# Prediction of Narrow $J^\pi = 1/2^+$, $3/2^+$, and $3/2^-$ states of $\Theta^+$ in a Quark model with Antisymmetrized Molecular Dynamics

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The exotic baryon $\Theta^+(uudd\bar{s})$ is studied with microscopic calculations in a quark model by using a method of antisymmetrized molecular dynamics. We predict that the lowest even state is $J^\pi = 1/2^+(I = 0)$ or $J^\pi = 3/2^+(I = 0)$, and the lowest odd state is $J^\pi = 3/2^-(I = 1)$. They nearly degenerate in the $uudd\bar{s}$ system. We discuss $K^+n$ decay widths and estimate them to be $\Gamma < 7$ for the $J^\pi = \{1/2^+, 3/2^+\}$, and $\Gamma < 1$ MeV for the $J^\pi = 3/2^-$ state.

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The evidence of an exotic baryon $\Theta^+$ has recently been reported by several experimental groups [1–9]. Since the quantum numbers determined from its decay modes of $\Theta^+$ indicate that the minimal quark content is $uudd\bar{s}$, this discovery proved the existence of the multiquark hadron. The study of pentaquarks became a hot subject in hadron physics.

The prediction of a $J^\pi = 1/2^+$ state of $uudd\bar{s}$ by a chiral soliton model [10] motivated the experiments of the first observation of $\Theta^+$ [1]. Their prediction of even parity is unnatural in the naive quark model. Theoretical studies were done to describe $\Theta^+$ by many groups [11–16], some of which predicted the opposite parity, $J^\pi = 1/2^-$ [14–16]. The problem of spin and parity of $\Theta^+$ is not only open but also essential to understand the dynamics of pentaquark systems.

In this paper we would like to clarify the mechanism of the existence of the pentaquark baryon. We try to extract a simple picture for the pentaquark baryon with its energy, width, spin, parity and also its shape from explicit calculation. In order to achieve this goal, we study the pentaquark with a flux-tube model [19,20] based on strong coupling QCD, by using a method of antisymmetrized molecular dynamics (AMD) [17,18]. In the flux-tube model, the interaction energy of quarks and anti-quarks is given by the energy of the string-like color-electric flux, which is proportional to the minimal length of the flux-tube connecting quarks and anti-quarks at long distances supplemented by perturbative one-gluon-exchange (OGE) interaction at short distances. For the $q^4\bar{q}$ system the flux-tube configuration has an exotic topology, Fig.1(c), in addition to an ordinary meson-baryon topology, Fig. 1(d), and the transition between different topologies takes place only in higher order of the strong coupling expansion. Therefore, it seems quite natural that the flux-tube model accommodates the pentaquark baryon. In 1991, Carlson and Pandharipande studied exotic hadrons in the flux-tube model [21]. They calculated for only a few $q^4\bar{q}$ states with very limited quantum numbers and concluded that pentaquark baryons are absent. We apply the AMD method to the flux-tube model. The AMD is a variational method to solve a finite many-fermion system. This method is powerful for the study of nuclear structure. One of the advantages of this method is that the spatial and spin degrees of freedom for all particles are independently treated. This method can successfully describe various types of structure such as shell-model-like structure and clustering (correlated nucleons) in nuclear physics. In the application of this method to a quark model, we take the dominant terms of OGE potential and string potential due to the gluon flux tube. Different flux-tube configurations are assumed to be decoupled. We calculate all the possible spin parity states of $uudd\bar{s}$ system, and predict low-lying states. By analysing the wave function, we discuss the properties of $\Theta^+$ and estimate the decay widths of these states with a method of reduced width amplitudes.

In the present calculation, the quarks are treated as non-relativistic spin-$\frac{1}{2}$ Fermions. We use a Hamiltonian as follows,

$$H = H_0 + H_I + H_f, \tag{1}$$

where $H_0$ is the kinetic energy of the quarks, $H_I$ represents the short-range OGE interaction between the quarks and $H_f$ is the energy of the flux tubes. For simplicity, we take into account the mass difference between the $ud$ quarks and the $s$ quark, $\Delta m_s$, only in the mass term of $H_0$ but not in the kinetic energy term. Then, $H_0$ is represented as follows;

$$H_0 = N_q m_q + N_s \Delta m_s + \sum_i \frac{p_i^2}{2m_q} - T_0, \tag{2}$$

where $N_q$ is the total number of quarks and $N_s$ is the total number of strange quarks. $T_0$ denotes the kinetic energy of the center-of-mass motion.

