

Secure Key Distribution by Swapping Quantum Entanglement

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We report two key distribution schemes achieved by swapping quantum entanglement. Using two Bell states, two bits of secret key can be shared between two distant parties that play symmetric and equal roles. We also address eavesdropping attacks against the schemes.

PACS numbers: 03.67.-a, 03.67.Dd

Cryptography has been one of the most fruitful applications coming out of quantum information theory and it appears to be practically implementible in the nearest future among quantum technology [1]. Since the first key distribution protocol using four quantum states was proposed in 1984 (called BB84) [2], a number of cryptographic methods based on quantum mechanics have been proposed [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. At the heart of quantum technology, including that of cryptography, lies entanglement. Quantum entanglement is a subtle nonlocal correlation between the parts of a quantum system and has no classical analog. In 1991, Ekert showed [3] that quantum entanglement can be useful in sharing private keys between the two parties. Suppose Alice and Bob share many maximally entangled pairs of qubits. They then make measurements in jointly determined random bases. After the measurements, Alice and Bob publicly announce which basis they have used. If they had measured in the same basis, the keys would be perfectly correlated. Instead of discarding the keys resulting from measuring in different bases, Alice and Bob use them to check whether or not Bell's inequality is satisfied. If it is, then Eve's presence is detected. If not, Eve is absent and they keep the perfectly correlated keys. Entanglement swapping [16] (also see [17, 18]) is a method that enables one to entangle two quantum systems that do not have direct interaction with one another. Based on entanglement swapping, quantum key distribution (QKD) protocols have been introduced [8, 13, 15] as well as the ones without alternative measurements [5, 8, 9, 12, 13] as seen in BB84 and Ekert's protocols.

In this paper, we report two QKD schemes using entanglement swapping which also do not require alternative measurements, thereby improving the rate of generated key bits per transmitted qubit, i.e. two bits per two Bell states. This rate of generated key bits is an improvement from the protocols introduced in [8, 13]. In order to illustrate entanglement swapping, we first define four Bell states as $\Phi^\pm \equiv (|00\rangle \pm |11\rangle)/\sqrt{2}$ and $\Psi^\pm \equiv (|01\rangle \pm |10\rangle)/\sqrt{2}$. Suppose two distant parties, Alice and Bob, share Φ_{12}^+ and Φ_{34}^+ where Alice has qubits 1 and 4, and Bob possesses 2 and 3. A measurement is performed on qubits 2 and 3 with the Bell basis, Φ^\pm and Ψ^\pm , then the total state is projected onto $|\eta_1\rangle = \Phi_{23}^+ \otimes \Phi_{14}^+$, $|\eta_2\rangle = \Phi_{23}^- \otimes \Phi_{14}^-$, $|\eta_3\rangle = \Psi_{23}^+ \otimes \Psi_{14}^+$, and $|\eta_4\rangle = \Psi_{23}^- \otimes \Psi_{14}^-$

with equal probability of 1/4 for each. Previous entanglement between qubits 1 and 2, and 3 and 4 are now swapped into entanglement between qubits 2 and 3, and 1 and 4. Although we considered entanglement swapping with the initial state $\Phi_{12}^+ \otimes \Phi_{34}^+$, similar results can be achieved with other Bell states. For example, when Alice and Bob initially share Φ_{12}^- and Ψ_{34}^+ , there are four possible measurement outcomes with equal probability. If Bob gets Φ^+ when qubits 2 and 3 are measured, then Alice will obtain Ψ^- for qubits 1 and 4. We denote this possibility as $\{\Psi_{14}^-, \Phi_{23}^+\}$. There are three other possibilities, $\{\Phi_{14}^+, \Psi_{23}^-\}$, $\{\Phi_{14}^-, \Psi_{23}^+\}$ and $\{\Psi_{14}^+, \Phi_{23}^-\}$. Table I shows Bell measurement outcomes for initial states of a different combination of four Bell states.

