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Charged Boson Stars and Vacuum Instabilities

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Abstract

We consider charged boson stars and study their effect on the structure of the vacuum. For very compact particle like "stars", with constituent mass m_* close to the Planck mass m_{Pl}, i.e. $m_*^2 = \mathcal{O}(\alpha m_{Pl}^2)$, we argue that there is a limiting total electric charge Z_c, which, primarily, is due to the formation of a pion condensate ($Z_c \simeq 0.5\alpha^{-1}e$, where α is the fine structure constant and e is the electric charge of the positron). If the charge of the "star" is larger than Z_c we find numerical evidence for a complete screening indicating a limiting charge for a very compact object. There is also a less efficient competing charge screening mechanism due to spontaneous electron-positron pair creation in which case $Z_c \simeq \alpha^{-1}e$. Astrophysical and cosmological abundances of charged compact boson stars are briefly discussed in terms of dark matter.

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1 Introduction

Compact objects play an important role in current astrophysical research. White dwarfs and neutron stars are examples of objects which involve physics on scales down to the one of nuclei and even of elementary particles. The recent developments in particle physics and cosmology suggest that scalar fields may have played an important role in the evolution of the early universe, for instance in primordial phase transitions, and that they may make up part of the dark matter (for a recent account see e.g. Ref.[1]). These facts motivated the study of gravitational equilibrium solutions of scalar fields, in particular for massive complex fields, which form so-called boson stars [2]. Recently, compact objects made of charged bosons have been considered and static spherically symmetric solutions were found [3]. For some charged boson star models their radius is extremely small, in which case a large electric charge can substantially modify the structure of the vacuum.

In the presence of extended heavy nuclei the perturbative vacuum of QED is unstable if the number of charges Z is larger than a certain critical value Z_c. For $Z > Z_c$ there is spontaneous production of electron-positron pairs [4] . For a positively (negatively) supercritically charged and sufficiently compact object, as compared e.g. with the Compton wavelength of the electron, pair-production is continued until the created electrons (positrons) shield the nucleus to an effective charge $Z_{eff} \approx Z_c$ accompanied by the emission of positrons (electrons) (for a review and detailed references on the subject see e.g. [5]). For a point-like charge it is, of course, well-known that $Z\alpha > 1$ makes the Dirac Hamiltonian non-selfadjoint and the energy eigenvalues become complex. For *extended* atomic nuclei the limiting charge has been estimated to be close to $Z_c = 173$. It has been argued that if the size of the extended object tends to zero Z_c approaches $1/\alpha$ [6].

For bosons and the Klein-Gordon equation similar conclusions hold, however with a limiting value $Z\alpha \geq 0.5$, the equality being valid for the point-limit case (see also in this context [7]). As the charge of the source becomes larger than the critical value Z_c pairs of particles antiparticles (pions) will be produced. The antiparticles (assumed to have the same sign for their charge as the source) will be emitted at infinity, whereas the

particles will be tightly bound to the nucleus. Due to Bose statistics a condensate with arbitrary many particles could be formed. However, one also has to take into account the mutual repulsive Coulomb interaction between the particles in the condensate. To add new particles cost a certain amount of energy and therefore limits their number in the condensate, which will screen the overcritical charge of the source [8].

In the present paper we confront these ideas about the existence of a limiting charge due to the instability of the vacuum to the charged boson stars. This might also be relevant for the stability of the charged boson stars, which has up to now been only investigated in terms of classical concepts [9]. A basic assumption is the existence of stable and superheavy charged scalar particles with mass $m_*^2 = \mathcal{O}(\alpha m_{Pl}^2)$, which could naturally appear owing to quantum fluctuations in the very early phases of the universe, when its density was close to $\rho \sim c^5/(G_N^2\hbar) = m_{Pl}^4$ [10] (in natural units $\hbar = c = 1$ which we will use from now on). It has been pointed out that the existence of very heavy particles, called maximons in Ref. [10], would unavoidably lead to strong violation of thermodynamic equilibrium in the very early universe [11]. A fact which is of importance for the generation of a baryon asymmetry [11]. If such fundamental particles exist, they may form new compact structures, i.e. boson stars.

We will discuss two limiting cases of the boson stars. In the case of a compact boson star, as compared to the Compton wavelength of the electron, these "stars" may have a typical size comparable to the Planck length (!) and a mass of the order of Planck mass or more. We will argue that these very compact objects , which we shall still call boson "stars", have a limiting total charge close to $0.5/\alpha$. The time-scale τ of charge screening due to the instability of the vacuum can be estimated to be of the same order as for supercritical atomic nuclei, i.e. $\tau \leq \mathcal{O}(10^{-19})\ s$ [5]. Expressed in terms of the parameter m_* one may say $m_* >> (m_*)_{cr}$, where $(m_*)_{cr} = m_{Pl}\sqrt{\alpha}$. In the other limiting case m_* will be very close to $(m_*)_{cr}$. Under such a condition the star might even have a macroscopic size and the screening mechanism discussed above will be no longer efficient. These considerations involve physics both at the Planck scale as well as at the scale of the electron or pion mass. The effective fine-structure constant will then vary over this

range of energy. This will, however, not change the qualitative picture and thus we will neglect such quantum effects.

In the effective-potential approach to quantum resonances in stationary geometries [12], like the Kerr-Newman geometry, the crossing of positive- and negative- root classical solutions of the equations of motion for a test particle with mass m signals particle-antiparticle production through the Klein process [13]. The gap between particle and antiparticles states, as described by the Dirac equation, is narrowed in the gravitational field of a collapsed star, such as neutron stars. When the star collapses to a black hole, the gap shrinks to zero at the Schwarzschild radius and the vacuum becomes unstable due to quantum tunneling [14] in analogy with the particle production mechanism in strong electric fields [15]. Similarly one is led to a limiting charge-to-mass ratio for a charged black-hole [14]. For the extended charged boson stars, which we consider here, the vacuum instability has a different character. In the effective-potential language we will, e.g., see that classical positive- and negative-root solutions will cross without a tunneling barrier unless the mass scale m is much less than m_*. Since the effective-potential language has been considered in quite a detail in this context, we will present some of these issues in the present paper.

It has been suggested that dark matter [16] may be composed of charged massive particles, so called CHAMPs [17]. If the mass of a CHAMP is of the order of Planck mass m_{Pl}, like *pyrgons* [18] or *maximons* [10], one may wonder if gravitational binding effects may be important. As we will argue, the compact boson "stars" can be considered to be a self-consistent model for such superheavy CHAMPs gravitationally bound and classically stable particle-like objects.

The present paper should be regarded as preliminary in confronting very compact charged boson stars with quantum mechanics. Since these objects may have a size comparable to the Planck length it is not clear what are the effects of quantum gravity. We do not, however, see a compelling reason why one should not pursue a study along the lines indicated above for these objects, as a first step towards a more detailed understanding of the physics of these objects. In section 2 we describe the basic features of charged boson

stars. Various scenarios of vacuum instabilities of very compact charged boson "stars" are discussed in section 3. The effective potentials for fermions (bosons), i.e. $V_{eff}^{D\pm}$ ($V_{eff}^{K\pm}$), are derived in section 3.1 (section 3.2), where we also present a numerical evaluation of them in the background fields of the compact "star". We also confront the physical picture as derived from these effective potentials with the energy spectra directly derived from the Dirac or Klein-Gordon equation in the external fields of the "star". In section 3.2 we study in some detail the formation of a charged bose condensate close to the compact "star" and present the numerical evaluation of the coupled non-linear equations of the bose condensate field and the electromagnetic field in the background fields of the "star". In the final section we discuss primordial cosmological production of charged compact "stars" by making use of conventional kinetic equations. We argue that a background of such CHAMPs may constitute at least a fraction of the dark matter of our universe.

