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Narrow Pentaquark States in a Quark Model with Antisymmetrized Molecular Dynamics

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Abstract

The exotic baryon $\Theta^+(uudd\bar{s})$ is studied with microscopic calculations in a quark model by using a method of antisymmetrized molecular dynamics(AMD). We predict narrow states, $J^\pi = 1/2^+(I=0)$, $J^\pi = 3/2^+(I=0)$, and $J^\pi = 3/2^-(I=1)$, which nearly degenerate in a low-energy region of the $uudd\bar{s}$ system. We discuss NK decay widths and estimate them to be $\Gamma < 7$ for the $J^\pi = \{1/2^+, 3/2^+\}$, and $\Gamma < 1$ MeV for the $J^\pi = 3/2^-$ state.

The evidence of an exotic baryon Θ^+ has recently been reported by several experimental groups. This discovery proved the existence of the multiquark hadron, whose minimal quark content is $uudd\bar{s}$ as given by the decay modes. The study of pentaquarks has become a hot subject in hadron physics. A chiral soliton model [1] predicted a narrow $\Theta^+(J^\pi = 1/2^+)$ state whose parity contradicts the naive quark model expectation. Theoretical studies were done to describe Θ^+ by many groups [2,3]. The spin parity of Θ^+ is not only a open problem but also a key property to understand the dynamics of pentaquark systems.

In this paper we would like to clarify the mechanism of the existence of narrow pentaquark states. We try to extract a simple picture for the pentaquark baryon with levels, width, spin-parity and structure from explicit calculation. In order to achieve this goal, we study the pentaquark with a flux-tube model [6,7] based on strong coupling QCD, by using a AMD method [4,5].

In the flux-tube model, the interaction energy of quarks and anti-quarks is given by the energy of the string-like color-electric flux, which is proportional to the minimal length of the flux-tube connecting quarks and anti-quarks at long distances supplemented by perturbative one-gluon-exchange (OGE) interaction at short distances. For the $q^4\bar{q}$ system the flux-tube configuration has an exotic topology, Fig.1(c), in addition to an ordinary meson-baryon topology, Fig. 1(d). An important point is that the transition between the different flux-tube topologies (c) and (d) is strongly suppressed because it takes place only in higher order. (In 1991, Carlson and Pandharipande studied exotic hadrons in the flux-tube model [8] and calculated a few $q^4\bar{q}$ states with very limited quantum numbers.)

We apply the AMD method to the flux-tube model and calculate the $uudd\bar{s}$ system. The AMD is a variational method to solve a finite many-fermion system. One of the advantages of this method is that the spatial and spin degrees of freedom for all particles are independently treated. This method can successfully describe various types of structure such as shell-model-like structure and clustering (correlated nucleons) in nuclear physics [4,5]. With the AMD method we calculate all the possible spin parity states of $uudd\bar{s}$ system, and analyze the wave function to estimate the decay widths of the obtained states with a method of reduced width amplitudes.

In the present calculation, the quarks are treated as non-relativistic spin-$\frac{1}{2}$ Fermions. We use a Hamiltonian as $H = H_0 + H_I + H_f$, where H_0 is the kinetic energy of the quarks, H_I represents the short-range OGE interaction between the quarks and H_f is the energy of the flux tubes. H_0 and H_I are represented as follows;

$$H_0 = \sum_i m_i + \sum_i \frac{p_i^2}{2m_q} - T_0, \tag{1}$$

$$H_I = \alpha_c \sum_{i<j} F_i^\alpha F_j^\alpha \left[\frac{1}{r_{ij}} - \frac{2\pi}{3m_i m_j} s(r_{ij}) \sigma_i \cdot \sigma_j \right], \tag{2}$$

where m_i(the i-th quark mass) is m_q for a u or d quark and m_s for a $\bar{s}$ quark, and T_0 denotes the kinetic energy of the center-of-mass motion. Here, we do not take into account the mass difference between ud and s in the second term of H_0, for simplicity. α_c is the quark-gluon coupling constant, and F_i^α is the generator of color $SU(3)$. In H_I, we take only the dominant

terms, Coulomb and color-magnetic terms, and omit other terms.