$H_I$ represents the short-range OGE interaction between quarks and consists of the Coulomb and the color-magnetic terms,

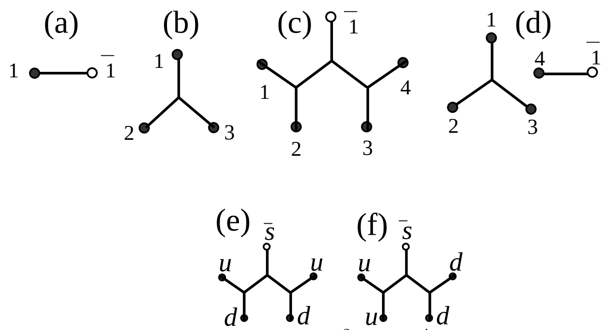

FIG. 1. Flux-tube configurations for confined states of $q\bar{q}$ (a), $q^3$ (b), $q^4\bar{q}$ (c), and disconnected flux-tube of $q^4\bar{q}$ (d). Figures (e) and (f) represent the flux tubes in the color configurations, $[ud][ud]\bar{s}$ and $[uu][dd]\bar{s}$, respectively.

$$H_I = \alpha_c \sum_{i<j} F_i F_j \left[ \frac{1}{r_{ij}} - \frac{2\pi}{3\bar{m}_i \bar{m}_j} s(r_{ij}) \sigma_i \cdot \sigma_j \right]. \tag{3}$$

Here, $\alpha_c$ is the quark-gluon coupling constant, and $F_i F_j$ is defined by $\sum_{\alpha=1,\cdots,8} F_i^\alpha F_j^\alpha$, where $F_i^\alpha$ is the generator of color $SU(3)$, $\frac{1}{2}\lambda_i^\alpha$ for quarks and $-\frac{1}{2}(\lambda_i^\alpha)^*$ for anti-quarks. $\bar{m}_i$ is the mass of $i$-th quark, which is $m_q$ for a $u$ or $d$ quark and $m_s$ for a $\bar{s}$ quark. The usual $\delta(r_{ij})$ function in the spin-spin interaction is replaced by a finite-range Gaussian, $s(r_{ij}) = \left[\frac{1}{2\sqrt{\pi}\Lambda}\right]^3 \exp\left[-\frac{r_{ij}^2}{4\Lambda^2}\right]$, as in Ref. [21]. Of course, the full OGE interaction contains other terms such as tensor and spin-orbit interactions. However, since our main interest here is to see the basic properties of the pentaquark, we do not include these minor contributions.

In the flux-tube quark model [19], the confining string potential is written a $H_f = \sigma L_f - M^0$, where $\sigma$ is the string tension, $L_f$ is the minimum length of the flux tubes, and $M^0$ is the zero-point string energy. $M^0$ depends on the topology of the flux tubes and is necessary to fit the $q\bar{q}$, $q^3$ and $q^4\bar{q}$ potential obtained from lattice QCD or phenomenology. In the present calculation, we adjust the $M^0$ to fit the absolute masses for each of three-quark and pentaquark.

For the meson and $3q$-baryon systems, the flux tube configurations are the linear line and the $Y$-type configuration with a junction as shown in Fig.1(a) and (b), respectively. The string potential given by the $Y$-type flux tube in a $3q$-baryon system is supported by Lattice QCD [22]. For the pentaquark system, the different types of flux-tube configurations appear as shown in Fig. 1.(e),(f), and (d), which correspond to the states, $|\Phi_{(e)}\rangle = |[ud][ud]\bar{s}\rangle$, $|\Phi_{(f)}\rangle = |[uu][dd]\bar{s}\rangle$, and $|\Phi_{(d)}\rangle = |(qqq)_1(q\bar{q})_1\rangle$, respectively. ($[qq]$ is defined by color anti-triplet of $qq$.) In the present calculation of the energy, we neglect the transition among $|\Phi_{(e)}\rangle$, $|\Phi_{(f)}\rangle$ and $|\Phi_{(d)}\rangle$ because they have different flux-tube configurations. It is reasonable in the first order approximation, as mentioned before. In the practical calculation of the expectation values of the string potential $\langle\Phi|H_f|\Phi\rangle$ with respect to a meson system($\Phi_{q\bar{q}}$), a three-quark system($\Phi_{q^3}$), and the pentaquark states $\Phi_{(e)}$, $\Phi_{(f)}$, $\Phi_{(d)}$, the minimum length of the flux tubes $L_f$ is approximated by a linear combination of two-body distances as,

