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Particle number in kinetic theory

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We provide a derivation for the particle number densities on phase space for scalar and fermionic fields in terms of Wigner functions. Our expressions satisfy the desired properties: for bosons the particle number is positive, for fermions it lies in the interval between *zero* and *one*, and both are consistent with thermal field theory. As applications we consider the Bunch-Davies vacuum and fermionic preheating after inflation.

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1. Introduction. The notion of particles is very intuitive, and at the classical level, in statistical physics, the dynamics is very successfully described by the classical Boltzmann equation for particle densities in phase space. In quantum physics however, the uncertainty principle seems to prohibit the use of phase space densities, and they are replaced by their closest analogues, the Wigner functions [1, 2]. Yet, strictly speaking they can neither be interpreted as particle numbers nor as probability distributions on phase space, since they may aquire negative values. Several attempts have been made to define particle number in relativistic quantum kinetic theory [3, 4], but so far there exists no result that would be applicable to general situations.

In spite of those difficulties, the dynamics of quantum fields and particle numbers in the presence of temporally varying background fields has been extensively studied and is well understood [5, 6, 7]. The particle number operator can be calculated by a Bogolyubov transformation rotating the Fock space to a new basis, which mixes positive and negative frequency solutions.

In the analysis presented in this Letter we show that the Wigner function, which we here take as an expectation value with respect to the ground state of the original basis, provides the necessary information about the rotated basis to calculate the particle number produced by the coupling to time-dependent external fields.

2. Scalars. As the first model case we consider a massive scalar field minimally coupled to gravity, such that in a conformal space-time, with the metric of the form $g_{\mu\nu} = a^2\eta_{\mu\nu}$, the Lagrangean is given by

$$\sqrt{-g}\mathcal{L}_\Phi = \frac{1}{2}a^2\eta^{\mu\nu}(\partial_\mu\Phi)(\partial_\nu\Phi) - \frac{1}{2}a^4m^2\Phi^2, \qquad (1)$$

where, $\eta^{\mu\nu} = \mathrm{diag}[1, -1, -1, -1]$ is the Minkowski (flat) metric, and $a = a(\eta)$ is the scale factor. For example, in inflation $a = -1/(H\eta)$ $(\eta < 0)$, while in radiation-matter era, $a = a_r\eta + a_m\eta^2$. Here η denotes conformal time, a_r and a_m are constants.

We quantize the theory (1) by promoting $\Phi(x)$ to an operator,

$$\Phi(x) \equiv \frac{\varphi}{a} = \frac{1}{aV}\sum_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}}\Big(\varphi_{\mathbf{k}}(\eta)a_{\mathbf{k}} + \varphi^*_{-\mathbf{k}}(\eta)a^\dagger_{-\mathbf{k}}\Big),$$

where V denotes the comoving volume, and the mode functions obey the Klein-Gordon equation

$$\big(\partial_\eta^2 + \omega^2 - a''/a\big)\varphi_{\mathbf{k}} = 0, \qquad (2)$$

where $' \equiv d/d\eta$, $\omega^2 = \mathbf{k}^2 + a^2m^2(\eta)$ defines the single particle (comoving) energy, and we take for the Wronskian

$$\varphi^*_{\mathbf{k}}\varphi'_{\mathbf{k}} - \varphi^{*\prime}_{\mathbf{k}}\varphi_{\mathbf{k}} = i. \qquad (3)$$

Throughout this Letter we assume that the modes $\varphi_{\mathbf{k}} = \varphi_k$ $(k \equiv |\mathbf{k}|)$ are homogeneous, which is justified when the mass is varying slowly in space, such that we can ignore its gradients. The field $\varphi = a\Phi$ obeys the canonical commutation relation,

$$[\varphi(\mathbf{x},\eta), \partial_\eta\varphi(\mathbf{x}',\eta)] = i\delta^3(\mathbf{x}-\mathbf{x}'), \qquad (4)$$

which implies $[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'}] = \delta_{\mathbf{k},\mathbf{k}'}$.