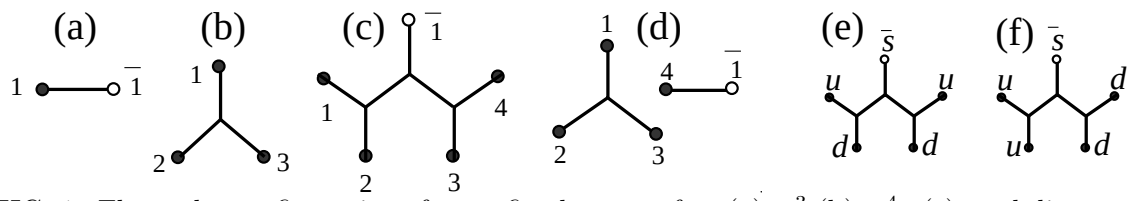

FIG. 1. Flux-tube configurations for confined states of $q\bar{q}$ (a), q^3 (b), $q^4\bar{q}$ (c), and disconnected flux-tube of $q^4\bar{q}$ (d). Figures (e) and (f) represent the flux tubes in the color configurations, $[ud][ud]\bar{s}$ and $[uu][dd]\bar{s}$, respectively. The string potentials given by the flux tubes (b) and (c) are supported by Lattice QCD [9].

In the flux-tube quark model [6], the confining potential is written as $H_f = \sigma L_f - M^0$, where σ is the string tension, L_f is the minimum length of the flux tubes, and M^0 is the zero-point energy. M^0 depends on the topology of the flux tubes and is necessary to fit the $q\bar{q}$, q^3 and $q^4\bar{q}$ potential. In the present calculation, we adjust the M^0 to fit the absolute masses for each of $3q$-baryon and pentaquark. For the meson and $3q$-baryon systems, the flux tube configurations are given as Fig.1(a) and (b). For the pentaquark system, the different types of flux-tube configurations appear as shown in Fig.1(e),(f), and (d), which correspond to the states, $|\Phi_{(e)}\rangle = |[ud][ud]\bar{s}\rangle$, $|\Phi_{(f)}\rangle = |[uu][dd]\bar{s}\rangle$, and $|\Phi_{(d)}\rangle = |(qqq)_1(qq)_1\rangle$, respectively ($[qq]$ is defined by color anti-triplet of qq). In the present calculation of energy variation, we neglect the transitions among $|\Phi_{(e)}\rangle$, $|\Phi_{(f)}\rangle$ and $|\Phi_{(d)}\rangle$ and solve $5q$ wave functions within the model space (e) or (f), which corresponds to the confined states. It is reasonable because the transitions are suppressed as mentioned before. In the practical calculations of the string potential $\langle\Phi|H_f|\Phi\rangle$, the minimum length of the flux tubes L_f is approximated by a linear combination of two-body distances as $L_f \approx \frac{1}{2}(r_{12} + r_{23} + r_{31})$ for a $3q$-baryon, and $L_f \approx \frac{1}{2}(r_{12}+r_{34})+\frac{1}{8}(r_{13}+r_{14}+r_{23}+r_{24})+\frac{1}{4}(r_{\bar{1}1}+r_{\bar{1}2}+r_{\bar{1}3}+r_{\bar{1}4})$ for $\Phi_{(e)}$ or $\Phi_{(f)}$ of the pentaquark systems. We note that the confinement is reasonably realized by the approximation for $\Phi_{(e,f)}$ as follows. The flux-tube configuration (e)(or (f)) consists of seven bonds and three junctions. In the limit that the length(R) of any one bond becomes much larger than other bonds, the approximated $\langle H_f\rangle$ behaves as a linear potential σR. It means that all the

quarks and anti-quarks are bounded by the linear potential with the tension σ. Therefore, the approximation for $\Phi_{(e)}$ or $\Phi_{(f)}$ is a natural extension of the usual approximation for $3q$-baryons. It is easily proved that the approximations are equivalent to $\langle\Phi|H_f|\Phi\rangle \approx \langle\Phi|\hat{O}|\Phi\rangle$, where $\hat{O} \equiv -\frac{3}{4}\sigma\sum_{i<j} F_i^\alpha F_j^\alpha r_{ij} - M^0$, within each of the flux-tube configurations.