$$L_f \approx r_{12} \quad \text{in } \langle\Phi_{q\bar{q}}|H_f|\Phi_{q\bar{q}}\rangle, \tag{4}$$

$$L_f \approx \frac{1}{2}(r_{12} + r_{23} + r_{31}) \quad \text{in } \langle\Phi_{q^3}|H_f|\Phi_{q^3}\rangle, \tag{5}$$

$$L_f \approx \frac{1}{2}(r_{12} + r_{34}) + \frac{1}{8}(r_{13} + r_{14} + r_{23} + r_{24}) + \frac{1}{4}(r_{\bar{1}1} + r_{\bar{1}2} + r_{\bar{1}3} + r_{\bar{1}4}) \quad \text{in } \langle\Phi_{(e,f)}|H_f|\Phi_{(e,f)}\rangle, \tag{6}$$

$$L_f \approx \frac{1}{2}(r_{12} + r_{23} + r_{31}) + r_{\bar{1}4} \quad \text{in } \langle\Phi_{(d)}|H_f|\Phi_{(d)}\rangle. \tag{7}$$

We note that the confinement is reasonably realized by the approximation in Eq.6 for $\Phi_{(e,f)}$ as follows. The flux-tube configuration (e)(or (f)) consists of seven bonds and three junctions. In the limit that the length($R$) of any one bond becomes much larger than other bonds, the string potential $\langle H_f\rangle$ approximated by the Eq.6 behaves as a linear potential $\sigma R$. It means that all the quarks and anti-quarks are bounded by the linear potential with the tension $\sigma$. In that sense, the approximation in Eq.6 for the connected flux-tube configurations is regarded as a natural extension of the approximation(Eq.5) for $3q$-baryons. It is convenient to introduce an operator $\mathcal{O} \equiv -\frac{3}{4}\sigma\sum_{i<j} F_i F_j r - M^0$. One can easily prove that the above approximations, 4,5,6,7, are equivalent to $\langle\Phi|H_f|\Phi\rangle \approx \langle\Phi|\mathcal{O}|\Phi\rangle$ within each of the flux-tube configurations because the proper factors arise from $F_i F_j$ depending on the color configurations of the corresponding $qq$ (or $q\bar{q}$) pairs.

TABLE I. Calculated masses (GeV) of the $q^3$ systems. The expectation values of the kinetic, string, Coulomb and color-magnetic terms are also listed.

| | $(uud)_1$ | $(uud)_1$ | $(uuu)_1$ |
|---|---|---|---|
| $S^\pi$ | $\frac{1}{2}^+$ | $\frac{1}{2}^-$ | $\frac{3}{2}^+$ |
| Kinetic($H_0$) | 1.74 | 1.87 | 1.66 |
| String($H_F$) | 0.02 | 0.27 | 0.07 |
| Coulomb | −0.65 | −0.52 | −0.62 |
| Color mag. | −0.17 | −0.09 | 0.14 |
| $E$ | 0.94 | 1.52 | 1.24 |
| exp. (MeV) | $N(939)$ | $N^*(1520), N^*(1535)$ | $\Delta(1232)$ |

We solve the eigenstates of the Hamiltonian with a variational method in the AMD model space [17,18]. We take a base AMD wave function in a quark model as follows.