2 Compact Charged Boson Stars

The equations which describe the charged boson star are obtained from the following action of a charged, massive boson field ϕ coupled to gravity and a $U(1)$ gauge field, i.e.

$$S = \int d^4x\sqrt{-g}\left[-\frac{R}{16\pi G_N} + g^{\mu\nu}(D_\mu\phi)^*(D_\nu\phi) - m_*^2|\phi|^2 - \frac{\lambda}{2}|\phi|^4 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right] \quad , \tag{2.1}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad , \tag{2.2}$$

and

$$D_\mu\phi = \partial_\mu\phi + ieA_\mu\phi \quad , \tag{2.3}$$

and $e > 0$ is the electric charge of the positron. If the $U(1)$-gauge symmetry is identified with the electromagnetic field, $\alpha \equiv e^2/4\pi \approx 1/137$ is the fine structure constant. We will consider the case $\lambda = 0$, but our results do not crucially depend on the self-coupling term. To the Lagrange density of Eq. (2.1) one may add a conformal coupling $R|\phi|^2/6$, where R is the scalar curvature, in order to preserve conformal symmetry in the limit $m_* \to 0$ [19]. The inclusion of this conformal coupling has the effect of "renormalizing"

the $|\phi|^4$-coupling but does not otherwise change the equations of motion [19, 20]. We therefore choose to disregard it.

A static spherically symmetric and classically stable solution of the coupled non-linear differential equations, as derived from the action Eq. (2.1), exists only if the gravitational attraction is larger than the Coulomb repulsion, i.e. if $\alpha < e^2_{crit}$, where $e^2_{crit} = G_N m_*^2$. The space-time metric is of Schwarzschild form

$$ds = B(r)dt^2 - A(r)dr^2 - r^2(d\theta^2 + sin^2\theta d\varphi^2) \quad , \tag{2.4}$$

and, furthermore,

$$A_\mu = (C(r), 0, 0, 0) \quad , \quad \phi(r,t) = \phi_0(r)e^{-i\omega t} \quad . \tag{2.5}$$

The eigenvalue ω corresponds physically to the energy of the last charged constituent particle added to the star. The non-linear coupled differential equations for $A(r), B(r),\ C(r)$ and $\phi_0(r)$, to be discussed in more detail in section 3.2, are solved numerically with the boundary conditions

$$A(0) = 1 \quad , \quad B(\infty) = 0 \quad , \quad dC(0)/dr = 0 \quad , \quad C(\infty) = 0 \quad , \tag{2.6}$$

and

$$\phi_0(0) = \text{constant} \quad , \quad d\phi_0(0)/dr = 0 \quad , \quad \phi_0(\infty) = d\phi_0(\infty)/dr = 0 \quad . \tag{2.7}$$

These boundary conditions describe a localized object. As discussed in Ref.[3], the solution is most conveniently described in terms of the rescaled variables, i.e.

$$\begin{aligned}
r &\rightarrow \tilde{r} = m_* r \quad , \\
\phi_0 &\rightarrow \tilde{\phi}_0 = (8\pi G_N)^{1/2}\phi_0 \quad , \\
C &\rightarrow \tilde{C} = (\omega - eC) \quad , \\
B &\rightarrow \tilde{B} = m_*^2 B \quad , \\
e^2 &\rightarrow \tilde{e}^2 = \frac{e^2 m_{Pl}^2}{8\pi m_*^2} = \frac{\alpha}{2}\frac{m_{Pl}^2}{m_*^2} \quad , \\
\lambda &\rightarrow \tilde{\lambda} = \frac{\lambda m_{Pl}^2}{8\pi m_*^2} \quad .
\end{aligned} \tag{2.8}$$

In terms of the total mass M and the particle number N (or, equivalently, the total charge $Q = eN$), the static solution has the asymptotic Reissner-Nordstrøm form, i.e.

$$A(\tilde{r}) = (1 - \frac{2\tilde{M}}{\tilde{r}} + \frac{2\tilde{N}^2\tilde{e}^2}{\tilde{r}^2})^{-1} \quad , \tag{2.9}$$

where $\tilde{M} = Mm_*/m_{Pl}^2$ and $\tilde{N} = Nm_*^2/m_{Pl}^2$. The radial component of the electric field has the asymptotic form

$$E(r) = -dC(r)/dr \approx Q/4\pi r^2 \quad . \tag{2.10}$$

The gravitational attraction overcomes the Coulomb repulsion if $\tilde{e}^2 < 0.5$, i.e. $m_*^2 > \alpha m_{Pl}^2$. In Fig.1 we exhibit a generic numerical solution for the coupled non-linear equations of the metric functions $A, \tilde{B}$, the scalar field $\tilde{\phi}_0$ and the radial component of the electric field. We notice that close to the critical charge $\tilde{e}_c^2 = 0.5$, or equivalently $m_* = (m_*)_{cr}$, we obtain for a solution without nodes that [3] the following maximal values of $\tilde{N}, \tilde{M}$ and $\tilde{R}$:

$$\begin{aligned} \tilde{N}_{max} &\approx \tilde{M}_{max} \approx 0.44 \cdot (\tilde{e}_c - \tilde{e})^{-1/2} \quad , \\ \tilde{R}_{max} &\approx 1.5 \cdot (\tilde{e}_c - \tilde{e})^{-1/2} \end{aligned} \tag{2.11}$$

for $\tilde{\phi}_0(0)$ such that

$$\tilde{\phi}_0(0) = \tilde{\phi}_c(0) \approx 0.0067 \cdot (\tilde{e}_c - \tilde{e})^{1/2} \quad . \tag{2.12}$$

Here $\tilde{R} = Rm_*$ is the rescaled mean radius R of the boson star, which is defined as follows

$$R = \frac{1}{eN} \int d^3x r J^0 \quad , \tag{2.13}$$

where

$$J^\mu = \sqrt{-g} g^{\mu\nu} \{ie[\phi^* \partial_\nu \phi - \phi \partial_\nu \phi^*] - 2e^2 A_\nu |\phi|^2\} \tag{2.14}$$

is the conserved electromagnetic current. If $\lambda \neq 0$ we obtain the following maximal values of $\tilde{M}$ and $\tilde{R}$:

$$\begin{aligned} \tilde{N}_{max} &\approx \tilde{M}_{max} \approx 0.226 \cdot (\tilde{e}_c - \tilde{e})^{-1/2} \frac{m_{Pl}}{m_*} \sqrt{\frac{\lambda}{8\pi}} \quad , \\ \tilde{R}_{max} &\approx 0.415 \cdot (\tilde{e}_c - \tilde{e})^{-1/2} \frac{m_{Pl}}{m_*} \sqrt{\frac{\lambda}{8\pi}} \end{aligned} \tag{2.15}$$

for $\tilde{\phi}_0(0)$ such that

$$\tilde{\phi}_0(0) = \tilde{\phi}_c(0) \approx 2.43 \cdot (\tilde{e}_c - \tilde{e})^{1/2} \quad . \tag{2.16}$$

We see that the scalar self-coupling does not change the overall picture very much, as long as $m_* \sim \mathcal{O}(m_{Pl})$. The mass-scale m_* is therefore essentially our only free parameter, if the $U(1)$-gauge symmetry is identified with the electromagnetic field.

The dynamical stability of spherically symmetric charged boson stars has been discussed in Ref.[9], where also the pulsation equation, which determines the normal modes of the radial oscillations, has been derived. The particle number N and the mass M, as a function e.g. of $\tilde{\phi}_0(0)$ or equivalently the central density, have their extrema, in particular their maximum, at the same value of $\tilde{\phi}_0(0)$. From this fact it follows that the pulsation equation has a zero mode, where N and M have their extrema. It has been shown that for the equilibrium solutions with a value of $\tilde{\phi}_0(0)$ bigger than a certain critical value $\tilde{\phi}_c(0)$, corresponding to the maximum mass, the pulsation equation has a negative mode. Therefore, these configurations are dynamically unstable. On the other hand for $\tilde{\phi}_0(0)$ less than $\tilde{\phi}_c(0)$ the equilibrium solutions are stable. Of course these results are based on a purely classical treatment of the stability analysis. They do not take into account e.g. quantum effects, such as tunneling among different configurations with the emission of particles or particle production in the strong electromagnetic and gravitational field which are generated by the bose stars.

