

Why isn't every physicist a Bohmian?

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This note collects, classifies and evaluates common criticism against the de Broglie-Bohm theory, including Ockham's razor, asymmetry in the de Broglie-Bohm theory, the "surreal trajectory" problem, the underdetermination of the de Broglie-Bohm theory and the question of relativistic and quantum field theoretical generalizations of the de Broglie-Bohm theory. We argue that none of these objections provide a rigorous disproof, they rather highlight that even in science theories can not solely be evaluated based on their empirical confirmation.

1. Introduction

In a previous article [74] we have argued that the de Broglie-Bohm theory can play an important role in teaching quantum mechanics since it provides an alternative view-point and illustrates the peculiar features of quantum phenomena. Of course most adherents of the de Broglie-Bohm theory would assign a more ambitious meaning to the theory and do rather claim its superiority to the ordinary formulation (or interpretation) of quantum mechanics. In this note we will examine some common objections raised against the de Broglie-Bohm theory.

In answering the question posed in the title one should first remark that the ongoing debate on the foundations of quantum mechanics has produced a vast number of different schools and interpretations. Presumably, the majority of physicists has lost track of this complex debate about the measurement problem, hidden variables, EPR, Bell etc. In a similar context this was strikingly expressed by David Mermin:

Contemporary physicists come in two varieties. Type 1 physicists are bothered by EPR and Bell's Theorem. Type 2 (the majority) are not, but one has to distinguish two subvarieties. Type 2a physicists explain why they are not bothered. Their explanations tend either to miss the point entirely (like Born's to Einstein) or to contain physical assertions that can be shown to be false. Type 2b are not bothered and refuse to explain why. (quoted from [83])

Even if one takes this remark with a pinch of salt, Mermin's observation that many physicists do not have a well founded standpoint in this affair seems to be correct.

Another reason for being rather indifferent to the de Broglie-Bohm theory is evidently the following. This theory can be viewed as a way to solve the conceptual problems of quantum mechanics. Those who are satisfied with the answers given by the standard interpretation (e.g. David Mermin [71]) or who favor other non-standard interpretations (like many-worlds, consistent histories, Floyd's trajectory interpretation or the like) are consequently not attracted by the de Broglie-Bohm theory.

However, our concern is with criticism and objections which are explicitly directed against the de Broglie-Bohm theory. It is rather popular among adherents of the de Broglie-Bohm theory to blame mainly historical and sociological reasons for the contempt of their theory [12,26,60]. We do not negate that these reasons may have played some role, although such a claim is hard to verify explicitly. In any case such a position renders the criticism as completely irrational and makes a sober discussion difficult. In fact there has been response to e.g. Bohm's paper from 1952. Wayne Myrvold, who has analyzed early objections against the de Broglie-Bohm theory, writes [73] :

Bohm's theory did not meet with the acceptance in the physics community that Bohm had hoped for. It was not, however, ignored; several prominent physicists, among which were Einstein, Pauli, and Heisenberg, wrote articles expressing their reasons for not accepting Bohm's theory.

In what follows we will also explore these early objections¹.

¹In fact, not all of the early reactions were hostile. For example in 1953 Joseph Keller from the New York University published a Physics Review paper in which he analyzed

The objections against the de Broglie-Bohm theory can roughly be divided into two classes². The first applies meta-theoretical considerations i.e. invokes criteria like symmetry or simplicity to discard the de Broglie-Bohm theory. Section 3 is devoted to these arguments. The other class of criticism seeks for a more textual or theory-immanent debate, like challenging the consistency or the ability of the de Broglie-Bohm theory to be generalized. This debate will be reviewed in Section 4.

For completeness we will give a brief summary of the de Broglie-Bohm theory in Sec. 2. A thorough discussion of the de Broglie-Bohm theory can be found e.g. in [14,23,26,60,75].

2. The de Broglie-Bohm theory

The de Broglie-Bohm theory describes a non-relativistic N -particle system by its wavefunction, ψ , and the position, Q_i , of the corresponding quantum objects (e.g. electrons, atoms or the like). The wavefunction, which is derived from the ordinary Schrödinger equation, guides the particle motion via the so-called guidance equation:

$$\frac{dQ_i}{dt} = \frac{1}{m_i} \nabla_i S(Q_1, \dots, Q_N) \quad (1)$$

Here m_i denotes the mass of particle i , ∇_i is the nabla operator applied to its coordinates and S the phase of the wavefunction in the polar representation $\psi = Re^{iS}$.

Since the guidance condition 1 is a first-order equation, one initial condition fixes the motion uniquely. Given a $\rho = |\psi|^2$ distribution as initial positions Equ. 1 will reproduce all predictions of ordinary quantum mechanics in terms of position distributions. Since all measurements can be expressed in terms of position (e.g. pointer positions) this amounts to full accordance with all predictions of ordinary quantum mechanics. Thereby the de Broglie-Bohm theory assigns a distinguished role to position and does not independently assign possessed-values to other observables. This ensures that the Kochen-Specker “no-go” theorem does not apply to the de Broglie-Bohm theory. What might be regarded as the values of quantum observables like spin, momentum or the like get established only in the context of a corresponding measurement-like experiment. From the viewpoint of the de Broglie-Bohm theory this “contextuality” amounts essentially to

the role of probability in Bohm’s interpretation [64]. He qualified Bohm’s work as an “interesting interpretation” of quantum mechanics.

²These classes are not completely disjoint and serve also the purpose to structure the presentation.

the observation, that the outcome of an experiment depends on the way it is performed.