$$\Phi(\mathbf{Z}) = (1 \pm P)\mathcal{A}\left[\phi_{Z_1}\phi_{Z_2}\cdots\phi_{Z_{N_q}}X\right], \tag{8}$$

$$\phi_{Z_i} = \left(\tfrac{1}{\pi b^2}\right)^{3/4} \exp\left[-\tfrac{1}{2b^2}(r-\sqrt{2}bZ_i)^2 + \tfrac{1}{2}Z_i^2\right], \tag{9}$$

where $1 \pm P$ is the parity projection operator, $\mathcal{A}$ is the anti-symmetrization operator, and the spatial part $\phi_{Z_i}$ of the $i$-th single particle wave function given by a Gaussian whose center is located at $Z_i$ in the phase space. $X$ is the spin-isospin-color function. For example, in case of the proton, $X$ is given as $X = (|\uparrow\downarrow\uparrow\rangle_S - |\uparrow\uparrow\downarrow\rangle_S) \otimes |uud\rangle \otimes \epsilon_{abc}|abc\rangle_C$. Here, $|m\rangle_S(m = \uparrow,\downarrow)$ is the intrinsic-spin function and $|a\rangle_C(a = 1,2,3)$ expresses the color function. Thus, the wave function of the $N_q$ quark system is described by the complex variational parameters, $\mathbf{Z} = \{Z_1, Z_2, \cdots, Z_{N_q}\}$. By using the frictional cooling method [17] the energy variation is performed with respect to $\mathbf{Z}$.

For the pentaquark system $(uudd\bar{s})$,

$$X = \sum_{m_1,m_2,m_3,m_4,m_5} c_{m_1m_2m_3m_4m_5}|m_1m_2m_3m_4m_5\rangle_S \otimes \{|udud\bar{s}\rangle \text{ or } |uudd\bar{s}\rangle\} \otimes \epsilon_{abg}\epsilon_{ceh}\epsilon_{ghf}|abce\bar{f}\rangle_C, \tag{10}$$

where $|udud\bar{s}\rangle$ and $|uudd\bar{s}\rangle$ correspond to the configurations $[ud][ud]\bar{s}$ and $[uu][dd]\bar{s}$ in Fig.1, respectively. Since we are interested in the confined states, we do not use the meson-baryon states, $(qqq)_1(q\bar{q})_1$. This assumption of decoupling between the reducible and irreducible configurations of the flux tubes can be regarded as a kind of bound-state approximation. The coefficients $c_{m_1m_2m_3m_4m_5}$ for the spin function are determined by diagonalization of Hamiltonian and norm matrices. After the energy variation, the intrinsic-spin and parity $S^\pi$ eigen wave function $\Phi(\mathbf{Z})$ for the lowest state is obtained for each $S^\pi$.

In the numerical calculation, the linear and Coulomb potentials are approximated by seven-range Gaussians. We use the parameters, $\alpha_c = 1.05$, $\Lambda = 0.13$ fm, $m_q = 0.313$ GeV, $\sigma = 0.853$ GeV/fm, and $\Delta m_s = m_s - m_q = 0.2$ GeV. The quark-gluon coupling constant $\alpha_c$ is chosen so as to fit the $N$ and $\Delta$ mass difference. The string tension $\sigma$ is adopted to adjust the excitation energy of $N^*(1520)$. The width parameter $b$ is chosen to be 0.5 fm.

In table.I, we display the calculated energy of $q^3$ states with $S^\pi = 1/2^+(N)$, $S^\pi = 3/2^+(\Delta)$, $S^\pi = 1/2^-(N^*)$. The zero-point energy $M^0$ of the string potential is chosen to be $M^0_{q^3} = 972$ MeV to fit the masses of $q^3$ systems, $N$, $N^*$ and $\Delta$. The contributions of the kinetic and each potential terms are consistent with the results of the Ref. [21]. We checked that the obtained states are almost eigen states of the angular momentum $L$ and the $L$ projection gives only minor effects on the energy.

Now, we apply the AMD method to the $uudd\bar{s}$ system. For each spin parity, we calculate energies of the $[ud][ud]\bar{s}$ and $[uu][dd]\bar{s}$ states and adopt the lower one. In table.II, the calculated results are shown. We adjust the zero-point energy of the string potential $M_0$ as $M^0_{q^4\bar{q}} = 2385$ MeV to fit the absolute mass of the recently observed $\Theta^+$. This $M^0_{q^4\bar{q}}$ for pentaquark system is chosen independently of $M^0_{q^3}$ for $3q$-baryon. If $M^0_{q^4\bar{q}} = \frac{5}{3}M^0_{q^3}$ is assumed as Ref. [21], the calculated mass of the pentaquark is around 2.2 GeV, which is consistent with the result of Ref. [21].