3 Decay of the Vacuum

The properties of the vacuum in the presence of a charged boson star are described by considering the spectrum of the single-particle Dirac and Klein-Gordon equations in the background of both gravitational and electromagnetic fields of the star. We will restrict ourselves to the behaviour of the lowest eigenvalue, corresponding to an $s-$wave. When the bound state energy of the electron dives into the negative-energy continuum the vacuum becomes unstable and pair production occurs [4, 5]. For charged bosons, as

described by the Klein-Gordon equation, the physics is quite different due to the formation of a Bose condensate, see section 3.2 below.

3.1 Vacuum Instabilities due to Fermions

The Dirac equation describing a stationary state of an electron (with electric charge -e) with energy E in the presence of a static background gravitational field and a $U(1)$ gauge potential A_μ, as given by Eqs.(2.4) and (2.5), can, in a straightforward manner, be reduced to the following set of two coupled differential equations

$$\begin{aligned} \frac{d}{dr}f(r) &= \sqrt{A(r)}\left(\frac{\kappa}{r}f(r) + m_e g(r) - \frac{1}{\sqrt{B(r)}}\Big(E + eC(r)\Big)g(r)\right) \quad , \\ \frac{d}{dr}g(r) &= \sqrt{A(r)}\left(m_e f(r) - \frac{\kappa}{r}g(r) + \frac{1}{\sqrt{B(r)}}\Big(E + eC(r)\Big)f(r)\right) \quad , \end{aligned} \tag{3.1}$$

where $\kappa = \pm n$ and n is an integer. We consider the ground state solution for which $\kappa = -1$. The functions f and g are the components of the two-component spinor Ψ, which is defined as follows

$$\Psi(r,t) = \sqrt[4]{\frac{A(r)}{B(r)}}\begin{pmatrix} f(r) \\ g(r) \end{pmatrix} e^{-iEt} \quad . \tag{3.2}$$

Ψ is normalized in such a way that

$$\int_0^\infty dr \Psi^\dagger(r,t)\Psi(r,t) = 1 \quad . \tag{3.3}$$

The boundary conditions correspond to normalizability and regularity at the origin, for which $df(0)/dr = dg(0)/dr = 0$.

It is clear from this equation that the natural length scale for the wavefunctions f and g is the Compton wavelength m_e^{-1}, where m_e is the electron mass. As discussed in the previous section, the mass of the particles forming the charged boson star is typically of order $m_{Pl}\sqrt{\alpha}$ and their total number in the star is $\mathcal{O}(\alpha^{-1})$. For such stars the effective radius is roughly $\mathcal{O}(m_{Pl}^{-1})$, which is much smaller than the Compton wavelength of the electron. This situation is similar to the one we get for pair production in the field of an

extended nucleus, in which case the critical value Z_c above which it occurs is increased with respect to $1/\alpha$. Z_c depends on the radius of the nucleus (see Ref.[6]). For atomic nuclei $Z_c \simeq 173$. In our case the radius of the boson star is several orders of magnitude smaller than the one of atomic nucleii and therefore we expect the value of Z_c practically to coincide with $1/\alpha$. In the pointlike limit, it turns out that for $Z_c = 1/\alpha$ the eigenvalues of the Dirac equation become imaginary. A fact which in our case will, however, not occur due to presence of a finite radius, even if extremely tiny. At the relevant scale, which is the Compton wavelength of the electron, the metric is practically flat and therefore we also do not expect any change in the value of Z_c induced by the background gravitational field.

It has been argued [12, 14] that an effective potential can be used to describe qualitatively the physics. We can derive an effective potential by transforming the Dirac equation (3.1) into a second order differential equation, i.e.

$$\left(\sqrt{\frac{B(r)}{A(r)}}\frac{d}{dr} + \sqrt{B(r)}M(r) + i\sigma_2(E - V(r))\right) \times$$
$$\left(\sqrt{\frac{B(r)}{A(r)}}\frac{d}{dr} - \sqrt{B(r)}M(r) - i\sigma_2(E - V(r))\right)\begin{pmatrix} f(r) \\ g(r) \end{pmatrix} = 0 \quad , \tag{3.4}$$

where the matrix $M(r)$ is given by

$$M(r) = \begin{pmatrix} \frac{\kappa}{r} & m_e \\ m_e & -\frac{\kappa}{r} \end{pmatrix} \quad , \tag{3.5}$$

and the potential $V(r)$ is

$$V(r) = -eC(r) \quad . \tag{3.6}$$

If we neglect derivatives in A, B and V, we can write

$$-\frac{d^2}{d^2 r_*}\begin{pmatrix} f(r) \\ g(r) \end{pmatrix} = (E - V_{eff}^{D+}(r))(E - V_{eff}^{D-}(r))\begin{pmatrix} f(r) \\ g(r) \end{pmatrix} \quad , \tag{3.7}$$

with

$$dr_*/dr = \sqrt{A(r)/B(r)} \quad . \tag{3.8}$$

Eq. (3.7) is in a suitable form for a WKB approximation. Then the effective potential $V_{eff}^{D\pm}$ for the Dirac equation is given by

$$\frac{V_{eff}^{D\pm}(r)}{m_*} = \frac{\tilde{C}(r) - \omega}{m_*} \pm \sqrt{B(r)} \left(\frac{m_e^2}{m_*^2} + \frac{\kappa^2}{\tilde{r}^2} \right)^{1/2} \quad . \tag{3.9}$$

In Fig.2a (2b) we exhibit $V_{eff}^{D\pm}$ as a function of $\tilde{r}$ for a positively charged boson star with $m_e/m_* \simeq 0$ ($m_e/m_* = 1$). If the minimum of V_{eff}^{D+} is less than $-m_e$, pair production occurs for a positively charged boson star. For dynamically stable charged boson equilibrium configurations without nodes, which is the case for $\tilde{\phi}_0(0)$ less than $\tilde{\phi}_c(0)$ corresponding to the maximal mass, we computed V_{eff}^{D+} and checked for which values of $\tilde{e}$ and $\tilde{\phi}_0(0)$ it becomes less than $-m_e$ starting from some value on the r axis. (In practice since $m_e^2/m_*^2 \ll 1$, it is accurate enough to see where V_{eff}^{D+} becomes negative.) The negatively charged boson star can be studied in a similar way.

It turns out that V_{eff}^{D+} has negative values whenever the value for the charge is nearby $Z = 1/\alpha$, and of course for higher values of Z (this can also bee seen by making use of the asymptotic form of Eq. (3.9) with $m_e/m_* \approx 0$, i.e. $V_{eff}^{D+}(r) \approx (-\alpha N + |\kappa|)/\tilde{r}$). Obviously since V_{eff}^{D+} is an approximation the so found value of Z is nearby the expected exact one, which is just slightly above $Z = 1/\alpha$, to within an accuracy of about 10%. In Fig. 3 we plotted together with the curves for the charged boson star equilibrium configuration also the borderline from which on V_{eff}^{D+} just starts having negative values. We see that the dynamical stable equilibrium configurations, which lie above this curve, are unstable against pair production. We expect for them pair production to occur and the *additional* charge above $Z_c = 1/\alpha$ to be completely screened.