In order to illustrate QKD based on entanglement swapping, we assign two bits to the measurement results as shown in table II. This assignment of two bits has a stipulation that one has to know both Alice and Bob's results in order to know the key bits. For instance, even if Eve knows Φ_{14}^+ without knowing Bob's result, there are still four different possible keys. We also introduce another basis, i.e. a rotated basis, $\omega^\pm \equiv (|0+\rangle \pm |1-\rangle)/\sqrt{2}$ and $\chi^\pm \equiv (|0-\rangle \pm |1+\rangle)/\sqrt{2}$ where $|+\rangle \equiv \sqrt{1/4}|0\rangle + \sqrt{3/4}|1\rangle$ and $|-\rangle \equiv \sqrt{3/4}|0\rangle - \sqrt{1/4}|1\rangle$. The first proposed QKD protocol, which we will call *Scheme I*, goes as follows (see Fig. 1):

- (S1) Alice prepares qubits 1 and 2 either in Bell basis, i.e. chosen from $\Psi_{12}^\pm, \Phi_{12}^\pm$, or on a rotated basis, $\omega_{12}^\pm, \chi_{12}^\pm$, known only to herself.
- (S2) Bob also prepares qubits 3 and 4 either in Bell basis chosen from $\Psi_{34}^\pm, \Phi_{34}^\pm$, or on a rotated basis of $\omega_{34}^\pm, \chi_{34}^\pm$, known only to himself.
- (S3) Alice sends qubit 2 to Bob and Bob transmits qubit 4 to Alice through public channels.
- (S4) Alice and Bob each publicly confirm that the other received the qubits.
- (S5) Alice and Bob also announce which basis have been used (i.e. either Bell basis or a rotated basis).
- (S6) If the received qubit had been prepared in the rotated basis, Alice (or Bob) rotates back the received qubit into Bell basis by applying $U_R = \sqrt{1/4}|0\rangle\langle 0| + \sqrt{3/4}|0\rangle\langle 1| + \sqrt{3/4}|1\rangle\langle 0| - \sqrt{1/4}|1\rangle\langle 1|$.

	Φ_{34}^+	Φ_{34}^-	Ψ_{34}^+	Ψ_{34}^-
Φ_{12}^+	Id	$+-$	$\Phi\Psi$	both
Φ_{12}^-	$+-$	Id	both	$\Phi\Psi$
Ψ_{12}^+	$\Phi\Psi$	both	Id	$+-$
Ψ_{12}^-	both	$\Phi\Psi$	$+-$	Id

TABLE I: Alice (on qubits 1 and 4) and Bob's (on qubits 2 and 3) Bell measurement results for Bell states 1 and 2, and 3 and 4. The symbol, "**Id**" means there are four possible outcomes for Alice and Bob's Bell measurement, $\{\Phi_{14}^+, \Phi_{23}^+\}$, $\{\Phi_{14}^-, \Phi_{23}^-\}$, $\{\Psi_{14}^+, \Psi_{23}^+\}$ and $\{\Psi_{14}^-, \Psi_{23}^-\}$ with equal probability of 1/4. Similarly, " $+-$ " \Rightarrow $\{\Phi_{14}^+, \Phi_{23}^-\}$, $\{\Phi_{14}^-, \Phi_{23}^+\}$, $\{\Psi_{14}^+, \Psi_{23}^-\}$, $\{\Psi_{14}^-, \Psi_{23}^+\}$, and " $\Phi\Psi$ " \Rightarrow $\{\Phi_{14}^+, \Psi_{23}^+\}$, $\{\Phi_{14}^-, \Psi_{23}^-\}$, $\{\Psi_{14}^+, \Phi_{23}^+\}$, $\{\Psi_{14}^-, \Phi_{23}^-\}$, and "both" \Rightarrow $\{\Phi_{14}^+, \Psi_{23}^-\}$, $\{\Phi_{14}^-, \Psi_{23}^+\}$, $\{\Psi_{14}^+, \Phi_{23}^-\}$, $\{\Psi_{14}^-, \Phi_{23}^+\}$.

	Id	$+-$	$\Phi\Psi$	both
"00"	$\{\Phi_{14}^+, \Phi_{23}^+\}$	$\{\Psi_{14}^-, \Psi_{23}^+\}$	$\{\Phi_{14}^-, \Psi_{23}^-\}$	$\{\Psi_{14}^+, \Phi_{23}^-\}$
"01"	$\{\Psi_{14}^-, \Psi_{23}^-\}$	$\{\Phi_{14}^+, \Phi_{23}^-\}$	$\{\Psi_{14}^+, \Phi_{23}^+\}$	$\{\Phi_{14}^-, \Psi_{23}^+\}$
"10"	$\{\Psi_{14}^+, \Psi_{23}^+\}$	$\{\Phi_{14}^-, \Phi_{23}^+\}$	$\{\Psi_{14}^-, \Phi_{23}^-\}$	$\{\Phi_{14}^+, \Psi_{23}^-\}$
"11"	$\{\Phi_{14}^-, \Phi_{23}^-\}$	$\{\Psi_{14}^+, \Psi_{23}^-\}$	$\{\Phi_{14}^+, \Psi_{23}^+\}$	$\{\Psi_{14}^-, \Phi_{23}^+\}$