The most striking point in the results is that the $S^\pi = 3/2^-$ and $S^\pi = 1/2^+$ states nearly degenerate with the $S^\pi = 1/2^-$ states. The $S^\pi = 1/2^+$ correspond to $J^\pi = 1/2^+$ and $3/2^+$ with $S = 1/2, L = 1$, the $S^\pi = 3/2^-$ is $J^\pi = 3/2^-(S = 3/2, L = 0)$. The lowest $S^\pi = 1/2^-(J^\pi = 1/2^-, L = 0)$ state appears just below the $S^\pi = 3/2^-$. However this state, as we discuss later, is expected to be much broader than other states. The $J^\pi = 1/2^+$ and $3/2^+$ exactly degenerate in the present Hamiltonian which does not contain the spin-orbit force. Other spin-parity states are much higher than these low-lying states.

TABLE II. Calculated masses(GeV) of the $uudd\bar{s}$ system. $M^0_{q^4\bar{q}}$=2385 MeV is used to adjust the energy of the lowest state to the observed mass. The expectation values of the kinetic, string, Coulomb, color-magnetic terms, and that of the color-magnetic term in $q\bar{q}$ pairs are listed. In addition to the lowest $1/2^-$ state with the $[uu][dd]\bar{s}$ configuration, we also show the results of the $/2^-$ state with $[ud][ud]\bar{s}$ configuration, which lies in the low-energy region.

| | $[uu][dd]\bar{s}$ | $[ud][ud]\bar{s}$ | $[ud][ud]\bar{s}$ | $[ud][ud]\bar{s}$ | $[uu][dd]\bar{s}$ | $[ud][ud]\bar{s}$ | $[ud][ud]\bar{s}$ |
|---:|---:|---:|---:|---:|---:|---:|---:|
| $S^\pi$ | $\frac{1}{2}^-$ | $\frac{3}{2}^-$ | $\frac{1}{2}^+$ | $\frac{1}{2}^-$ | $\frac{5}{2}^-$ | $\frac{3}{2}^+$ | $\frac{5}{2}^+$ |
| Kinetic($H_0$) | 3.23 | 3.22 | 3.36 | 3.19 | 3.19 | 3.36 | 3.33 |
| String($H_F$) | −0.67 | −0.66 | −0.55 | −0.64 | −0.64 | −0.56 | −0.54 |
| Coulomb | −1.05 | −1.04 | −0.99 | −1.03 | −1.03 | −0.99 | −0.98 |
| Color mag. | −0.01 | 0.01 | −0.25 | 0.04 | 0.19 | −0.06 | 0.17 |
| $q\bar{q}$Color mag. | −0.06 | −0.01 | 0.00 | 0.02 | 0.06 | 0.02 | 0.04 |
| $E$ | 1.50 | 1.53 | 1.56 | 1.56 | 1.71 | 1.75 | 1.98 |

Next, we analyze the spin structure of these states, and found that the $J^\pi = \{1/2^+, 3/2^+\}(S = 1/2, L = 1)$ states consist of two spin-zero $ud$-diquarks, while the $J^\pi = 3/2^-$ consists of a spin-zero $ud$-diquark and a spin-one $ud$-diquark. Since the spin-zero $ud$-diquark has the isospin $I = 0$ and the spin-one $ud$-diquark has the isospin $I = 1$ because of the color asymmetry, the isospin of the $J^\pi = 3/2^-$ state is $I = 1$. The lowest even-parity states have isopin $I = 0$. The $J^\pi = 1/2^+$ state corresponds to the $\Theta^+(1530)$ in the flavor $\overline{10}$-plet predicted by Diakonov et al. [10]. It is surprising that the odd-parity state, $J^\pi = 3/2^-$ has the isopin $I = 1$, which means that this state is a member of the flavor 27-plet and belong to a new family of $\Theta$ baryon. We denote the $J^\pi = \{1/2^+, 3/2^+\}, I = 0$ states by $\Theta^+_0$, and the $J^\pi = 3/2^-, I = 1$ state by $\Theta^+_1$. The mass difference $E(\Theta^+_0) - E(\Theta^+_1)$ is about 30 MeV in the present work.