Since the size of the radius of the charged boson "stars" is extremely tiny the electrostatic potential becomes very deep, such that also heavier particles besides the electrons will be overcritical, leading thus to their pair production. This effect, by considering a nucleus with shrinking radius, has been studied in ref. [5], where the influence of the heavier leptons, like muons and taus, has been taken into account. It turns out that the overcritical nucleus, with $Z > 1/\alpha$, gets sorrounded by shells of different leptons, but the fact that the charge above $1/\alpha$ gets completely screened remains unaffected. Notice that

to be completely selfconsistent one should also take into account the fact that the heavier leptons are not stable. Similar mechanism of pair production of heavier particles will of course also occur for charged overcritical boson "stars". The net effect remains however that the charge above $Z_c = 1/\alpha$ gets completely screened, as mentioned above, and as long as such particles have not masses of the order of the constituent particles itself we also do not expect important back-reaction effects, which could alter the structure of the star itself.

As an illustrative example we have also computed the spectrum of the Dirac equation for a fermionic particle of mass equal to m_* rather than the mass of the electron m_e. In Fig. 4 we show E/m_* as a function of $\tilde{e}^2$, or equivalently $\alpha m_{Pl}^2/2m_*^2$, for various values of $\tilde{\phi}_0(0)$ for the $1s_{1/2}$ state ($\kappa = -1$). We see that in this case E decreases by increasing $\tilde{e}$ and can even become negative. However, not as much as $E/m_* = -1$ and therefore pair production *does not* occur. This because the mass m_* is too heavy. For this case the presence of gravity plays an important role, since now the typical length scale of the wavefunctions f and g is of order $1/m_*$, a distance at which the metric is still far from being flat. The solution of the Dirac equation in the external fields of the compact charged boson "star" leads to a physical picture in agreement with the one obtained by making use of the effective potential $V_{eff}^{D\pm}$.

3.2 Vacuum Instabilities due to Bosons

One may also consider pair production of charged bose particles, like for instance pions. For atomic nuclei at normal nuclear density the critical value Z_c above which pion pair production occurs is large: $Z_c \sim 3000$ [5]. As for fermions it depends on the radius of the nucleus. It decreases by decreasing the nuclear size. Since the charged boson stars are extremely small the corresponding value Z_c is much smaller and is actually very close to $0.5/\alpha$ [21]. In Fig.5 we illustrate this approach to the critical charge Z_c by considering the ground state energy $E(Z, R)$ of the Klein-Gordon equation in the external field of an extended uniformly charged sphere with radius R (in Ref.[21] one considers for a radius smaller than R a constant potential). Analytic solution can be obtained for $r \leq R$ and

$r \geq R$. These solutions are then numerically linked together at $r = R$ in a standard manner [22]. We notice that $\partial E(Z,R)/\partial Z$ tends to minus infinity as R tends to zero for Z close to Z_c.

Once the lowest bound state of the Klein-Gordon equation dives into the continuum pair production of pions occurs. Due to Bose statistics a condensate of pions is then formed. The number of pions in the condensate can be quite large. It is limited, however, due to the Coulomb repulsion among the pions, such that above a certain critical number it is energetically no more possible to add new pions to the condensate [5].

The presence of such a bose condensate, which is described by the negatively charged scalar field η of the pion or any other similar charged scalar field, can be conveniently incorporated by including an additional term S_{con} to the action Eq. (2.1), i.e.

$$S_{con} = \int d^4x \sqrt{-g} \left[g^{\mu\nu} (D_\mu \eta)^* D_\nu \eta - m^2 |\eta|^2 \right] \quad , \tag{3.10}$$

where now

$$D_\mu = \partial_\mu - ieA_\mu \quad . \tag{3.11}$$

Higher order terms in the field η can be added to this action if there are additional interactions among the η fields as for instance a $\lambda\eta^4$ term [8]. Here, however, for clarity we restrict ourselves to this form. The electromagnetic and gravitational fields couple to both η and ϕ. Variation of the various fields leads now to the following equations of motion, i.e. the two Einstein equations

$$\begin{aligned} \frac{A'}{A^2 r} + \frac{1}{r^2}(1 - \frac{1}{A}) &= 8\pi G_N \left[(\frac{(\omega - eC)^2}{B} + m_*^2)\phi_0^2 + (\frac{(E + eC)^2}{B} + m^2)\eta_0^2 \right. \\ &+ \left. \frac{\phi_0'^2}{A} + \frac{\eta_0'^2}{A} + \frac{C'^2}{2AB} \right] \quad , \end{aligned} \tag{3.12}$$

and

$$\begin{aligned} \frac{B'}{ABr} - \frac{1}{r^2}(1 - \frac{1}{A}) &= 8\pi G_N \left[(\frac{(\omega - eC)^2}{B} - m_*^2)\phi_0^2 + (\frac{(E + eC)^2}{B} - m^2)\eta_0^2 \right. \\ &+ \left. \frac{\phi_0'^2}{A} + \frac{\eta_0'^2}{A} - \frac{C'^2}{2AB} \right] \quad , \end{aligned} \tag{3.13}$$

the Maxwell equation

$$C'' + (\frac{2}{r} - \frac{A'}{2A} - \frac{B'}{2B})C' + 2e\phi_0^2 A(\omega - eC) - (E + eC)2eA\eta_0^2 = 0 \quad , \tag{3.14}$$

and the scalar wave equations for ϕ_0

$$\phi_0'' + (\frac{2}{r} - \frac{A'}{2A} + \frac{B'}{2B})\phi_0' + A(\frac{(\omega - eC)^2}{B} - m_*^2)\phi_0 = 0 \quad , \tag{3.15}$$

where a prime denotes differentiation with respect to r. For the field η we have assumed

$$\eta(r,t) = \eta_0(r)e^{-iEt} \quad , \tag{3.16}$$

and hence we also obtain

$$\eta_0'' + (\frac{2}{r} - \frac{A'}{2A} + \frac{B'}{2B})\eta_0' + A(\frac{(E + eC)^2}{B} - m^2)\eta_0 = 0 \quad . \tag{3.17}$$

In order to solve this Klein-Gordon equation we impose the boundary conditions

$$\eta_0(0) = const \quad , \quad d\eta_0(0)/dr = 0 \quad , \quad \eta_0(\infty) = d\eta_0(\infty)/dr = 0 \quad . \tag{3.18}$$

Similarly to the Dirac case one can also find an effective potential $V_{eff}^{K\pm}$. If we neglect derivatives of the metric functions A and B, the Klein-Gordon equation (3.17) leads to

$$-\frac{d^2u(r)}{d^2r_*} = (E - V_{eff}^{K+}(r))(E - V_{eff}^{K-}(r))u(r) \quad , \tag{3.19}$$

where

$$\eta_0(r) = u(r)/r \quad , \tag{3.20}$$

and dr_* is defined as in eq.(3.8). The effective potential for the Klein-Gordon equation is then given by

$$\frac{V_{eff}^{K\pm}(r)}{m_*} = \frac{\tilde{C}(r) - \omega}{m_*} \pm \sqrt{B(r)}\frac{m}{m_*} \quad , \tag{3.21}$$

where $\tilde{C}(r)$ is defined in Eq.(2.8). We found, however, that it is difficult to numerically determine the critical charge with hig accuracy by making use of this effective potential.

We notice that $V_{eff}^{K\pm} = V_{eff}^{D\pm}|_{\kappa=0}$ [4]. In Fig.6a (6b) we exhibit $V_{eff}^{K\pm}(\tilde{r})$ for a positively charged boson star with $m/m_* = 0.05$ ($m/m_* = 1$). In Fig.6a we see that the bound state of the negatively charged particle dives into the negative continuum, corresponding to anti-particle states, without the presence of a tunneling barrier. If $m/m_* = 1$ there is a finite energy barrier between the spectrum of particle and anti-particle.