We solve the eigenstates of the Hamiltonian with a variational method in the AMD model space [4,5]. We take a base AMD wave function in a quark model as follows.

$$\Phi(\mathbf{Z}) = (1 \pm P)A\left[\phi_{Z_1}\phi_{Z_2}\cdots\phi_{Z_{N_q}}X\right], \qquad \phi_{Z_i} \propto e^{-\frac{1}{2b^2}(r-\sqrt{2}bZ_i)^2}, \tag{3}$$

where $1 \pm P$ is the parity projection operator, A is the anti-symmetrization operator, and the spatial part ϕ_{Z_i} of the i-th single-particle wave function is written by a Gaussian with the center Z_i(Z_i is a complex parameter). X is the spin-isospin-color function. For the pentaquark($uudd\bar{s}$) system, X is expressed as

$$\begin{aligned} X = \ & \textstyle\sum_{m_1,m_2,m_3,m_4,m_5} c_{m_1m_2m_3m_4m_5}|m_1m_2m_3m_4m_5\rangle_S \\ & \otimes \{|udud\bar{s}\rangle \text{ or } |uudd\bar{s}\rangle\} \otimes \epsilon_{abg}\epsilon_{ceh}\epsilon_{ghf}|abce\bar{f}\rangle_C, \end{aligned} \tag{4}$$

where $|udud\bar{s}\rangle$ and $|uudd\bar{s}\rangle$ correspond to the configurations $[ud][ud]\bar{s}$ and $[uu][dd]\bar{s}$ in Fig.1, respectively. Here, $|a\rangle_C(a = 1, 2, 3)$ denotes the color function, and $|m\rangle_S(m =\uparrow, \downarrow)$ is the intrinsic-spin function. Since we are interested in the confined states, we adopt those model space for the color configurations $(qq)_{\bar{3}}(qq)_{\bar{3}}\bar{q}$, but do not use the meson-baryon configurations $(qqq)_1(q\bar{q})_1$. The variational parameters are $\mathbf{Z} = \{Z_1, Z_2, \cdots, Z_5\}$ and $c_{m_1m_2m_3m_4m_5}$ which specify the spatial and spin configurations. The energy variation for $\mathbf{Z}$ is performed by a frictional cooling method, and the coefficients $c_{m_1m_2m_3m_4m_5}$ are determined by diagonalization of Hamiltonian and norm matrices. After the energy variation, the intrinsic-spin and parity S^π eigen wave function $\Phi(\mathbf{Z})$ for the lowest state is obtained for each S^π.

In the numerical calculation, the linear and Coulomb potentials are approximated by seven-range Gaussians. We use the parameters, $\alpha_c = 1.05$, $\Lambda = 0.13$ fm, $m_q = 0.313$ GeV, $\sigma = 0.853$ GeV/fm, and $\Delta m_s = m_s - m_q = 0.2$ GeV. The quark-gluon coupling constant α_c is chosen so as to fit the N and Δ mass difference. The string tension σ is adopted to

adjust the excitation energy of $N^*(1520)$. The width parameter b is chosen to be 0.5 fm. By choosing M_0 as $M^0_{q^3} = 972$ MeV, the masses of N, $N^*(1520)$ and Δ are fitted [10], and the masses of Λ, Σ and Σ^*1385 are well reproduced with these parameters.

Now, we apply the AMD method to the $uudd\bar{s}$ system. For each spin parity, we calculate energies of the $[ud][ud]\bar{s}$ and $[uu][dd]\bar{s}$ states and adopt the lower one. In table.I, the calculated results are shown. We adjust the zero-point energy of the string potential M_0 as $M^0_{q^4\bar{q}} = 2385$ MeV to fit the absolute mass of the recently observed Θ^+. This $M^0_{q^4\bar{q}}$ for pentaquark system is chosen independently of $M^0_{q^3}$ for $3q$-baryon. If $M^0_{q^4\bar{q}} = \frac{5}{3}M^0_{q^3}$ is assumed as Ref.[8],the calculated mass of the pentaquark is around 2.2 GeV, which is consistent with the result of Ref.[8].