The fundamental quantity of quantum kinetic theory is the two-point Wightman function, which we here write for the ground state $|0\rangle$ annihilated by $a_{\mathbf{k}}$, $a_{\mathbf{k}}|0\rangle = 0$. With the rescaling suitable for conformal space-times, it reads

$$i\bar{G}^<(u,v) \equiv \frac{iG^<(u,v)}{a(u)a(v)} = \langle 0|\varphi(v)\varphi(u)|0\rangle\,, \qquad (5)$$

and its Wigner transform is defined as

$$iG^<(x,k) = \int d^4r e^{ik\cdot r} iG^<(x+r/2, x-r/2).$$

The Wigner function satisfies simultaneously the kinetic and constraint equations [8, 9],

$$\Big(k_0\partial_\eta + \bar{m}^2(\eta)\sin\big(\frac{1}{2}\overleftarrow{\partial}_\eta\partial_{k_0}\big)\Big)i\bar{G}^< = 0 \qquad (6)$$

$$\Big(\frac{1}{4}\partial_\eta^2 - k^2 + \bar{m}^2(\eta)\cos\big(\frac{1}{2}\overleftarrow{\partial}_\eta\partial_{k_0}\big)\Big)i\bar{G}^< = 0, \qquad (7)$$

where $\bar{m}^2 = a^2m^2 - a''/a$. It is then useful to define the n-th moments of the Wigner function,

$$f_n(x,\mathbf{k}) \equiv \int \frac{dk_0}{2\pi} k_0^n i\bar{G}^<(x,k), \tag{8}$$

such that the 1st and 0th moments of Eqs. (6-7) are [8, 9]

$$f_2' - \frac{1}{2}(\bar{m}^2)' f_0 \;=\; 0 \tag{9}$$

$$\frac{1}{4}f_0'' - f_2 + \bar{\omega}^2 f_0 \;=\; 0, \tag{10}$$

with $\bar{\omega}^2 = \mathbf{k}^2 + \bar{m}^2$. Eliminating f_2 from (10) yields [9]

$$f_0''' + 4\bar{\omega}^2 f_0' + 2(\bar{\omega}^2)' f_0 = 0. \tag{11}$$

This can be integrated once to give

$$\bar{\omega}^2 f_0^2 + \frac{1}{2}f_0{}'' f_0 - \frac{1}{4}f_0'^2 = \frac{1}{4}\,, \tag{12}$$

where the integration constant is obtained by making use of $f_0 = |\varphi_\mathbf{k}|^2$ (*cf.* Eq. (17) below), Eq. (2) and the Wronskian (3).

3. Bogolyubov transformation. The Hamiltonian density corresponding to the Lagrangean (1) reads

$$\begin{aligned} H \;&=\; \frac{1}{2V}\sum_\mathbf{k}\Big\{\Omega_k(a_\mathbf{k}a_\mathbf{k}^\dagger + a_\mathbf{k}^\dagger a_\mathbf{k}) + (\Lambda_k a_\mathbf{k} a_{-\mathbf{k}} + \text{h.c.})\Big\} \\ \Omega_k \;&=\; |\varphi_k' - (a'/a)\varphi_k|^2 + \omega^2|\varphi_k|^2 \\ \Lambda_k \;&=\; \Big(\varphi_k' - \frac{a'}{a}\varphi_k\Big)^2 + \omega^2\varphi_k^2, \end{aligned} \tag{13}$$

and it can be diagonalised by the homogeneous Bogolyubov transformation,

$$\begin{pmatrix} \hat{a}_\mathbf{k} \\ \hat{a}_{-\mathbf{k}}^\dagger \end{pmatrix} = \begin{pmatrix} \alpha_k & \beta_k^* \\ \beta_k & \alpha_k^* \end{pmatrix}\begin{pmatrix} a_\mathbf{k} \\ a_{-\mathbf{k}}^\dagger \end{pmatrix}, \tag{14}$$

where α_k and β_k are given by

$$\frac{1}{2}\left(\left|\frac{\alpha_k}{\beta_k}\right| + \left|\frac{\beta_k}{\alpha_k}\right|\right) = \frac{\Omega_k}{|\Lambda_k|}, \qquad |\alpha_k|^2 - |\beta_k|^2 = 1. \tag{15}$$

