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SUPERLUMINALITY IN QUANTUM THEORY

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Abstract. In the present work we investigate the possibility of superluminal information transmission in quantum theory. We give simple and general arguments to prove that the general structure (Hilbert's space plus instantaneous state reduction) of the theory allows the existence of superluminal communication. We discuss how this relates with existing no-signalling theorems.

Introduction

Since the EPR paradox appeared [1] the question of which were the sense of non-locality in quantum theory was a very fundamental question to address [9]. One main problem is to know if quantum non-locality implies superluminal communication between two separated parts A and B. Several arguments are given to exclude this hypotheses [5] [7] [3]: they form well-known no-signalling theorems. However these theorems have all the common hypotheses that operators associated with the two separated parts commute: [A,B]=0. We show possible gedanken experiments which violate this hypotheses: in effect we give two examples that prove that the general structure (Hilbert's space plus instantaneous state reduction) of quantum theory allows the existence of superluminal communication.

Superluminal communication

In this section we propose two different protocols for superluminal communication and we discuss how they circumvent no-signalling theorems. Suppose that a quantum system, say a particle, is in the state

$$\frac{1}{\sqrt{2}}(|A\rangle + |B\rangle) \tag{1}$$

where $|A\rangle$ and $|B\rangle$ represent two long distance separated spatial localization of the particle. We assume that trough all the duration of the protocol dispersion of the particle will be not relevant. So $|A\rangle$ represents the particle localized in a finite volume in region A e so does $|B\rangle$. If we make a measurement testing if particles is in the state $|A\rangle$ this action is represented by the projector $|A\rangle\langle A|$ where $\langle A|$ has the properties

$$\langle A|\, A\rangle = 1; \langle A|\, B\rangle = 0 \tag{2}$$

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In this way observing the state (1) using $|A\rangle\langle A|$ will transform the state of the system in $|A\rangle$ or in with equal probability. Analogously we can define a projector

$$|B\rangle\langle B| \tag{3}$$

Let construct a new projector

$$\frac{1}{\sqrt{2}}|A+B\rangle\frac{1}{\sqrt{2}}\langle A+B| \tag{4}$$

defining

$$\frac{1}{\sqrt{2}}\langle A+B| = \frac{1}{\sqrt{2}}(\langle A| + \langle B|)$$

where $\frac{1}{\sqrt{2}}\langle A+B|$ has the properties

$$\frac{1}{\sqrt{2}}\langle A+B|\frac{1}{\sqrt{2}}(A+B)\rangle = 1$$
$$\frac{1}{\sqrt{2}}\langle A+B|\frac{1}{\sqrt{2}}(A-B)\rangle = 0$$

In this way observing the state $|A\rangle$ using projector (4) will transform the state of the system in $\frac{1}{\sqrt{2}}(|A\rangle + |B\rangle)$ or in $\frac{1}{\sqrt{2}}(|(A\rangle - |B\rangle)$ with equal probability. In fact $|A\rangle = \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}}(|A\rangle + |B\rangle) + \frac{1}{\sqrt{2}}(|A\rangle - |B\rangle))$ where $\frac{1}{\sqrt{2}}(|A\rangle + |B\rangle)$ and $\frac{1}{\sqrt{2}}(|A\rangle - |B\rangle)$ are the eigen-vectors of projector $\frac{1}{\sqrt{2}}|A+B\rangle\frac{1}{\sqrt{2}}\langle A+B|$. Making now a projection with $|B\rangle\langle B|$ and whatever was the state leaved by $\frac{1}{\sqrt{2}}|A+B\rangle\frac{1}{\sqrt{2}}\langle A+B|$ we will find the particle in the state $|B\rangle$ and so with probability $\frac{1}{2}$ we teleportate the particle from A to B.
Using a large collection of particle will permit us to raise the probability that at least one particle reaches B close as we want to 1.
Summarizing we have a particle in the localized state $|A\rangle$, part A makes a measurement projecting onto $\frac{1}{\sqrt{2}}(|A\rangle + |B\rangle)$ then part B makes a second measurement projecting onto $|B\rangle$ in a finite amount of time. This protocol leads with probability $\frac{1}{2}$ to teleportate particle from A to B.
The key feature of this protocol is the use of projector (4). We see that this projector doesn't commute with the one associated with B (3). In fact

$$[\frac{1}{\sqrt{2}}|A+B\rangle\frac{1}{\sqrt{2}}\langle A+B||B\rangle\langle B|, |B\rangle\langle B|\frac{1}{\sqrt{2}}|A+B\rangle\frac{1}{\sqrt{2}}\langle A+B|] =$$
$$\frac{1}{2}|A+B\rangle\langle B| - \frac{1}{2}|B\rangle\langle A+B| = \frac{1}{2}(|A\rangle\langle B| - |B\rangle\langle A| \neq 0$$