The most striking point in the results is that the $S^\pi = 3/2^-$ and $S^\pi = 1/2^+$ states nearly degenerate with the $S^\pi = 1/2^-$ states. The $S^\pi = 1/2^+$ correspond to $J^\pi = 1/2^+$ and $3/2^+$ with $S = 1/2, L = 1$, and the $S^\pi = 3/2^-$ is $J^\pi = 3/2^-(S = 3/2, L = 0)$. The lowest state $J^\pi = 1/2^-(S^\pi = 1/2^-, L = 0)$ exists just below the $J^\pi = 3/2^-$ state, however, this state, as we discuss later, is expected to be much broader than other states. Other spin-parity states are much higher than these low-lying states.

The LS-partners, $J^\pi = 1/2^+$ and $3/2^+$ exactly degenerate in the present Hamiltonian where the spin-orbit and tensor terms are omitted. If we introduce the spin-orbit force into the Hamiltonian the LS-splitting is small in the diquark structure because the effect of the spin-orbit force from the spin-zero diquarks is very weak as discussed in Ref. [12]. As shown later, since the present results show that the diquark structure is realized in the $J^\pi = 1/2^+$ and $3/2^+$ states, the LS-splitting should not be large in the $uudd\bar{s}$ system.

Next, we analyze the spin structure of these states, and found that the $J^\pi = \{1/2^+, 3/2^+\}(S = 1/2, L = 1)$ states consist of two spin-zero ud-diquarks, while the $J^\pi = 3/2^-$ consists of a spin-zero ud-diquark and a spin-one ud-diquark. Since the spin-zero ud-diquark has the isospin $I = 0$ and the spin-one ud-diquark has $I = 1$ because of the color asymmetry, the isospin of the $J^\pi = 3/2^-$ state is $I = 1$, while the even-parity states $J^\pi = 1/2^+, 3/2^+$ are $I = 0$. We consider that the $J^\pi = 1/2^+$ state corresponds to

	$[uu][dd]\bar{s}$	$[ud][ud]\bar{s}$	$[ud][ud]\bar{s}$	$[ud][ud]\bar{s}$	$[uu][dd]\bar{s}$
S^π	$\frac{1}{2}^-$	$\frac{3}{2}^-$	$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{5}{2}^-$
Kinetic(H_0)	3.23	3.22	3.36	3.19	3.19
String(H_F)	−0.67	−0.66	−0.55	−0.64	−0.64
Coulomb	−1.05	−1.04	−0.99	−1.03	−1.03
Color mag.	−0.01	0.01	−0.25	0.04	0.19
$q\bar{q}$Color mag.	−0.06	−0.01	0.00	0.02	0.06
E	1.50	1.53	1.56	1.56	1.71

TABLE I. Calculated masses(GeV) of the $uudd\bar{s}$ system. The expectation values of the kinetic, string, Coulomb, color-magnetic terms, and that of the color-magnetic term in $q\bar{q}$ pairs are listed. The $S^\pi = 3/2^+$ and $S^\pi = 5/2^+$ states are higher than the $S^\pi = 5/2^-$ state.

the $\Theta^+(1530)$ in the flavor $\overline{10}$-plet predicted by Diakonov et al. [1]. The odd-parity state, $J^\pi = 3/2^-$ has $I = 1$, which means that this state is a member of the flavor 27-plet. We denote the $J^\pi = 1/2^+, 3/2^+(I = 0)$ by Θ_0^+, and the $J^\pi = 3/2^-(I = 1)$ by Θ_1^+.