Note that Eq. (15) permits solutions when $\Omega_k \geq |\Lambda_k|$. In terms of the new operators the Hamiltonian is simply

$$H = \frac{1}{2V}\sum_\mathbf{k}\omega(\hat{a}_\mathbf{k}\hat{a}_\mathbf{k}^\dagger + \hat{a}_\mathbf{k}^\dagger\hat{a}_\mathbf{k}), \qquad [\hat{a}_\mathbf{k}, \hat{a}_{\mathbf{k}'}^\dagger] = \delta_{\mathbf{k},\mathbf{k}'},$$

and the particle number is then (*cf.* Refs. [5, 10])

$$n_\mathbf{k} \;=\; \langle 0|\hat{a}_\mathbf{k}^\dagger\hat{a}_\mathbf{k}|0\rangle = |\beta_k|^2 = \frac{\Omega_k}{2\omega} - \frac{1}{2}. \tag{16}$$

In deriving these expressions we used $\Omega_k^2 - |\Lambda_k^2| = \omega^2$, which can be obtained by making use of the Wronskian (3).

If at very early times, $\eta \to -\infty$, the system is in the adiabatic regime, the mode functions can be written as

$$\varphi_k \;\to\; (2\omega)^{-1/2}\big(\alpha_0 \mathrm{e}^{-i\omega\eta} + \beta_0 \mathrm{e}^{i\omega\eta}\big),$$

where $|\alpha_0|^2 - |\beta_0|^2 = 1$, with $\alpha_0 = \alpha_k(-\infty)$, $\beta_0 = \beta_k(-\infty)$. The particle number corresponding to this state is $n_\mathbf{k}^{(0)} = |\beta_0|^2$.

4. Particle number in scalar kinetic theory. It is now a simple matter to calculate the particle number in terms of Wigner functions. Making use of (5) and (8) we find

$$|\varphi_k|^2 \;=\; f_0, \qquad |\varphi'_k|^2 = \frac{1}{2}f_0'' + \bar{\omega}^2 f_0, \tag{17}$$

from which it follows

$$\Omega_k = 2\left(\omega^2 f_0 + \frac{1}{4}f_0''\right) - \frac{d}{d\eta}\left(\frac{a'}{a}f_0\right). \tag{18}$$

We then insert (18) into (16) to get

$$n_\mathbf{k} \;=\; \omega f_0 + \frac{1}{4\omega}f_0'' - \frac{1}{2} - \frac{1}{2\omega}\frac{d}{d\eta}\left(\frac{a'}{a}f_0\right). \tag{19}$$

This is our main result for scalars. The last term is of course absent in a nonexpanding universe.

We now apply (19) to the Bunch-Davies vacuum [12],

$$\varphi_k = \frac{1}{\sqrt{2k}}\Big(1 - \frac{i}{k\eta}\Big)\mathrm{e}^{-ik\eta}, \tag{20}$$

which corresponds to the mode functions of a minimally coupled massless scalar field in de Sitter inflation, $a = -1/H\eta$ (*cf.* Eq. (2)), for which $f_0 = (2k)^{-1}\big(1+1/(k\eta)^2\big)$, leading to the particle number (*cf.* Ref. [13])

$$n_\mathbf{k} \;=\; \frac{1}{4k^2\eta^2} = a^2\Big(\frac{H}{2k}\Big)^2. \tag{21}$$

Integration then gives the particle number per unit (physical) volume,

$$\frac{N}{V_\text{ph}} \;=\; \frac{1}{Va^3}\sum_\mathbf{k} a^2\Big(\frac{H}{2k}\Big)^2 = \frac{H^2}{8\pi^2}\int^{a\Lambda}\frac{dk}{a}, \tag{22}$$

where this is regularised by a constant ultraviolet physical cutoff $k_\text{ph} \equiv k/a \leq \Lambda$, which corresponds to taking the generation mechanism operative on scales $k_\text{ph} \leq \Lambda$. When $\Lambda \leq M_\text{Pl}$, the gravitational backreaction from the Bunch-Davies vacuum can be absorbed by rescaling H.