A protocol that uses only statistical correlations between spin systems could be find in [6].
There is however a physical objection that may be done to measurements of the kind (4). In effect to do this measurement part A would physically operate on both region of space A and B because this measurement test a non-local properties of the system. We can test if the particle is in A or in B just operating locally in A (or in B) but it is not clear if measurement (4) may be physically achieved by part A operating locally in A (or in B) or even non-locally in A and in B. Even if this operator is formally a well defined self-adjoint operator it is hard to imagine

a physically (local or non-local) realization of such a measurement. Anyway this problem doesn't seems to affect the next example of superluminal communication. Suppose now part A has a particle in the state $|A\rangle$. So the particle is strictly localized around A and probability that B detect the particle is very near to 0. Part A performs (locally) a precision measurement on the momentum of the particle. This "collapse" the wave function from $|A\rangle$ to $|A'\rangle$. The new state $|A'\rangle$, in momentum domain, is strictly localized around a random value λ that is the result of precision momentum measurement. Obviously, as indetermination relation between momentum and position $2\Delta x\Delta p \geq \hbar$ requires, the new state $|A'\rangle$ is more spread in space than old state $|A\rangle$ so that now probability that part B detects the particle will be increased. So part A could send instantaneous information to B. Again the protocol circumvent the no-signalling theorems because measurement operators of part A and part B doesn't commute. We do now some formal calculations to validate this protocol.

Suppose part A and B laying on a line. Part A is located at the origin and B is located at a distance d from A. Part B detects if the particle is located in the region [d-k,d+k]. So the starting state $|A\rangle$ written in space domain is

$$|A(x)\rangle = \frac{1}{\sqrt[4]{\pi\sigma^2}}\ e^{-\frac{x^2}{2\sigma^2}} \tag{5}$$

($\hbar = 1$) with the variance small enough to let probability that part B detect the particle near to 0

$$\int_{d-k}^{d+k} \parallel \frac{1}{\sqrt[4]{\pi\sigma^2}}\ e^{-\frac{x^2}{2\sigma^2}} \parallel^2 dx \approx 0 \tag{6}$$

Fourier transform of state $|A\rangle$ will give us the wave function written in momentum domain

$$F(|A\rangle) = |A(p)\rangle = \sqrt[4]{\frac{\sigma^2}{\pi}}\ e^{-\frac{\sigma^2 p^2}{2}} \tag{7}$$

After the precision momentum measurement, supposed we got the random value $p = \lambda$, the state $|A'\rangle$ written in momentum domain will be

$$|A'(p)\rangle = \sqrt[4]{\frac{\overline{\sigma}^2}{\pi}}\ e^{-\frac{\overline{\sigma}^2(p-\lambda)^2}{2}} \tag{8}$$

with $\overline{\sigma} \gg \sigma$. In the space domain state $|A'\rangle$ will be

$$|A'(x)\rangle = F^{-1}(A'(p)) = \frac{1}{\sqrt[4]{\pi\overline{\sigma}^2}}\ e^{-\frac{x^2}{2\overline{\sigma}^2}-\frac{i\lambda x}{2}} \tag{9}$$

The probability that part B will detect the particle is now

$$\int_{d-k}^{d+k} \parallel \frac{1}{\sqrt[4]{\pi\overline{\sigma}^2}}\ e^{-\frac{x^2}{2\overline{\sigma}^2}-\frac{i\lambda x}{2}} \parallel^2 dx = \int_{d-k}^{d+k} \frac{1}{\sqrt[2]{\pi\overline{\sigma}^2}}\ e^{-\frac{x^2}{\overline{\sigma}^2}} dx > 0 \tag{10}$$

and we see that doesn't depend from the λ measured momentum.

Conclusion

Even if superluminal velocity in unitary evolutions of quantum system was already known [4] [8] we think that we have here provides some clear examples of how hypotheses of instantaneous wave collapse in quantum theory will permit superluminal communication. This is not in conflict with existing no-signalling theorems

because we don't use commuting operators. Our main conclusion is that quantum theory doesn't peacefully coexist with special relativity and no-signalling requirements will impose new postulates to quantum theory. In effect it turns out that a sufficient condition for no-signalling [2] is $[A, B] = 0$ where A are B are measurement operators associated with part A and B but , as we have seen, we can imagine several situations in which $[A, B] \neq 0$.

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