We investigate now the effect of a pion condensate on a charged boson star in which case $m^2/m_*^2 = \mathcal{O}(m_\pi^2/\alpha m_{Pl}^2)$, which, of course, is a very small number. In our case the compact object can therefore be considered as a charged point source. In fact the typical length scale of the pion condensate is the Compton wavelength of a pion, i.e. the compact object has a size much smaller than the bose condensate as follows from Eq.(3.17). Therefore, the fields $\phi_0(r)$, $A(r)$ and $B(r)$, as described by the equations of motion Eqs.(3.10)-(3.17), will be very localized as compared to variations of the fields $\eta_0(r)$ and $C(r)$ (see Fig.1a and 1b). It is reasonable to assume that under such circumstances we can neglect the effect of the bose condensate on the compact object itself.

We checked this by solving numerically the full set of Eqs.(3.12)-(3.17) for some values of the ratio m/m_*. It turns out that the back-reaction of the condensate on the boson star itself is small even for relatively large mass ratios. In fact the total mass, particle number and size of the star is very little changed by the presence of the condensate. We illustrate this feature for a dynamically stable charged boson star with $\tilde{e}^2 = 0.25$ and $\tilde{\phi}_0(0) = 0.2$, which corresponds to about $N \approx 66$ particles for the compact object itself when there is no condensate present. If we now include a condensate with a mass ratio $m^2/m_*^2 = 1/1000$, we find that the boson star remains essentially unaffected (the particle number as well as its mass changes only within less than one percent) apart from its electromagnetic properties at large distances. As seen in Fig.7, the charge distribution

[4] The two effective potentials are equal as far as non-relativistic effects are concerned. This can be seen by making use of the the so called Langer modification of the effective potential [23]: we perform the shift $m/m_* \to \sqrt{(m/m_*)^2 + (l+1/2)^2/r^2}$ in $V_{eff}^{K\pm}$, where l is the angular momentum quantum number, and in $V_{eff}^{D\pm}$ the shift $\kappa \to \kappa + 1/2$. V_{eff}^{K+} and V_{eff}^{D+} lead to the correct spectrum in the non-relativistic limit by making use of the WKB-approximation. With the Langer modification V_{eff}^{K+} will change sign for some value of $\tilde{r}$ at $Z = Z_c = 0.5/\alpha$ within an accuracy of about 15% if $m/m_* \ll 1$.

of the condensate is localized mostly outside the compact object and contains $N_c \approx 10$ particles, i.e. at large distances the star, however, appears to have a net charge $Q \approx 56e$. We have verified numerically that the back-reaction of the condensate on the boson star gets even less when m^2/m_*^2 decreases.

We now return to the issue concerning a pion condensate on a charged boson star, in which case m is the mass of the pion. At distances much larger as compared to m_*^{-1} we therefore regard the term in Maxwell's equation

$$\rho_{ext}(r) = 2e\phi_0(r)^2 A(r)(\omega - eC(r)) \tag{3.22}$$

as a localized external charge distribution for which we can put $A(r) \approx 1$. Let C_0 be a solution to

$$-\left(C_0'' + \frac{2}{r}C_0'\right) = \rho_{ext} \quad , \tag{3.23}$$

i.e. at large distances

$$C_0 = \frac{Q}{4\pi r} \quad . \tag{3.24}$$

We now write $C = C_0 + C_b$. At distances which are large compared to m_*^{-1}, Maxwell's equation becomes

$$\left(\tilde{C}_b'' + \frac{2}{r}\tilde{C}_b'\right) = \left(E + eC_0 + eq\tilde{C}_b\right) 2e\tilde{\eta}_0^2 \quad , \tag{3.25}$$

and the Klein-Gordon equation

$$\tilde{\eta}_0'' + \frac{2}{r}\tilde{\eta}_0' + \left((E + eC_0 + eq\tilde{C}_b)^2 - m^2\right)\tilde{\eta}_0 = 0 \quad . \tag{3.26}$$

Here we rescaled the fields as follows:

$$\tilde{C}_b = C_b/q \quad , \quad \tilde{\eta}_0 = \eta_0/\sqrt{q} \quad , \tag{3.27}$$

where q is choosen in such a way that

$$2e^2 \int d^3x \; \tilde{\eta}_0^2 \left(E + eC_0 + eq\tilde{C}_b\right) = 1 \quad . \tag{3.28}$$

The $\tilde{\eta}_0$-field is normalized to a unit charge. The parameter $-q$ corresponds thus to the total electric charge of the condensate. The physical interpretation of these equations is

now clear. The set of eqs (3.26) and (3.28) is analogous to the Thomas-Fermi equation, which describes in a self-consistent way the electron wave functions in the field of a nucleus. The repulsive interaction between the pions is crucial to ensure the stability of the system. Otherwise there would be production of an arbitrarily large number of particle antiparticle pairs.

The eigenvalue E corresponds to the energy of the last particle added to the condensate [5]. A natural choice is $E = -m$, where m is the rest mass of the pion. With this choice $\tilde{\eta}_0$ does no longer decrease exponentially for $r \to \infty$, as can be seen from eq. (3.26). As a result it turns out that the normalization integral, eq. (3.28), is divergent. This fact may indicate that the mean field approximation breaks down, or that one has to take into account other interactions among the pions, as for instance a $\lambda\eta^4$ term.

However, if weak decays of the pions are taken into account the first source of instability is the "inverse β decay" [21]: $N \to (N, \pi^-) + e^+ + \nu_e$. N denotes the "star" (positively charged) and (N, π^-) a π^- bound state in the field of the "star". This process sets in for $Z \geq Z_c$, as soon as the ground state energy of the Klein-Gordon equation is lower than $-m_e$ (the energy of ν_e can be arbitrarily small), rather than $-m$. As we have seen above when the radius of the "star" is sufficiently small compared to the Compton wavelength of an electron or a pion and $Z \geq Z_c$, the ground state energy of the Klein-Gordon equation becomes very rapidly negative, see Fig.5. In this case one can set $E = -m_e$ in eq.(3.26). Then $\tilde{\eta}_0$ falls off exponentially for $r \to \infty$ and thus the integral (3.28) is finite.

We have numerically solved the above Eqs.(3.25)-(3.28) with $E = -m_e$ for the electromagnetic field $\tilde{C}_b$ and the condensate $\tilde{\eta}_0$ using an iterative method as discussed in Ref.[5]. For our purposes it is sufficient to consider a uniformly positively charged solid sphere with radius R. The critical charge Z_c is first determined by solving the corresponding Klein-Gordon equation with $q = 0$ for a negatively charged particle with binding energy E close to zero. The corresponding solution is then inserted into Eq.(3.25) as initial data for $\tilde{\eta}_0$ in the iterative method, where we now solve for $\tilde{C}_b$. With it we solve again for $\tilde{\eta}_0$ in Eq.(3.26) with q as an eigenvalue and renormalize the solution according to Eq.(3.28). This iterative procedure converges rapidly and it is interrupted when the desired numeri-

cal accuracy in q is reached. In Fig.8 we present the result of such a numerical calculation for three different values of R in terms of an effective charge $Z_{eff} = Z - q$ of the "star". It turns out that, as already discussed to some extent in Ref.[5], the number of particles q/e in the charged condensate depends on δZ ($\delta Z = Z - Z_c$ is the amount of charge above the critical value) and also on the radius R of the "star". For very compact objects, with a radius smaller than pion's Compton wavelength, our numerical calculations show that the absolute value of q is bigger than δZ. This tendency increases by shrinking the radius. It may even turn out that in the point-like limit, for $Z \geq 0.5/\alpha$, the pion condensate will completely screen the object, i.e. it becomes neutral. We would therefore obtain a limiting charge $Z_c = 0.5/\alpha$ for a point-like charge defined as the limit of an extended charge distribution. Recently Gribov and Nyiri [24] have reached a similar conclusion, however in the approximation of a massless "pion". We intend to return to this issue elsewhere.

