Comment on "London Theory for Superconducting Phase Transitions in External Magnetic Fields: Application to UPt$_3$"

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The theory of the equilibrium flux lattice in UPt_3 developed in Ref. [1] is valid for the superconducting A-phase near the phase transition to the normal state, where the symmetry of the superconducting state is mostly dictated by one of the two components of the order parameter $\boldsymbol{\eta} = (\eta_1, \eta_2)$. An attempt to spread the analysis [1] towards the phase transition between the superconducting A and B states has been undertaken in the recent Letter [2]. In our opinion, the approach and the conclusions of Ref. [2] are wrong and here we present why it is so.

The Ginzburg-Landau equations for $H \parallel z$ after taking just for brevity $\beta_2 = 0$ are [3]

$$\left(\alpha_0\tau_1 + 2\beta_1\left(|\eta_1|^2 + |\eta_2|^2\right) + \left(K_{123}D_x^2 + K_1 D_y^2\right)\right)\eta_1 = \\ -\left(K_2 D_x D_y + K_3 D_y D_x\right)\eta_2, \tag{1}$$

$$\left(\alpha_0\tau_2 + 2\beta_1\left(|\eta_1|^2 + |\eta_2|^2\right) + \left(K_{123}D_y^2 + K_1 D_x^2\right)\right)\eta_2 = \\ -\left(K_3 D_x D_y + K_2 D_y D_x\right)\eta_1, \tag{2}$$

where $\tau_1 = (T - T_1)$, $\tau_2 = (T - T_2)$ with the temperatures $T_1 > T_2$. Near the phase transition to the normal state, that is when $H \to H_{c2} = -\Phi_0\alpha_0\tau_1/2\pi\sqrt{K_1K_{123}}$ the component η_2 is expressed through the first component η_1 [1]:

$$\eta_2 = -\frac{\left(K_3 D_x D_y + K_2 D_y D_x\right)\eta_1}{\alpha_0\left(T - T_{c2}\right)}, \tag{3}$$

and η_1 is solution of the nonlinear Eq. (1) at $\eta_2 = 0$ (here $T_{c2} = T_2 - 2\beta_1\langle|\eta_1|^2\rangle/\alpha_0$).

It is clear that in the region of the neighbourhood of the AB transition ($T \to T_{c2}$), the simple linear but nonlocal relationship (3) between the two order parameter components is divergent and out of applicability. The grow of the amplitude of η_2 near the transition between the A and B states is limited in fact principally by the nonlinear (and local) terms in the free energy. The mixing gradient terms (right hand side of Eq (1)-(2)) transform the AB and AC phase transition lines into regions of crossover between the superconducting states with the same symmetry. The crossovers are certainly not characterized by a divergence of the corresponding coherence length as it has to be in the case of a real second order phase transition.

The authors of the Letter [2] disregard the contribution of the nonlinear term $2\beta_1|\eta_2|^2\eta_2$ in Eq. (2) by considering for $H \sim H_{c1}$ (with $K_2 = K_3 \ll K_1$) the relationship

$$\left(\alpha_0\left(T - T_{c2}\right) + K_1\mathbf{D^2}\right)\eta_2 = -K_2\left(D_xD_y + D_yD_x\right)\eta_1. \tag{4}$$

The divergence at $T \to T_{c2}$ is avoided here by the presence of the differential operator acting on η_2. Substituting Eq. (4) back to the gradient part of the free energy, Agterberg and Dodgson have found an effective nonlocal (4-th order in gradients) London energy functional for fields and currents originating from η_1. These additional nonlocal terms in the London energy starts to be important when $T \to T_{c2}$. As the results, the authors of Ref. [2] have derived many conclusions concerning the structures of the Abrikosov lattice in vicinity of the AB transition.

It is possible to estimate under which conditions the nonlinear term is negligible at $T \sim T_{c2}$. Using Eq. (4), we have

$$\frac{\beta_1\langle|\eta_2|^4\rangle}{K_1\langle|\mathbf{D}\eta_2|^2\rangle} \sim \left(\frac{K_2}{K_1}\right)^2\kappa^2\left(\frac{\lambda}{d}\right)^4, \tag{5}$$

where $\kappa \approx 60$ is the Ginzburg-Landau parameter for UPt_3, d the distance between vortices and λ the London penetration depth. As $H \to H_{c1}$, $d \sim \lambda(\ln\kappa)^{-1/2}$ so that the nonlinear term is found negligible when

$$\frac{K_2}{K_1} < (\kappa\ln\kappa)^{-1}. \tag{6}$$

On the other hand, the approach of Ref. [2] does not consider the usual nonlocal terms originating from the 4-th order gradient terms acting on η_1 in the free energy, which is valid if

$$K_2/K_1 > \kappa^{-1}. \tag{7}$$

Taking account of (6) and (7), we thus conclude that the nonlinear terms are important in the vicinity of the AB crossover.

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[1] T. Champel and V.P. Mineev, Phys. Rev. Lett. **86**, 4903 (2001).
[2] D.F. Agterberg and M.J.W. Dodgson, Phys. Rev. Lett. **89**, 017004 (2002).
[3] J.A. Sauls, Adv. Phys. **43**, 113 (1994).