TABLE II: Assignment of key bits for Alice and Bob's measurement results for *Scheme I*. In order to discover key bits, one has to know both Alice and Bob's measurement results. For *Scheme II*, we assign two bits to the initial Bell state preparation, thereby the subscripts 14 and 23 are replaced by 12 and 34, respectively, in the table.

- (S7) Alice and Bob perform Bell measurements on 1 and 4, and 2 and 3, respectively.
- (S8) Alice and Bob announce which initial Bell states (If the initial state were prepared on a rotated basis, then it would correspond to one of four Bell states after rotating it back, using the same local unitary operation U_R in (S6)) each had prepared.
- (S9) Now knowing the initial Bell states prepared by each other and their own measurement result, they could determine which Bell measurement result the other had obtained.
- (S10) Finally, with the given Bell measurement result, Alice and Bob share two key bits according to table II.

For example, suppose Alice initially prepared Ψ_{12}^+ (or χ_{12}^+) while Bob prepared Φ_{34}^- (or ω_{34}^-). After they publicly announce their prepared states as in (S8), from table I, they could find out that there are four possible Bell measurement outcomes: $\{\Phi_{14}^+, \Psi_{23}^-\}$, $\{\Phi_{14}^-, \Psi_{23}^+\}$, $\{\Psi_{14}^+, \Phi_{23}^-\}$, and $\{\Psi_{14}^-, \Phi_{23}^+\}$. Therefore, Alice and Bob, knowing their own measurement results, could determine which correlated measurement result both are sharing. If Alice got Φ_{14}^- and Bob obtained Ψ_{23}^+ , then they will share the key bits "01" according to table II.

In order to describe the second QKD scheme, we will assign two bits to the initial state preparations. Just as

we assigned two bits to Alice and Bob's measurement results, we will assign the same two bits where initial state for qubits 1 and 2 replacing Alice's measurement result and initial Bell state of qubits 3 and 4 replacing Bob's measurement result in table II. As in *Scheme I*, the key bits for ω^\pm (χ^\pm) will be same as Φ^\pm (Ψ^\pm). The second protocol is very similar to the first one, and it proceeds the same as the first one until (S7) of *Scheme I*. Then it goes as follows:

- (S8') Alice and Bob announce the result of their measurements.
- (S9') Now each knowing the measurement results for both and their own prepared state, Alice and Bob could determine the initial state preparation by each other from table III.
- (S10') This enables Alice and Bob to share two key bits according to table II.

For example, if Alice's measurement result is Φ_{14}^- and Bob obtains Ψ_{23}^- , then from table III, there are four possible state preparation: $\{\Phi_{12}^+, \Psi_{34}^+\}$, $\{\Phi_{12}^-, \Psi_{34}^-\}$, $\{\Psi_{12}^+, \Phi_{34}^+\}$, and $\{\Psi_{12}^-, \Phi_{34}^-\}$. If Alice initially prepared Ψ_{12}^+ and Bob prepared Φ_{34}^+ then they will share the key bits "01" according to table II.

Although in the QKD schemes described above assert that Alice and Bob publicly confirm whether the other received the qubits and announce initial state preparation or measurement result, in practice, this communication will be through a private channel. In order to prevent Eve from listening and altering these classical messages, encrypted messages can be used. Note that two proposed QKD schemes assume equal roles played by Alice and Bob.