As already mentioned in the previous section on fermion instabilities, due to the extremely small radius of the "star" also particles heavier than the pions could contribute to the condensate. Like in the fermion case, we do not expect this fact to alter significantly the result for the screening of the overcritical charge, since this depend primarily from the electromagnetic charges involved and not from the the particle mass. As mentioned above we solved numerically the full set of eqs. (3.25) - (3.28) for large values of the the ratio m/m_* (in particular for $\approx 1/32$) in order to study possible back-reaction effects on the star itself. We find that such effects are negligible. For larger m, comparable to m_*, the vacuum is however no longer overcritical (see Fig. 4) and thus such particle will not be produced. We conclude thus that within our Thomas-Fermi like approach back-reactions are not important. Of course the actual composition of the condensate may change from the simple one particle-type solution presented here, but not the net screening effect. This last point is what is most important for our following astrophysical applications.

In this section we have shown that the vacuum is unstable against pair production of fermions or bosons if the charged bose-"star" is overcritical, i.e. if $Z \geq Z_c$. The time-scale, t_c, of the destabilization of the vacuum can be obtained from the structure of the bound state wave function of the Dirac equation or Klein-Gordon equation close to the edge of

the negative continuum, the negative continuum states themselves and the structure of the potential C in terms of an overlap integral of these quantities, as far as one considers only one state diving into the negative continuum (c.f. chapter 6 in Ref.[5]). The natural scale determining the relevant time-scale, therefore, corresponds to the scale of the bound state wave functions, i.e. the electron mass or the pion mass. For our purposes t_c can be estimated, within a few orders of magnitude, by making use of the probability density per unit time, ω, for pair production of particles with mass m and spin s in a strong electric field E [15], i.e. ($\hbar = c = 1$)

$$\omega = (2s+1)\frac{\alpha E^2}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}(-1)^{(2s+1)(n+1)}\exp\left(-\frac{n\pi m^2}{eE}\right) \quad , \tag{3.29}$$

where we use $eE \simeq N\alpha/r_C^2$ and $r_C = 1/m$. We can then estimate the change in the screening charge, dQ/dt, by writing (c.f. chapter 21 in Ref.[5])

$$\frac{1}{e}\frac{dQ}{dt} \simeq r_C^3\omega \quad . \tag{3.30}$$

The time-scale t_c for a process for which the change in particle number is of order one, i.e. $\Delta Q/e \simeq 1$, is

$$t_c \simeq \frac{8\pi^3 r_C}{\alpha^2 N^2(2s+1)A} \quad , \tag{3.31}$$

where $A \approx 0.022(\approx 0.021)$ for fermions (bosons) and we used $N \simeq 1/\alpha$. For the charged and very compact bose "stars" we thus obtain $t_c \simeq 10^{-17}s$ for the pair production of electrons and positrons, whereas for pions we get a time-scale which is three orders of magnitude smaller, i.e. $t_c \simeq 10^{-20}s$. From these values we conclude that the pion condensate most likely will form first and is thus an efficient screening mechanism once the radius of the bose "star" is less than $0.1 fermi$.

4 Final Remarks

The very compact charged boson "stars" taken as a self-consistent model for CHAMPs involve physics at Planck scale, a fact this which makes it difficult to perform reliable estimates of their possibile cosmological relic density. Nevertheless, below we suggest some

plausible arguments in favour of such a relic abundance which may even be observable and make up a substantial fraction of the dark matter present in the universe.

A bound on the mass M of CHAMPs is found by considering their cosmic relic number density, n_M, along the lines discussed in Ref.[17]. We assume that CHAMPs, in the form of compact boson "stars" with mass M, were formed in the very early universe. At a temperature $T = T_*$, such that $M/T_* \simeq 40$ [25], the superheavy CHAMPs (and anti-CHAMPs) will freeze out due to the equality between the Hubble expansion rate and the annihilation rate of particles and anti-particles. Then, a bound on the mass M emerges by equating the relic energy density to the critical density $\rho_c = 3H_0^2/8\pi G_N \approx 10^{-29} gcm^{-3}$, where $H_0 \approx 100 kms^{-1}Mpc^{-1}$ is the Hubble parameter, i.e. we assume that the superheavy CHAMPs constitute the dark matter of the universe [16]. If only electromagnetic interactions are taken into account, we would get $M \simeq \mathcal{O}(TeV)$. For larger values of M the universe becomes matter dominated. Such an estimate is based on the assumption that the CHAMPs are in *thermal equilibrium* at sufficiently high temperatures as compared to their rest mass. Since in our case $M \simeq \mathcal{O}(1/\sqrt{\alpha})m_{Pl}$, this may be a doubtful assumption.

One may instead assume that CHAMPs are thermally produced in the early universe starting with a very small or even a vanishing initial density. This way we get a *lower* bound on their present relic abundance for a given ratio M/T_i. T_i is the temperature where the termal production is assumed to begin. The thermal production (and annihilation) in an expanding universe is assumed, in analogue with thermal production of grand unified magnetic monopoles [26], to be described by a Boltzmann equation

$$\frac{df(x)}{dx} = Z\left(f^2(x) - g(x)f_\gamma^2(x)\right) \quad , \tag{4.1}$$

where

$$x = T/M \quad , \quad f(x) = n_M(T)/T^3 \quad , \quad f_\gamma(x) = n_\gamma(T)/T^3 \quad . \tag{4.2}$$

Here $n_\gamma(T)$ is the number density of photons at temperature T. Since we consider a radiation dominated phase for the early universe, we have the following relation between

the temperature $T(t)$ and the time t:

$$T(t)^2 = \sqrt{\frac{45}{16\pi^3 G_N N_{eff}}}\ \frac{1}{t} \quad , \tag{4.3}$$

where $N_{eff} \simeq 427/4$ is the effective number of degrees of freedom for the standard model well above the electro-weak scale. We also have that $R(t)T(t)$ remains constant in time, where $R(t)$ is the cosmic scale factor. The first term in Eq.(4.1) describes the annihilation process of CHAMPs (and anti-CHAMPs) and $-Zgf_\gamma^2$ describes all possible production processes. The parameter Z characterizes the electromagnetic annihilation process and is given by

$$Z = 2M < \sigma v > m_{Pl}\sqrt{\frac{45}{16\pi^3 N_{eff}}} \quad . \tag{4.4}$$

$< \sigma v >$ is an average annihilation cross section, which we assume to have the form

$$< \sigma v > \simeq \pi N^4 \alpha^2 / M^2 \simeq \pi/\alpha^2 M^2 \quad , \tag{4.5}$$

where $M \simeq N m_*$ or $N \simeq 1/\alpha$. If the particles whose reactions produce CHAMPs are in thermal equilibrium themselves, it was shown by Turner [26] that for $x < 1$, $g(x)$ is given by an equilibrium distribution, i.e.

$$g(x) \simeq \frac{1}{x^3}\exp(-\frac{2}{x}) \quad . \tag{4.6}$$

If we neglect the annihilation term in Eq.(4.1) we obtain

$$\frac{d}{dx}\left(\frac{n_M(T)}{T^3}\right) = -a\frac{1}{x^3}\exp(-\frac{2}{x}) \quad , \tag{4.7}$$

where $a = \mathcal{O}(10)$ if $M = \mathcal{O}(1/\sqrt{\alpha})m_{Pl}$, i.e.

$$\frac{n_M(T_*)}{T_*^3} = \frac{a}{2}\exp(-\frac{2}{x_i})\left(\frac{1}{x_i} + 0.5\right) + \frac{n_M(T_i)}{T_i^3} \quad , \tag{4.8}$$

if $T_* << T_i$. Here $x_i = T_i/M$. This solution actually overestimates $n_M(T)$, since we have neglected the annihilation process. However, by inspection we now see that the annihilation term can be neglected in comparison with the production term in eq.(4.1). If the initial temperature T_i is such that $x_i \approx 0.028$ and the initial density fulfills $n_M(T_i) = 0$, then we would get at $T = T_*$, e.g. the today temperature of the cosmic background

radiation, a relic density compatible with the cosmological critical mass density ρ_c. Since we should require $T_i \leq T_{Pl}$, which is easily fulfilled for the CHAMPs under consideration, we would then obtain an unobservable small relic density for heavier CHAMPs, corresponding to $M \gg m_{Pl}/\sqrt{\alpha}$, unless the initial abundance $n_M(T_i)$ differs from zero and is such that $n_M(T_i)/n_\gamma(T_i) = (n_M(T_i)/n_\gamma(T_i))_{crit} \geq \mathcal{O}(10^{-28})$. This critical abundance corresponds to ρ_c. We conclude that CHAMPs with $M \simeq m_{Pl}/\sqrt{\alpha}$, could be thermally produced in the very early universe with no initial abundance and thereby leading to a critical density.