As a consistency check, we now apply (19) to thermal equilibrium, where the Wigner function is (*cf.* Ref. [11])

$$i G^< = 2\pi\text{sign}(k_0)\delta(k^2 - m^2)\frac{1}{\mathrm{e}^{\beta k_0} - 1},$$

which indeed implies the standard Bose-Einstein distribution, $n_\mathbf{k} = 1/(e^{\beta\omega} - 1)$.

5. Fermions. Provided the fields are rescaled as $a^{3/2}\psi \to \psi$ and the mass as $am \to m$, the fermionic Lagrangean reduces to the standard Minkowski form,

$$\sqrt{-g}\mathcal{L}_\psi \to \bar{\psi}\, i\partial\!\!\!/\, \psi - \bar{\psi}(m_R + i\gamma^5 m_I)\psi,$$

where, for notational simplicity, we omitted the rescaling of the fields and absorbed the scale factor in the mass term. Note that the complex mass term $m = m_R(\eta) + im_I(\eta)$ induces CP-violation (*cf.* Ref. [14]).

The fermionic Wigner function,

$$iS^<(k,x) = -\int d^4r e^{ik\cdot r}\langle 0|\bar{\psi}(x-r/2)\psi(x+r/2)|0\rangle,$$

satisfies the corresponding Dirac equation which, in the Wigner representation, reads

$$\Big(\not{k} + \frac{i}{2}\gamma^0\partial_t - (m_R + i\gamma^5 m_I)e^{-\frac{i}{2}\overleftarrow{\partial}_t\partial_{k_0}}\Big)iS^< = 0, \quad (23)$$

where $(i\gamma^0 S^<)^\dagger = i\gamma^0 S^<$ is hermitian. The helicity operator in the Weyl representation $\hat{h} = \hat{\mathbf{k}}\cdot\gamma^0\boldsymbol{\gamma}\gamma^5$ commutes with the Dirac operator in (23), such that we can make the helicity diagonal ansatz for the Wigner function (*cf.* Ref. [14])

$$-i\gamma_0 S_h^< = \frac{1}{4}\big(\mathbb{1} + h\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}\big)\otimes\rho^a g_{ah}, \quad (24)$$

where $\hat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|$ and σ^a, ρ^a $(a = 0,1,2,3)$ are the Pauli matrices. Taking the appropriate traces of the real part of (23) and integrating over k_0, yields the kinetic equations for the 0th moments of g_{ah},

$$\dot{f}_{0h} = 0 \quad (25)$$

$$\begin{aligned}\dot{f}_{1h} + 2h|\mathbf{k}|f_{2h} - 2m_I f_{3h} &= 0\\ \dot{f}_{2h} - 2h|\mathbf{k}|f_{1h} + 2m_R f_{3h} &= 0\\ \dot{f}_{3h} - 2m_R f_{2h} + 2m_I f_{1h} &= 0.\end{aligned} \quad (26)$$

Eq. (25) expresses the conservation of the Noether vector current. The moments f_{ah} can be related to the positive and negative frequency mode functions, $u_h(\mathbf{k},t)$ and $v_h(\mathbf{k},t) = -i\gamma^2(u_h(\mathbf{k},t))^*$, respectively. They form a basis for the Dirac field,

$$\psi(x) = \frac{1}{V}\sum_{\mathbf{k}h} \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}}\big(u_h a_{\mathbf{k}h} + v_h b^\dagger_{-\mathbf{k}h}\big), \quad u_h = \begin{pmatrix} L_h \\ R_h\end{pmatrix}\otimes\xi_h,$$

where ξ_h is the helicity two-eigenspinor, $\hat{h}\xi_h = h\xi_h$. The Dirac equation then decomposes into

$$\begin{aligned} i\partial_0 L_h - h|\mathbf{k}|L_h &= m_R R_h + im_I R_h\\ i\partial_0 R_h + h|\mathbf{k}|R_h &= m_R L_h - im_I L_h.\end{aligned} \quad (27)$$