As in BB84, Alice and Bob can detect eavesdropping by comparing the shared information publicly. They will take out a sample and compare by publicly announcing both the correlated measurement results and the initial states. Comparing measurement result and initial states rather than key bits gives extra safety since there are four different possible measurement results (or initial preparations for *Scheme II*) for each key. Let us consider an eavesdropping scenario for *Scheme I* as shown in Fig. 2. Eve prepares Φ_{56}^+ while Alice and Bob, as before, prepare initial states. Alice sends her qubit 2, and Eve intercepts it and sends qubit 6 to Bob instead. Suppose Alice and Bob use only the Bell basis rather than using both Bell and the rotated basis. Eve could perform Bell measurement on qubits 5 and 4 and perform local unitary operation, $\mathbf{1}$ for Φ_{54}^+ , σ_z for Φ_{54}^- , σ_x for Ψ_{54}^+ , and $\sigma_x\sigma_z$ for Ψ_{54}^- , on qubit 2 and return it to Alice. After Alice and Bob announce initial preparation, Eve could find out the key bits Alice and Bob are sharing. However, this would not be possible since Alice and Bob prepared their initial states in both Bell and the rotated bases and announced the choice of basis after they confirmed the other received the qubit. Therefore, it is important for Alice and Bob not to reveal which state had been prepared initially and

Measurement results	States prepared
Id	$\Phi_{12}^+ \Phi_{34}^+, \Phi_{12}^- \Phi_{34}^-, \Psi_{12}^+ \Psi_{34}^+, \Psi_{12}^- \Psi_{34}^-$
+-	$\Phi_{12}^+ \Phi_{34}^-, \Phi_{12}^- \Phi_{34}^+, \Psi_{12}^+ \Psi_{34}^-, \Psi_{12}^- \Psi_{34}^+$
$\Phi\Psi$	$\Phi_{12}^+ \Psi_{34}^+, \Phi_{12}^- \Psi_{34}^-, \Psi_{12}^+ \Phi_{34}^+, \Psi_{12}^- \Phi_{34}^-$
both	$\Phi_{12}^+ \Psi_{34}^-, \Phi_{12}^- \Psi_{34}^+, \Psi_{12}^+ \Phi_{34}^-, \Psi_{12}^- \Phi_{34}^+$

TABLE III: Possible state preparations for given measurement results for *Scheme II*. When Alice and Bob obtain the measurement result as in the first column, there are four possible initial Bell state preparation (If the initial state were prepared on a rotated basis, then it would correspond to one of four Bell states after rotating it back, using the same local unitary operation U_R in (S6)) as shown in the second column. The symbols in the “Measurement results” column are the same as in Table I.

perform measurements after they confirm the other received the qubits.

Practical feasibility of the proposed schemes can be sought in experiments that use Bell operator measurements, such as teleportation [19] and entanglement swapping [20]. Successful Bell type measurements have been performed using two photons, which were both path- and polarization entangled. Although there is an improvement of the key bit generation rate compared to the protocol introduced by Zhao *et. al.* in [13], practical implementation of the proposed scheme will have difficulty with classical information needed to perform necessary local unitary operations before performing a Bell measurement for each photon received.

The author thanks E. Knill for pointing out an error in the previous version of the proposed protocol and making many helpful comments. The author is also grateful to R. Boisvert, G. Brennen, and R. Kuhn for helpful discussions.

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- [1] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, *Rev. Mod. Phys.* **74**, 145 (2002).
- [2] C.H. Bennett and G. Brassard, in *Proceedings of the IEEE International Conference on Computers, Systems and Signal Processings, Bangalore, India* (IEEE, New York, 1984), p. 175.
- [3] A.K. Ekert, *Phys. Rev. Lett.* **67**, 661 (1991).
- [4] C.H. Bennett, *Phys. Rev. Lett.* **68**, 3121 (1992).
- [5] L. Goldenberg and L. Vaidman, *Phys. Rev. Lett.* **75**, 1239 (1995).
- [6] D. Bruß, *Phys. Rev. Lett.* **81**, 3018 (1998).
- [7] H. Bechmann-Pasquinucci and N. Gisin, *Phys. Rev. A* **59**, 4238 (1999).
- [8] A. Cabello, *Phys. Rev. A* **61**, 052312 (2000); *Phys. Rev. A* **64**, 024301 (2001).
- [9] A. Cabello, *Phys. Rev. Lett.* **85**, 5635 (2000).
- [10] G.L. Long and X.S. Liu, *Phys. Rev. A* **65**, 032302 (2002).
- [11] G.-P. Guo, C.-F. Li, B.-S. Shi, J. Li, and G.-C. Guo, *Phys. Rev. A* **64**, 042301 (2001).
- [12] A. Beige, B.-G. Englert, Ch. Kurtseifer, and H. Weinfurter, *Acta. Phys. Pol. A* **101**, 357 (2002).
- [13] Z. Zhao, T. Yang, Z.-B. Chen, J. Du, and J.-W. Pan, *quant-ph/0211098*
- [14] J.-W. Lee, E.K. Lee, Y.W. Chung, H.-W. Lee, and J. Kim, *Phys. Rev. A* **68**, 012324 (2003).
- [15] J. Lee, S. Lee, J. Kim, and S.D. Oh, *quant-ph/0309185*.
- [16] M. Zukowski, A. Zeilinger, M.A. Horne and A.K. Ekert, *Phys. Rev. Lett.* **71**, 4287 (1993).
- [17] S. Bose, V. Vedral and P.L. Knight, *Phys. Rev. A* **57**, 822 (1998).
- [18] L. Hardy and D. Song, *Phys. Rev. A* **62**, 052315 (2000).
- [19] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, *Phys. Rev. Lett.* **80**, 1121 (1998).
- [20] J.-W. Pan, D. Bouwmeester, H. Weinfurter, and A. Zeilinger, *Phys. Rev. Lett.* **80**, 3891 (1998).

