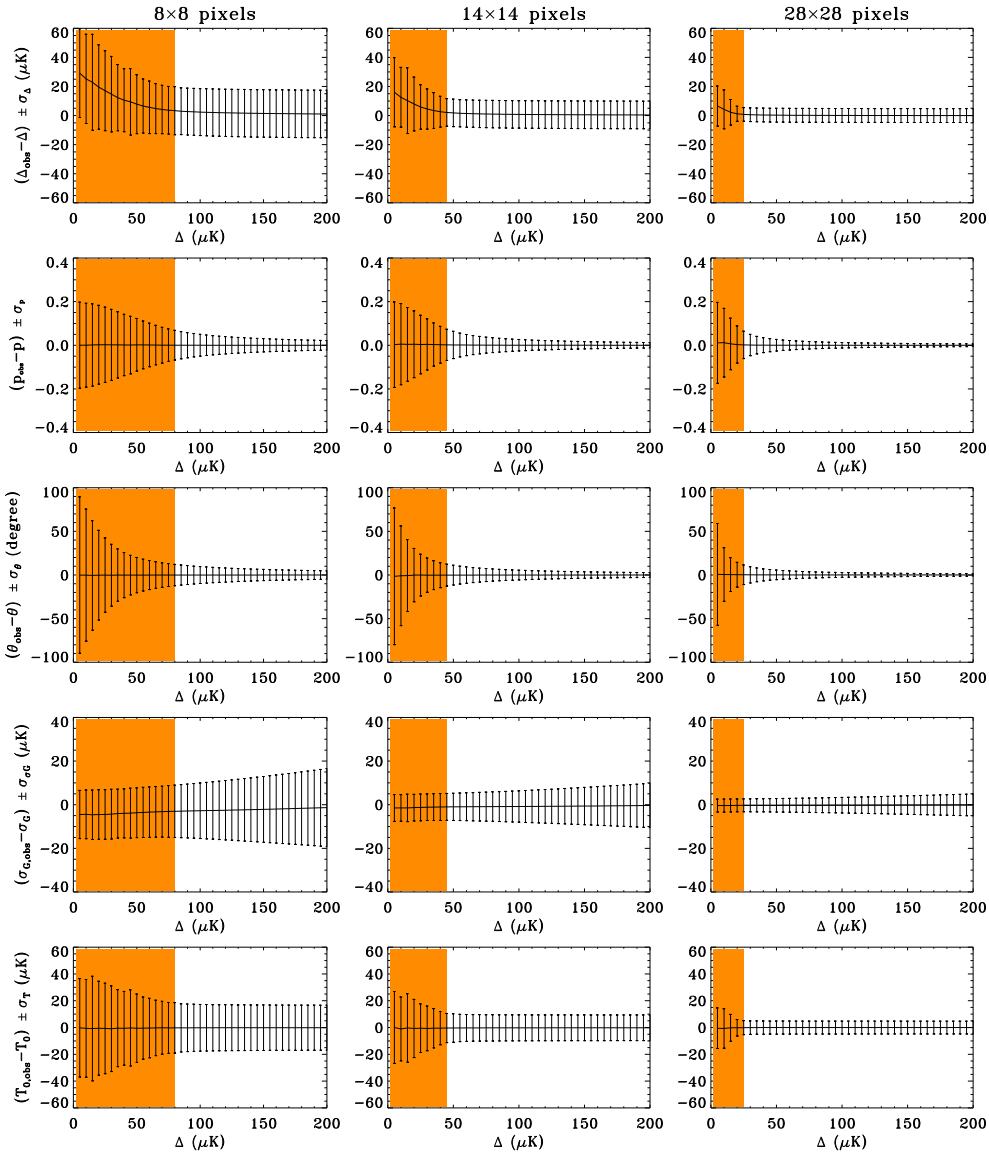

Fig. 2.— Evolution of errors of characteristic parameters as functions of Δ with input values of parameters ($T_0 = 10.0\mu K$, $p = 0.5$, $\theta = 0$, $\sigma_G = 116.0\mu K$). *Left panels*: 8×8 pixels in a square. *Middle panels*: 14×14 pixels. *Right panels*: 28×28 pixels. For each size of patch, the number of simulations performed is 10,000. The shaded regions indicate the ranges of Δ for which the step feature in a patch is not obvious compared to its background fluctuation ($\sigma_G = 116.0$ μK). The result on Δ_{obs} is biased upward because the simulations with erroneous values of Δ_{obs} ($\Delta_{obs} < 0$) were not included in the statistics.

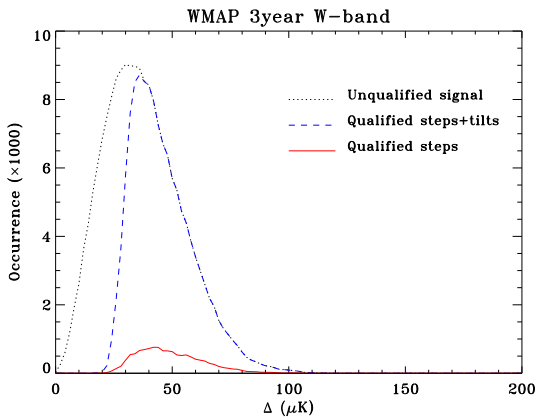

Fig. 3.— Distributions of Δ_{obs} detected in *WMAP* 3 year W-band data set. The number of step signals found are 193,160 unqualified signals (*dotted curve*), 129,049 qualified steps+tilts (*dashed curve*), and 12,330 qualified steps (*solid curve*).