Note that these equations incorporate CP-violation and thus generalise the analysis of Refs. [6, 7, 15]. Now, from (27) one can derive (25)-(26) by multiplying with L_h and R_h and employing

$$\begin{aligned} f_{0h} &= |L_h|^2 + |R_h|^2, \qquad f_{3h} = |R_h|^2 - |L_h|^2\\ f_{1h} &= -2\Re(L_h R_h^*), \qquad f_{2h} = 2\Im(L_h^* R_h).\end{aligned} \quad (28)$$

The Hamiltonian density reads

$$H = \frac{1}{V}\sum_{\mathbf{k}h}\Big\{\Omega_{kh}\big(a^\dagger_{\mathbf{k}h}a_{\mathbf{k}h} + b^\dagger_{-\mathbf{k}h}b_{-\mathbf{k}h}\big) + (\Lambda_{kh}b_{-\mathbf{k}h}a_{\mathbf{k}h} + \text{h.c.})\Big\}$$

where

$$\begin{aligned}\Omega_{kh} &= hk\left(|L_h|^2 - |R_h|^2\right) + mL_h^* R_h + m^* L_h R_h^*\\ \Lambda_{kh} &= 2kL_h R_h - hm^* L_h^2 + hmR_h^2,\end{aligned} \quad (29)$$

with $\{\hat{a}_{\mathbf{k}h}, \hat{a}^\dagger_{\mathbf{k}'h'}\} = \delta_{h,h'}\delta_{\mathbf{k},\mathbf{k}'}$, $\{\hat{b}_{\mathbf{k}h}, \hat{b}^\dagger_{\mathbf{k}'h'}\} = \delta_{h,h'}\delta_{\mathbf{k},\mathbf{k}'}$. We now use the Bogolyubov transformation

$$\begin{pmatrix}\hat{a}_{\mathbf{k}h}\\ \hat{b}^\dagger_{-\mathbf{k}h}\end{pmatrix} = \begin{pmatrix}\alpha_{kh} & \beta_{kh}\\ -\beta^*_{kh} & \alpha^*_{kh}\end{pmatrix}\begin{pmatrix}a_{\mathbf{k}h}\\ b^\dagger_{-\mathbf{k}h}\end{pmatrix},$$

to diagonalise the Hamiltonian, where α_{kh} and β_{kh} are

$$\frac{1}{2}\left(\left|\frac{\alpha_k}{\beta_k}\right| - \left|\frac{\beta_k}{\alpha_k}\right|\right) = \frac{\Omega_k}{|\Lambda_k|}, \quad |\alpha_k|^2 + |\beta_k|^2 = 1, \quad (30)$$

leading to the particle number density on phase space,

$$n_{\mathbf{k}h} = |\beta_{kh}|^2 = \frac{1}{2} - \frac{\Omega_{kh}}{2\omega}. \quad (31)$$

To construct the initial mode functions in the adiabatic domain, $\eta \to -\infty$, we use the positive frequency solution and its charge conjugate,

$$\psi_{\mathbf{k}} \to \begin{pmatrix}\alpha_0 L_h^+ + \beta_0 L_h^-\\ \alpha_0 R_h^+ + \beta_0 R_h^-\end{pmatrix}, \qquad |\alpha_0|^2 + |\beta_0|^2 = 1.$$

From the Dirac equation under adiabatic conditions it follows

$$\begin{aligned} L_h^+ &= \sqrt{\frac{\omega + hk}{2\omega}}, \qquad & L_h^- &= -i\frac{m}{|m|}\sqrt{\frac{\omega - hk}{2\omega}}\\ R_h^+ &= \frac{m^*}{\sqrt{2\omega(\omega + hk)}}, \qquad & R_h^- &= i\frac{|m|}{\sqrt{2\omega(\omega - hk)}}.\end{aligned}$$

These mode functions correspond to an initial particle number $n^{(0)}_{\mathbf{k}} = |\beta_0|^2$.