If the initial abundance is much larger than $(n_M(T_i)/n_\gamma(T_i))_{crit}$ and $m_*^2/m_{pl}^2 \simeq \mathcal{O}(\alpha)$, we expect the freeze-out temperature T_d to be below the Planck scale within a few orders of magnitude. One can then imagine a completely different scenario for the cosmic production of superheavy CHAMPs as compared to the analysis of Ref.[17]. A mechanism of diluting an *early* matter dominated phase of the universe has been discussed in quite a detail by Polnarev and Khlopov [27]. We will thus not discuss that scenario in great detail. We would like, however, to point out that the most simple aspects of this scenario may lead to an upper bound on m_*, which is not in contradiction with the requirement that $\alpha \leq (m_*/m_{Pl})^2$. The physical picture is again an early radiation dominated phase with subsequent thermal production of CHAMPs. Let $\nu_d = \nu(T_d)$, where $\nu(T) = n_M(T)/n_r(T)$, be the ratio of the number densities of CHAMPs (n_M) and relativistic degrees of freedom (n_r) at the freeze out temperature $T = T_d$. The universe will then develop into a matter dominated phase at the temperature [27]

$$T \simeq \nu_d M \quad , \tag{4.9}$$

where we demand that ν_d is much smaller than one. Small initial metric perturbations can grow large in this matter dominated phase and thereby convert the primordial gas of CHAMPs and anti-CHAMPs into primordial black holes (PBH), provided the early matter dominated phase lasts sufficiently long. (PBHs with a mass $\leq 10^{15} g$ would have been evaporated today by Hawking radiation. The time-scale τ for such an evaporation process is $\tau \simeq 10^{10} years (M_{BH}/10^{15})^3$, where M_{BH} is the mass of the black hole in units

of g.) In this scenario the early matter dominated stage is assumed to end by a gradual transition into a radiation dominated phase. Besides the formation of PBHs, it may be that a certain amount of CHAMPs survive. Their abundance will then, of course, be bounded by the critical density ρ_c. The production of PBHs will, however, put limits on M and hence on m_*, due to the observed spectrum of PBHs [27, 28]. For this scenario to work it requires that the relic abundance of CHAMPs at $T = T_d$ does not dominate the energy density of the universe: $n_M M \leq n_r T_d \leq n_r T_{Pl}$, i.e. $\nu_d M/m_{Pl} \leq 1$. The Boltzmann equation, which determines the temperature T_d at decoupling and the relic abundance of charged boson particles with mass M, is [25]

$$\frac{df(x)}{dx} = Z\left(f^2(x) - f_{eq}^2(x)\right) \quad , \tag{4.10}$$

with $x = T/M$ and Z is again given by Eq.(4.4). Here we have, although this is questionable, allowed for an initial condition corresponding to thermal equilibrium in order to get an estimate of $f(x_d)$, where $x_d = T_d/M$.

In the expression for Z, $< \sigma v >$ is an average annihilation cross-section, where for the relative particle anti-particle velocity v we insert a typical virial velocity taken from the PBH-formation process and which is of the form $v \simeq \delta_0^{1/2}$ [27]. Here the initial metric perturbation $\delta(\tilde{M})$ on a mass-scale $\tilde{M}$ is parametrized by δ_0 and n, with

$$\delta(\tilde{M}) = \delta_0 \left(\frac{\tilde{M}}{\tilde{M}_0}\right)^{-n} \quad , \tag{4.11}$$

where $\tilde{M}_0$ is the mass inside the horizon at the beginning of the early matter dominated phase. The annihilation cross section for the superheavy CHAMPs is estimated similarly as for the magnetic monopoles, i.e. the dominant process is the emission of dipole radiation. Thus we get [29]

$$< \sigma v > \simeq \pi\alpha^2 \frac{v^{-9/5}}{M^2} N^4 \quad . \tag{4.12}$$

The equilibrium distribution $f_{eq}(x)$ in Eq.(4.10) is now

$$f_{eq}(x) = \frac{1}{2\pi^2}\int_0^\infty dy \frac{y^2}{\exp(\sqrt{y^2 + 1/x^2}) - 1} \quad . \tag{4.13}$$

The freeze-out temperature T_d corresponds to the temperature when the expansion rate $R^{-1}(t)dR(t)/dt$ is comparable to the reaction rate $< \sigma v > n_{eq}$. This temperature can approximatively be determined by considering

$$\frac{df_{eq}(x_d)}{dx_d} = Z f_{eq}^2(x_d) \quad . \tag{4.14}$$

For $x_d \ll 1$, Eq.(4.14) leads to $1/x_d + 0.5\log(1/x_d) \simeq \ log(Z/(2\pi)^{3/2})$. We then find that

$$f(x_d) \simeq \sqrt{2} f_{eq}(x_d) \simeq \frac{\sqrt{2}}{Z x_d^2} \quad . \tag{4.15}$$

For metric perturbations in the range $\mathcal{O}(10^{-5}) \leq \delta_0 \leq \mathcal{O}(10^{-3})$ we can use $1/x_d \simeq \mathcal{O}(15)$. The bound $\nu_d M/m_{Pl} \leq 1$ leads then, within an order of magnitude, to

$$\frac{1}{\alpha}\left(\frac{m_*}{m_{Pl}}\right) \leq \frac{1}{\alpha^2}\delta_0^{-0.9} x_d^2 \quad . \tag{4.16}$$

In Fig.9 we present the numerical evaluation of the Boltzmann equation Eq.(4.10) in the range $0.001 \leq x \leq 0.5$ where, for our purpose, we consider as initial data $f(x_i) = f_{eq}(x_i)$ with $x_i = 0.5$ and $Z = \mathcal{O}(10^6)$. The numerical value of $f(x_d) \simeq \mathcal{O}(10^{-5})$ does not depend strongly on the actual value of x_i. The results obtained numerically are in good agreement with the qualitative solution presented above. Notice that the bound, Eq.(4.16), is not in contradiction with the condition $\alpha \leq (m_*/m_{Pl})^2$. For a scale invariant initial metric perturbation, i.e. $n = 0$ in Eq.(4.11), with $\delta_0 \simeq 10^{-3}$ and following the analysis for magnetic monopoles of Ref.[27] one would get $M \leq 10^{17} GeV$. As pointed out in Ref.[27] such a limit does, however, not apply if the initial metric perturbation is to small, i.e. if $\delta_0 \leq (\delta_0)_{min} = 6 \times 10^{-4}$. The recent *COBE* data [30] give $\delta_0 \simeq \mathcal{O}(10^{-5})$ and thus we would not get such a strong bound on m_* in this way. The scenario discussed above, therefore, suggests that an early matter dominated phase can be diluted by the process of primordial black hole formation. We are then lead to the thermal production scenario with a small initial abundance, in which case we have argued that, for the CHAMPs under consideration, the today cosmic abundance will be small but nevertheless may lead to a critical density.