Although it is naively expected that unnatural spin parity states are much higher than the natural spin-parity $1/2^-$ state, the results show the abnormal level structure of the $(udud\bar{s})$ system, where the high spin state, $J^\pi = 3/2^-$, and the unnatural parity states, $J^\pi = \{1/2^+, 3/2^+\}$, nearly degenerate just above the $J^\pi = 1/2^-$ state. By analysing the details of these states, the abnormal level structure can be easily understood with a simple picture as follows. As shown in table.I, the $J^\pi = \{1/2^+, 3/2^+\}(S = 1/2, L = 1)$ states have larger kinetic and string energies than the $J^\pi = 3/2^-(S = 3/2, L = 0)$ and $J^\pi = 1/2^-(S = 1/2, L = 0)$ states, while the former states gain the color-magnetic interaction. It indicates that the degeneracy of the even-parity states with the odd-parity states is realized by the balance of the loss of kinetic and string energies and the gain of the color-magnetic interaction. In the $J^\pi = \{1/2^+, 3/2^+\}$ and the $3/2^-$ states, the competition of the energy loss and gain can be simply understood from the point of view of the diquark structure as follows. As already mentioned by Jaffe and Wilczek [2], the relative motion between two

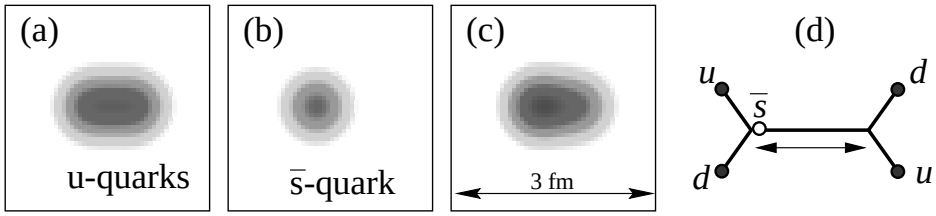

FIG. 2. q and $\bar{q}$ density distribution in the $J^\pi = 1/2^+, 3/2^+ (S = 1/2, L = 1)$ states of $Theta^+$. The u-quark density (a), $\bar{s}$ density (b), and total quark-antiquark density (c) of the intrinsic state before parity projection are shown. The d-quark density is same as the u-quark density. The root-mean-square radius of q and $\bar{q}$ is 0.63 fm (the nucleon size is 0.5 fm).

spin-zero diquarks must have the odd parity ($L = 1$) because of Pauli blocking between the two identical diquarks. In the $J^\pi = 3/2^-$ state, one of the spin-zero ud-diquarks is broken to be a spin-one ud-diquark to avoid the Pauli blocking, then, the $L = 0$ is allowed because diquarks are not identical. The $L = 0$ is energetically favored in the kinetic and string terms, and the energy gain cancels the color-magnetic energy loss of a spin-one ud-diquark. Although we can not describe the $J^\pi = 1/2^-$ state by such a simple diquark picture, the competition of energy loss and gain in this state is similar to the $J^\pi = 3/2^-$.

We remark that the existence of two spin-zero ud-diquarks in the $J^\pi = \{1/2^+, 3/2^+\}$ states predicted by Jaffe and Wilczek [2] is actually confirmed in our *ab initio* calculations. We found that the component with two spin-zero ud-diquarks is 97% in the present $J^\pi = \{1/2^+, 3/2^+\}$ state. In Fig.2, we show the quark and anti-quark density distributions in the $J^\pi = \{1/2^+, 3/2^+\}$ states. In the intrinsic state before parity projection, we found the spatial development of ud-uds clustering, which causes a parity-asymmetric shape (Fig.2 (c)).

We estimate the KN-decay widths of these states by using a method of reduced width amplitudes [11]. The decay width Γ is estimated by the product $\Gamma^0_L \times S_{fac}$, where $\Gamma^0_L(a, E_{th})$ is given by the penetrability of the barrier [10], and $S_{fac}(a)$ is the probability of the decaying particle at the channel radius a. In the following discussion, we use the channel radius $a = 1$ fm and the threshold energy $E_{th} = 100$ MeV. We here estimate the maximum values of the

widths, by taking into account only quark degrees of freedom. We omit the suppression of the transition between the confined state and the meson-baryon state due to the rearrangement of flux-tubes, which makes S_{fac} small in general.