We now make use of (28) to express Ω_{kh} in terms of the Wigner functions,

$$\Omega_{kh} = -(hkf_{3h} + m_R f_{1h} + m_I f_{2h}), \quad (32)$$

which implies our main result for fermions,

$$n_{\mathbf{k}h} = \frac{1}{2\omega}\left(hkf_{3h} + m_R f_{1h} + m_I f_{2h}\right) + \frac{1}{2}. \quad (33)$$

Note that in the limit $m \to 0$, this expression reduces to the phase space density of axial particles.

As an application of Eq. (33) we consider particle production at preheating [10, 15], in which the fermionic mass is generated by an oscillating inflaton condensate. Assuming that the inflaton oscillates as a cosine function results in a fermion production shown in figure 1. Observe that, even for a relatively small imaginary (pseudoscalar) mass term, particle production of the opposite helicity states is completely different, implying a nonperturbative enhancement of a CP-violating particle density, $n_{\mathbf{k}+} - n_{\mathbf{k}-}$, which may be of relevance for baryogenesis.

When applied to thermal equilibrium, where (*cf.* [11])

$$iS^< = -(\not{k} + m_R - i\gamma_5 m_I)\delta(k^2 - |m|^2)\mathrm{sign}(k_0)\frac{2\pi}{\mathrm{e}^{\beta k_0} + 1},$$

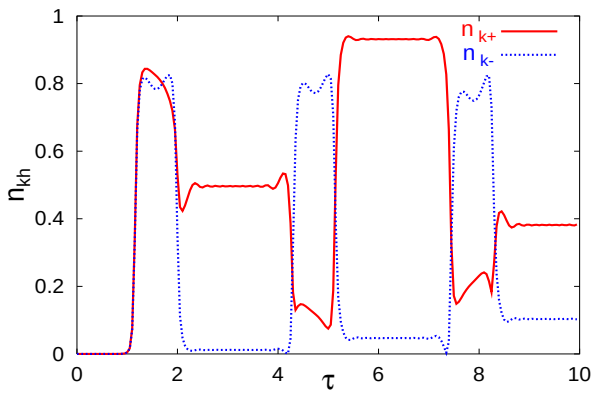

FIG. 1: The number of produced fermions as a function of time with helicity $h = +$ (*solid*) and $h = -$ (*dotted*), mass $m/\omega_I = 10 + 15\cos(2\tau) - i\sin(2\tau)$, $|\mathbf{k}| = \omega_I$, $\tau = \omega_I t$, where ω_I denotes the frequency of the inflaton oscillations.

Eq. (33) yields the Fermi-Dirac distribution, $n_{\mathbf{k}h} = 1/(e^{\beta\omega} + 1)$.

6. Discussion. We have derived general expressions for the particle number densities on phase space for scalars (19) and fermions (33) in terms of the appropriate Wigner functions. Both of these expressions are strictly positive, and moreover, the number of fermions is bounded from above by *unity*, as required by the Pauli principle. In order to incorporate the effect of self-energy into (19) and (33), one needs to include this correction into the dispersion relation, $\omega = \omega(\mathbf{k}, x)$. When the single particle picture breaks down it is not clear whether a sensible definition of particle number can be constructed. Our analysis can be extended to include gauge fields by coupling them canonically to scalars and fermions.

In the derivation of our results, we considered pure quantum states, yet showed explicitly their applicability to thermal states. More generally, our definitions are valid if one requires the density matrix ϱ to satisfy $\langle a_{\mathbf{k}} a_{\mathbf{k}}\rangle_\varrho = \langle a^\dagger_{\mathbf{k}} a^\dagger_{\mathbf{k}}\rangle_\varrho = 0$. These relations hold *e.g.* for eigenstates of the particle number operator $\hat{N}_{\mathbf{k}} \equiv a^\dagger_{\mathbf{k}} a_{\mathbf{k}}$, and, as pointed out in Ref. [16], for random phase states, a special case of which is the canonical ensemble. States of this kind can be treated as a linear superposition of the particle number eigenstates which we considered above.