Although it is naively expected that unnatural spin parity states are much higher than the natural spin-parity $1/2^-$ state, the results show the abnormal level structure of the $(udud\bar{s})$ system, where the high spin state, $J^\pi = 3/2^-$, and the unnatural parity states, $J^\pi = \{1/2^+, 3/2^+\}$, nearly degenerate just above the $J^\pi = 1/2^-$ state. By analysing the details of these states, the abnormal level structure can be easily understood with a simple picture as follows. As shown in table.II, the $J^\pi = \{1/2^+, 3/2^+\}(S = 1/2, L = 1)$ states have larger kinetic and string energies than the $J^\pi = 3/2^-(S = 3/2, L = 0)$ and $J^\pi = 1/2^-(S = 1/2, L = 0)$ states, while the former states gain the color-magnetic interaction. It indicates that the degeneracy of parity-odd states and parity-even states is realized by the balance of the loss of the kinetic and string energies and the gain of the color-magnetic interaction. In the $J^\pi = \{1/2^+, 3/2^+\}$ and the $3/2^-$ states, the competition of the energy loss and gain can be understood by a simple picture from the point of view of the diquark structure as follows. As already mentioned by Jaffe and Wilczek [11], the relative motion between two spin-zero diquarks must have the odd parity $(L = 1)$ because the $L = 0$ is forbidden between the two identical diquarks due to the color antisymmetry. In the $J^\pi = 3/2^-$ state, one of the spin-zero $ud$-diquarks is broken to be a spin-one $ud$-diquark, and the $L = 0$ is allowed because two diquarks are not identical. The $L = 0$ is energetically favored in the kinetic and string terms, and the energy gain cancels the color-magnetic energy loss of a spin-one $ud$-diquark. Although we can not describe the $J^\pi = 1/2^-$ state by such a simple di-quark picture, the competition of energy loss and gain in this state is similar to the $J^\pi = 3/2^-$, for each contribution of the kinetic, string and potential energies is almost the same between the $J^\pi = 1/2^-$ and the $J^\pi = 3/2^-$ as shown in table II. It means that the gain of the kinetic energy of the $L = 0$ state compete with the color-magnetic energy loss in the $J^\pi = 1/2^-$ as well as the $J^\pi = 3/2^-$.

We remark that the existence of two spin-zero $ud$-diquarks in the $J^\pi = \{1/2^+, 3/2^+\}$ states predicted by Jaffe and Wilczek [11] is actually confirmed in our *ab initio* calculations. In fact, the component with two spin-zero $ud$-diquarks is 97% in the present $J^\pi = \{1/2^+, 3/2^+\}$ state. In Fig.2, we show the quark and anti-quark density distribution in the $J^\pi = \{1/2^+, 3/2^+\}$ states. In the intrinsic state before parity projection, we found the spatial development of $ud$-$uds$ clustering, as seen in the density. As a result, it has a parity-asymmetric shape. In the intrinsic wave function, the Gaussian centers of two diquarks are located far from each other and the $\bar{s}$ stays in the vicinity of one of diquarks. After the parity projection, the $\bar{s}$ is exchanged between two diquarks.

We give a comment on the $LS$-splitting between $J^\pi = 1/2^+$ and $3/2^+(S = 1/2, L = 1)$. In the present calculation, where the spin-obit force is omitted, the $J^\pi = 1/2^+$ and $3/2^+$ states exactly degenerate. Even if we introduce the spin-orbit force into the Hamiltonian, the $LS$-splitting should not be large in this diquark structure because the effect of the spin-orbit force from the spin-zero diquarks is very weak as discussed in Ref. [24].