Since the CHAMPs we considered are superheavy, they will not affect big-bang nucleosynthesis. A likely scenario is that the compact charged boson stars formed will not bind

to nuclei as is the case for the CHAMPs discussed in Ref.[17]. Thus at least a fraction of the dark matter of the universe could be made of compact boson stars.

As discussed in the previous section there is a limiting charge for such objects, i.e. $Z \leq 0.5\alpha^{-1}$. With regard to possible detection of such a dark matter candidate we notice that compact bose stars with large Z will, with regard to their ionization properties, behave like superheavy magnetic monopoles. Supermassive electrically charged particles have recently been looked for by making use of plastic track detectors sensitive to masses $M \geq 10^{-7} m_{Pl}$ [31]. The bound on their number density, n_M, relative to the number density of the cosmic background radiation $n_\gamma \simeq 400(T_\gamma/2.7K)^3 cm^{-3}$ was found to be $n_M/n_\gamma \leq 10^{-29}$. Even such a small relative fraction of supermassive CHAMPs can lead to a critical density for the universe, as we already pointed out above. It is e.g. sufficient to consider compact charged boson stars with a mass $M \simeq 250 m_{Pl}$. Another completely different scenario of diluting an early abundance is, of course, inflation which we have not considered here.

Strongly interacting microgram dark matter has been discussed recently [32]. For that case one may replace in the above calculation the annihilation cross-section with a suitably scaled strong-interaction annihilation cross-section as in Ref.[17]. The previous analysis can be repeated for such CHAMPs with a resulting less restrictive bound on m_*. If the physical meaning of the $U(1)$ charge of the CHAMPs under consideration is different from the electromagnetic coupling, then alternative astrophysical scenarios may be possible. This has been discussed by Madsen and Liddle [33].

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Figure Captions

Fig.1 The metric functions $A(\tilde{r})$ and $B(\tilde{r})$ (upper and lower curve respectively in Fig.1a), the scalar field $\tilde{\phi}_0(\tilde{r})$ and the electric field $E(\tilde{r})$ (Fig.1b) as a function of $\tilde{r} = m_* r$ for a typical "star" with $\tilde{\phi}_0(0) = 0.1$, $\tilde{e}^2 = 0.4$, $M \approx 166 m_*$ and $N \approx 168$.

Fig.2 The effective potential V_{eff}^{D+}, upper curve, (V_{eff}^{D-}, lower curve) for electrons (positrons) with mass m_e, i.e. $m_e/m_* \simeq 0$, as a function of $\tilde{r} = m_* r$ for a positively charged "star" with $\tilde{\phi}_0(0) = 0.1$, $\tilde{e}^2 = 0.4$ and $Q \approx 168e$ (Fig.2a). For a sufficiently large value of $\tilde{r}$, V_{eff}^{D+} becomes negative . This signals that the bound states dive into the negative continuum of the Dirac sea and that the vacuum becomes unstable against e^+e^--pairs production. In Fig.2b we show the corresponding effective potentials if $m_e = m_*$. In this case the bound states will not dive into the negative continuum. (V_{eff}^{D+}: upper curve; V_{eff}^{D-}: lower curve.)

Fig.3 The mass $\tilde{M}$ of the charged boson star in units of m_{Pl}^2/m_* as a function of $\tilde{\phi}_0(0)$ for various values of the effective charge $\tilde{e}$. The line (the dashed one) going through the maxima of the mass is drawn. Equilibrium configurations to the left of this line are dynamically stable, whereas the other ones are unstable. The lines above which the vacuum becomes unstable against pion pair production (dotted line), which is determined by $Z = 0.5/\alpha$, and electron positron pair production (dotted dashed line) are also drawn.

Fig.4 The lower branch of the curves is the bound state spectrum of the Dirac equation for negatively charged fermions of mass m_* in a $1s_{1/2}$ state ($\kappa = -1$) in the field of a positively charged boson star as a function of $\tilde{e}^2$ with $\tilde{\phi}_0(0) = 0.2$ (continuous line) and $\tilde{\phi}_0(0) = 0.5$ (dashed line). The latter choice corresponds to a dynamically unstable configuration. The upper branch of the curves correspond to the anti-fermion bound states. The binding energy becomes zero nearby the critical value of the $\tilde{e}$-coupling. The corresponding spectrum for the Klein-Gordon equation agrees, within the accuracy of our results, with that of the Dirac equation.

Fig.5 The Klein-Gordon equation ground-state spectrum for negatively charged bosons with mass m in the field of a positively charged "star" approximated by a uniformly charged sphere of radius R and charge Ze. As R tends to zero the spectrum dives faster

into the negative continuum corresponding to anti-particle states. In the point-like limit ($R \to 0$), we reach the critical value $Z_c = 0.5/\alpha$.

Fig.6 The effective potential V_{eff}^{K+}, upper curve, (V_{eff}^{K-}: lower curve) for "pions" (anti-"pions") with mass m, where we consider as an illustrative example $m/m_* = 0.05$, as a function of $\tilde{r} = m_* r$ for a positively charged "star" with $\tilde{\phi}_0(0) = 0.1$, $\tilde{e}^2 = 0.4$ and $Q \approx 168e$ (Fig.6a). V_{eff}^{K+} is negative and less than -0.05 for sufficiently small values of $\tilde{r}$. This signals that the bound states dive into the negative continuum of the anti-"pions" and that the vacuum becomes unstable against particle production. In Fig.6b we show the corresponding effective potentials if $m = m_*$. In this case the bound states do not dive into the negative continuum. (V_{eff}^{K+}: upper curve; V_{eff}^{K-}: lower curve.)

Fig.7 The particle number density $n(\tilde{r}) = \frac{4\pi}{e} J^0(\tilde{r})\tilde{r}^2$, such that $N = \int_0^\infty n(\tilde{r}) d\tilde{r}$, for a dynamically stable boson star with $\tilde{\phi}_0(0) = 0.2$, $\tilde{e}^2 = 0.25$ corresponding to $N \approx 66$ in the presence of an induced charged condensate. The solid curve is the number density of the star itself . The dashed curve corresponds to the particle number density of the condensate (multiplied with a factor 50) in which we use $m^2/m_*^2 = 10^{-3}$ as an illustrative example. The mean number of particles in the condensate is $N_c \approx 10$. With regard to the gravitational properties of the boson star the back-reaction of the condensate on the star is negligible. On the scale of the figure, the particle number density of the star in the absence of the condensate actually coincides with the solid curve.

Fig. 8 The effective charge of the bose "star", $Z_{eff} = Z - q$, where $-q$ is the total charge of the pion condensate. The "star" is approximated by a positive uniformly charged sphere with radius R and charge Ze. The critical charge Z_c of the "star" is determined for fixed R by finding Z such that the binding energy $E = -m_e$, which is very close to zero so that it can practically be taken equal to zero. As R tends to zero the screening becomes more efficient.

Fig. 9 The number density $n_M(T)$ of charged boson stars with $M \simeq m_{Pl}/\sqrt{\alpha}$ (solid line) as derived from the Boltzmann equation valid for a radiation dominated universe for the range $0.001 < x < 0.5$ and $Z = \mathcal{O}(10^6)$. The initial data is such that $n_M(T_i) = n_{eq}(T_i)$, where we chose $T_i/M = 0.5$. As a comparison we also plot (dashed line) $n_{eq}(T_i)/T_i^3 \approx$

0.053, which corresponds to the solution of the Boltzmann equation for charged bosons with mass $M = m_* \simeq m_{Pl}\sqrt{\alpha}$ and an initial equilibrium distribution, i.e. in this case the bosons remain in thermal equilibrium.

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