While the fermionic particle number definition (33) is generally applicable, the scalar one (19) fails however when $\omega^2 = k^2 + m^2 < 0$, which can happen at phase transitions. Then $\Omega_k < |\Lambda_k|$ in (13), and the Bogolyubov transformation (15) does not have a solution. Nevertheless, even in this case, the energy density on phase space Ω_k in Eq. (18) is well defined, and should be considered as a fundamental quantity of kinetic theory. Another important quantity is $\Lambda^*_k = \langle \mathbf{k}, -\mathbf{k}|H|0\rangle$, the transition amplitude for particle pair creation with the momenta $\{\mathbf{k}, -\mathbf{k}\}$; and likewise Λ_k is the transition amplitude for pair annihilation. The appropriate description in this case is in terms of squeezed states. For an account of the inverted harmonic oscillator in terms of squeezed states see *eg.* Ref. [17].

Our definition of particle number can be used for studies of quantum-to-classical transition, decoherence and entropy calculations of *eg.* cosmological perturbations [17, 18, 19]. Moreover, when suitably normalised, the particle density $n_{\mathbf{k}}$ can be used to define a density matrix on phase space, $\varrho_{\mathbf{k}} = n_{\mathbf{k}}/\sum_{\mathbf{k}'} n_{\mathbf{k}'}$.

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[1] E. P. Wigner, Phys. Rev. **40** (1932) 749.
[2] M. Hillery, R. F. O'Connell, M. O. Scully and E. P. Wigner, Phys. Rept. **106** (1984) 121.
[3] Y. Kluger, E. Mottola and J. M. Eisenberg, Phys. Rev. D **58** (1998) 125015 [hep-ph/9803372].
[4] G. Aarts and J. Berges, Phys. Rev. D **64** (2001) 105010 [hep-ph/0103049].
[5] L. Parker, Phys. Rev. **183** (1969) 1057.
[6] S. G. Mamaev, V. M. Mostepanenko and V. M. Frolov, Sov. J. Nucl. Phys. **23** (1976) 592.
[7] D. J. Chung, E. W. Kolb, A. Riotto and I. I. Tkachev, Phys. Rev. D **62** (2000) 043508 [hep-ph/9910437].
[8] P. f. Zhuang and U. W. Heinz, Phys. Rev. D **57** (1998) 6525 [hep-ph/9801221].
[9] D. Bödeker, K. Kainulainen and T. Prokopec (unpublished).
[10] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D **56** (1997) 3258 [hep-ph/9704452].
[11] M. Le Bellac, "Thermal Field Theory," Cambridge University Press (1996).
[12] T. S. Bunch and P. C. Davies, Proc. Roy. Soc. Lond. A **360** (1978) 117.
[13] Our result differs from M. Mijić, Phys. Rev. D **57** (1998) 2138 [gr-qc/9801094];
L. H. Ford, Phys. Rev. D **35** (1987) 2955.
[14] K. Kainulainen, T. Prokopec, M. G. Schmidt and S. Weinstock, Phys. Rev. D **66** (2002) 043502 [hep-ph/0202177];
JHEP **0106** (2001) 031 [hep-ph/0105295].
[15] M. Peloso and L. Sorbo, JHEP **0005** (2000) 016 [hep-ph/0003045].
[16] H. E. Kandrup, Phys. Rev. D **38** (1988) 1773.
[17] A. Albrecht, P. Ferreira, M. Joyce and T. Prokopec, Phys. Rev. D **50** (1994) 4807 [astro-ph/9303001].
[18] D. Polarski and A. A. Starobinsky, Class. Quant. Grav. **13** (1996) 377 [gr-qc/9504030].
[19] R. H. Brandenberger, V. Mukhanov and T. Prokopec, Phys. Rev. Lett. **69** (1992) 3606 [astro-ph/9206005].