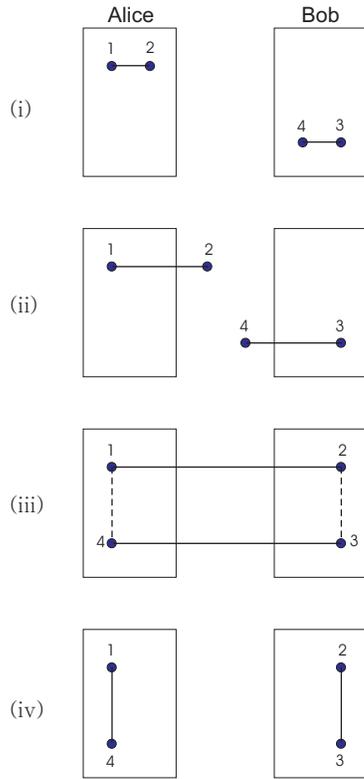


FIG. 1: QKD using entanglement swapping. The bold line means qubits are entangled and the dotted line implies a Bell measurement is performed on the qubits. (i) Alice and Bob each prepare arbitrary states, either in Bell or on a rotated bases, known only to themselves. (ii) Each sends one qubit of the states to the other party. (iii) After they confirm publicly the other person received the qubit and announce the choice of basis for initial preparation, Alice and Bob perform Bell measurements on 1 and 4, and 2 and 3, respectively. (iv) Alice and Bob now announce which Bell states had been prepared (or their measurement result for *Scheme II*), and they are able to find out the correlated measurement results (or initial state preparation for *Scheme II*).

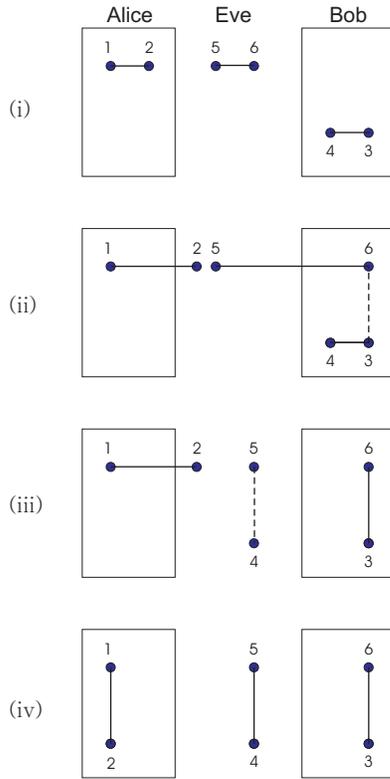


FIG. 2: Eavesdropping scheme on the proposed entanglement swapping QKD. As in Fig. 1, the bold line means the qubits are entangled and the dotted line implies a Bell measurement is performed on the qubits. (i) While Alice and Bob prepare arbitrary states, Eve also prepares Φ_{56}^+ . (ii) Eve intercepts qubit 2 sent by Alice and sends qubit 6 to Bob instead. (iii) Eve performs a Bell measurement on qubits 5 and 4. (iv) Eve then performs a local operation on qubit 2 and sends it back to Alice.