It is important that in the decay mode, $\Theta^+_1(J^\pi = 3/2^-) \to KN$, the $D$ wave is dominant, which makes the width of $\Theta^+_1$ narrower than than that of $\Theta^+_0(J^\pi = 1/2^+, 3/2^+)$ because of higher centrifugal barrier. We estimate the $KN$-decay widths of these states by using a method of reduced width amplitudes [23]. In this method, the decay width $\Gamma$ is estimated by the product $\Gamma^0_L \times S_{fac}$, where $\Gamma^0_L(a, E_{th})$ is given by the penetrability of the barrier as a

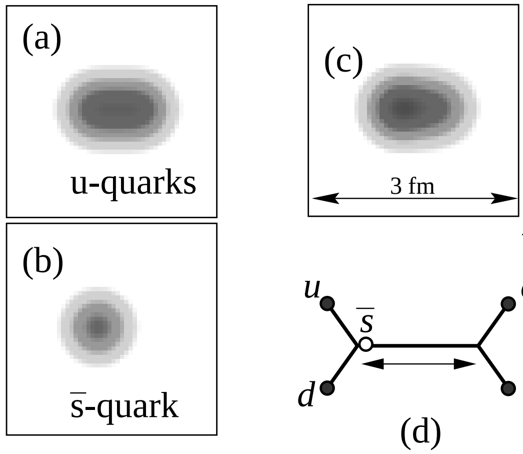


FIG. 2. The $q$ and $\bar{q}$ density distribution in the $J^\pi = 1/2^+, 3/2^+ (S = 1/2, L = 1)$ states of the $uudd\bar{s}$ system. The $u$ density (a), $\bar{s}$ density (b), and total quark-antiquark density (c) of the intrinsic state before parity projection are shown. The schematic figure of the corresponding flux-tube configuration is illustrated in (d).

function of the threshold energy $E_{th}$ and the channel radius $a$, as $\Gamma^0_L(a, E_{th}) \equiv \frac{\hbar^2 k}{\mu} \frac{1}{j^2_L(ka)+n^2_L(ka)}$ ($\mu$ is the reduced mass, $k$ is the wave number, and $j_L(n_L)$ is the regular(irregular) spherical Bessel function). $S_{fac}$ is the probability of the decaying particle at the channel radius $a$. In the following discussion, we choose the channel radius $a = 1$ fm and $E_{th} = 100$ MeV. Since the transitions between the different flux tube configurations, a confined state $[ud][ud]\bar{s}$ and a decaying state $(udd)_1(u\bar{s})_1$, are of higher order, the $S_{fac}$ should be small in general when the suppression by the flux-tube transition is taken into account. Here, we discuss the maximum values of the widths, by considering just the simple overlap with respect to the quark degrees of freedom.

In case of even parity $J^\pi = 1/2^+, 3/2^+$ states, the $KN$ decay modes are the $P$-wave, which gives $\Gamma^0_{L=1} \approx 100$ MeV fm$^{-1}$. By assuming $(0s)^2$ and $(0s)^3$ harmonic-oscillator wave functions for $K^+$ and $n$, we calculate the overlap between the obtained pentaquark wave function and the $K^+n$ state. The probability $S_{fac} = 0.034$ fm$^{-1}$ is evaluated by the overlap. Roughly speaking, the main factors in this meson-baryon probability are the factor $\frac{1}{3}$ from the color configuration, the factor $\frac{1}{4}$ from the intrinsic spin part, and the other factor which arises from the spatial overlap. By using the probability $S_{fac} = 0.034$, the $K^+n$ partial decay width is evaluated as $\Gamma < 3.4$ MeV. Since the $K^0p$ decay width is the same as the $K^+n$ decay, the total width of the $J^\pi = 1/2^+, 3/2^+$ states is estimated to be $\Gamma < 7$ MeV. This is consistent with the discussion in Ref. [25]. For more quantitative discussions, the coupling with the $KN$ continuum states is important. We should point out that, in introducing the coupling, one should not treat only the quark degrees of freedom but should take into account the suppression due to the rearrangement of flux-tube topologies between the meson-baryon states and the confined states.