In case of even parity $J^\pi = 1/2^+, 3/2^+$ states, the KN decay modes are the P-wave, which gives $\Gamma^0_{L=1} \approx 100$ MeV fm^{-1}. We calculate the overlap between the obtained pentaquark wave function and the K^+n state, and evaluate the probability as $S_{fac} = 0.034$ fm^{-1}. Roughly speaking, the main factors in this meson-baryon probability are the factor $\frac{1}{3}$ from the color configuration, the factor $\frac{1}{4}$ from the intrinsic spin part, and the other factor which arises from the spatial overlap. By using this value, the total width for K^+n and K^0p decays of the $J^\pi = 1/2^+, 3/2^+$ states is estimated to be $\Gamma < 7$ MeV. For more quantitative discussions, it is important to treat the coupling with the KN continuum states, where one must take into account the suppression due to the rearrangement of flux-tube topologies.

It is interesting that the KN decay width of the $J^\pi = 3/2^-$ state is extremely small due to the D-wave centrifugal barrier. In fact, $\Gamma^0_{L=2} \approx 30$ MeV fm^{-1} is much smaller than the P-wave case. Moreover, the $J^\pi = 3/2^-(S^\pi = 3/2^-, L = 0)$ has no D-wave component, therefore, no overlap with the $KN(L = 2)$ states in the present calculation. Even if we introduce the spin-orbit or tensor forces, the KN probability(S_{fac}) in the $J^\pi = 3/2^-$ pentaquark state is expected to be minor. Consequently, the $J^\pi = 3/2^-$ state should be extremely narrow. If we assume the S_{fac} in the $J^\pi = 3/2^-$ to be half of that in the $J^\pi = 1/2^+, 3/2^+$ states, the KN decay width is estimated to be $\Gamma < 1$ MeV. Contrary to the narrow features of the $J^\pi = 3/2^-$ state, in case of $J^\pi = 1/2^-$, S-wave($L = 0$) decay is allowed and this state should be much broader.

In conclusion, we proposed a quark model in the framework of the AMD method, and applied it to the $uudd\bar{s}$ system. The level structure of the the $uudd\bar{s}$ system and the properties of the low-lying states were studied. We predicted that the narrow $J^\pi = \{1/2^+, 3/2^+\}(\Theta_{I=0})$ and $J^\pi = 3/2^-$ $(\Theta_{I=1})$ states nearly degenerate. The widths of Θ^+_0 and Θ^+_1 are estimated to be $\Gamma < 7$ MeV and $\Gamma < 1$ MeV, respectively. Two spin-zero diquarks are found in the Θ^+_0, which confirms Jaffe-Wilczek picture. The origin of the novel level structure is the $5q$

dynamics of the confined system bounded by the connected flux-tubes. We consider that the present results for the $J^\pi = \{1/2^+, 3/2^+\}(\Theta^+_{I=0})$ states correspond to the experimental observation of Θ^+, while the $\Theta_{I=1}$ is not observed yet. The existence of many narrow states, $J^\pi = 1/2^+$, $3/2^+$, and $3/2^-$, may give an light to further experimental observations.

Concerning other pentaquarks, we give a comment on $\Xi(ddss\bar{u})$. The AMD calculations indicate that the diquark structure disappears in the $ddss\bar{u}(1/2^+)$ due to the $SU(3)$-symmetry breaking in the color-magnetic interaction. As a result, the estimated width of the $ddss\bar{u}(1/2^+)$ state is $\Gamma \approx 100$ MeV, which is much broader than $\Theta^+(1/2^+)$. Also the $3/2^-$ state is not so narrow because a S-wave decay channel $\Xi^*(1530)\pi$ is open.

Finally, we would like to remind the readers that the absolute masses of the pentaquark in the present work are not predictions. We have an ambiguity of the zero-point energy of the string potential, which depends on the flux-tube topology in each of meson, $3q$-baryon, pentaquark systems. We adjust that for the pentaquarks to reproduce the observed Θ^+ mass. To confirm the zero-point energy, experimental information for other pentaquark states are desired.

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