It is interesting that the $KN$ decay width of the $J^\pi = 3/2^-$ state is strongly suppressed by the $D$-wave centrifugal barrier, which makes $\Gamma^0_L$ smaller $\Gamma^0_{L=2} \approx 30$ MeV fm$^{-1}$ than the $P$-wave case. Moreover, the $J^\pi = 3/2^-$ in the present calculation is the state with $S^\pi = 3/2^-$ and $L = 0$, which has no overlap with the $KN$ $3/2^-$ states ($S^\pi = 1/2^-$ and $L = 2$). Therefore, even if we introduce the spin-orbit or tensor forces, the $KN$ probability($S_{fac}$) in the $J^\pi = 3/2^-$ pentaquark state is expected to be rather suppressed than that in the $J^\pi = 1/2^+, 3/2+$ states. Consequently, the $J^\pi = 3/2^-$ state should be extremely narrow. If we assume the $S_{fac}$ in the $J^\pi = 3/2^-$ to be half of that in the $J^\pi = 1/2^+, 3/2^+$ states, the $KN$ decay width is estimated to be $\Gamma < 1$ MeV. Contrary to the narrow width of the $J^\pi = 3/2^-$ state, in the $J^\pi = 1/2^-$, $S$-wave($L = 0$) decay is allowed and this state should be much broader than other states because of the absence of the centrifugal barrier.

The present results for the $J^\pi = \{1/2^+, 3/2^+\}(\Theta^+_{I=0})$ states are consistent with the experimental observation of $\Theta^+$, while $J^\pi = 3/2^-(\Theta_{I=1})$ is not observed yet. One should pay attention to the properties of these states, because the production rates depend on their spin, parity and widths. The existence of many narrow states, $J^\pi = 1/2^+$, $3/2^+$, and $3/2^-$, for the $\Theta^+_0$ and $\Theta^+_1$ may give an answer to the inconsistent mass positions of the $\Theta^+$ among the different experiments. The widths of the $\Theta^{++}_1$ are too narrow to be observed in the $K^+p$ scattering. The data of the invariant $K^+p$ mass in the photo-induced reactions [5,9,26] do not seem to exclude the posibility of $\Theta^{++}_1$ peaks.

Within the present framework, the mass differences between $\Theta^+$ and other pentaquark systems are given by the effect of $s$ and $ud$ mass difference $\Delta m_s$ on the mass term and color-magnetic interactions. Other mechanism beyond simple quark models should be necessary for more detailed discussions of systematics of pentaquark masses. However, the $J^\pi = \{1/2^+, 3/2^+\}$ and $J^\pi = 3/2^-$ states may degenerate also in other pentaquark systems, because the mechanism of the degeneracy is simple as mentioned before. Concerning the prediction of another pentaquark $N(1710)$ by Diakonov et al., it is worth mentioning that the $N(1700)(J^\pi = 3/2^-)$, $N(1710)$ $(J^\pi = 1/2^+)$ and $N(1720)(J^\pi = 3/2^+)$ can be candidates of the degenerated $J^\pi = 3/2^-$, $J^\pi = 1/2^+$ and $J^\pi = 3/2^+$ states in pentaquark in the nucleon sector.

In conclusion, we proposed a quark model in the framework of the AMD method, and applied it to the $uudd\bar{s}$ system. The level structure of the the $uudd\bar{s}$ system and the properties of the low-lying states were studied. We predicted that the narrow $J^\pi = \{1/2^+, 3/2^+\}(\Theta_0)$ and $J^\pi = 3/2^-$ $(\Theta_1)$ states nearly degenerate. The widths of $\Theta_0^+$ and $\Theta_1^+$ are estimated to be $\Gamma < 7$ MeV and $\Gamma < 1$ MeV, respectively. Two spin-zero diquarks are found in the $\Theta_0^+$, which confirms Jaffe-Wilczek picture. The degeneracy of $\Theta_0^+$ and $\Theta_1^+$ is realized by the balance of the kinetic and string energies and the color-magnetic interaction. The origin of the novel level structure is due to the color structure in the confined five quark system bounded by the connected flux-tubes.

Finally, we would like to remind the readers that the absolute masses of the pentaquark in the present work are not predictions. We have an ambiguity of the zero-point energy of the string potential, which depends on the flux-tube topology in each of meson, three-quark baryon, pentaquark systems. In the present calculation of the pentaquark, we phenomenologically adjust it to reproduce the observed mass of the $\Theta^+$. In order to predict absolute masses of unknown multiquarks with new flux-tube topologies, it is desirable to determine the zero-point energy more theoretically.

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