As mentioned above the de Broglie-Bohm theory reproduces all predictions of ordinary quantum theory provided that the initial positions of particles described by the wavefunction ψ are $|\psi|^2$ distributed. The motivation of this so-called quantum equilibrium hypothesis has been explored for example in [36,89]. Most important, the quantum mechanical continuity equation (Equ. 2) ensures that this condition is consistent i.e. any system will stay $|\psi|^2$ distributed if the quantum equilibrium hypothesis holds initially.

$$\frac{\partial |\psi|^2}{\partial t} + \nabla \cdot \left(|\psi|^2 \cdot \frac{\nabla S}{m} \right) = 0. \quad (2)$$

It follows that in a universe being in quantum equilibrium it is not possible to control the initial positions beyond the $|\psi|^2$ distribution. Hence the de Broglie-Bohm theory does not allow for an experimental violation of Heisenberg’s uncertainty principle [89]. While ordinary quantum mechanics assumes that probability enters on a fundamental level, the de Broglie-Bohm theory is deterministic and probability enters only as an expression of ignorance. However, given the quantum equilibrium hypothesis this ignorance holds in principle. Thus the fundamental determinism is turned into predictive indeterminism.

The important feature of Equ.1 is its non-locality. The guidance equation links the motion of every particle to the configuration of the whole system, no matter how distant its different parts are. Technically expressed this follows from the fact, that the wavefunction ψ (hence its phase S) at a given time is a function on the configuration space \mathbb{R}^{3N} . It is exactly this non-locality which allows the de Broglie-Bohm theory to violate the Bell inequalities [9] as demanded by experiment³. However, this non-locality vanishes if the wavefunction factorizes in the contributions of the different particles.

The guidance condition 1 can be motivated in different ways and its precise status gives rise to different interpretations of the de Broglie-Bohm theory. The starting point of Bohm’s original presentation of the theory⁴ in 1952 [18] was the decomposition of the Schrödinger equation for the

³Note, that the two “no-go” theorems of Bell and Kochen-Specker actually strengthen the position of the de Broglie-Bohm theory. The first shows that “hidden variable” theories have to be non-local in order to agree with the predictions of quantum mechanics. The second rules out that a mapping of all observables to possessed values can be achieved. Hence, the corresponding properties of the de Broglie-Bohm theory (namely the non-locality and contextuality) are no shortcomings but necessities.

⁴We refer to this theory as de Broglie-Bohm theory since

wavefunction $\psi = Re^{\frac{i}{\hbar}S}$ into a set of two equations for the real functions R and S . The resulting equation for S has a structure similar to the classical Hamilton-Jacobi equation for the action S , which implies $p = \nabla S$. The only difference is the appearance of an extra term which Bohm named “quantum potential”:

$$U_{\text{quant}} = -\frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} \quad (3)$$

Bohm (and later also e.g. Hiley [22] and Holland [60]) regard the quantum potential as the key ingredient of the de Broglie-Bohm theory and derive all its novelty from it. The guidance equation 1 is only viewed as a “special assumption” [20] or a “consistent subsidiary condition” [19].

In contrast to this position another school of the de Broglie-Bohm theory regards the guidance condition as the fundamental equation and avoids emphasizing the quantum potential. The main proponents of this school are Dürr et al. [14,29,36,38] who have named their version of the de Broglie-Bohm theory “Bohmian mechanics”. This view was anticipated by John Bell in his work on the de Broglie-Bohm theory [11]. In fact the guidance equation can be motivated without appeal to the Hamilton-Jacobi equation from symmetry arguments alone [36]. According to this position the quantum potential deserves no special attention and is rather viewed as an artefact which enters the discussion when the classical limit of the theory is treated.

One should not mistake this discussion as only quibbling over a mathematical ambiguity in the formulation of the theory. In fact these different interpretations of Equ.1 are related to a substantial difference of view on e.g. the role of “observables” other than position or the meaning of the wavefunction. Our discussion of objections against the de Broglie-Bohm theory is in part complicated by this debate on the interpretation. If some criticism applies more strongly or solely to one specific interpretation of the de Broglie-Bohm theory, it does not undermine the concept as a whole. Likewise the different interpretations provide different replies to the objections. The different interpretation of the de Broglie-Bohm theory will be disentangled elsewhere [76].

Since the rest of our note will be concerned with the objections against the de Broglie-Bohm theory we should balance the discussion by some brief remarks on its merits. The supporters of the de Broglie-Bohm theory emphasize its “clear ontology” i.e. that the vague notion of “complementarity” and wave-particle duality becomes dis-

pensable. Within the de Broglie-Bohm theory one can consistently entertain the notion of particle trajectories. However, this should not be misunderstood as adherence to classical prejudices but provides an elegant solution of the measurement problem. The superposition of the wavefunction at the end of a measurement causes no difficulty since the actual position of the system corresponds to the actual outcome. In addition the de Broglie-Bohm theory provides means to deal non-ambiguously with the question of tunneling time or time-of-arrival [65–67]. Some authors also suggest that the de Broglie-Bohm theory has conceptual advantages over quantum mechanics in connecting quantum mechanics to other theories such as chaos theory and classical mechanics [28] or when dealing with CP violation [63].

3. The meta-theoretical debate

Most authors accept that the de Broglie-Bohm theory and ordinary non-relativistic quantum mechanics make identical predictions i.e. that no experiment can decide which one to prefer⁵. Even Wolfgang Pauli admitted in a letter to Bohm from December 1951:

I do not see any longer the possibility of any logical contradiction as long as your results agree completely with those of the usual wave mechanics and as long as no means is given to measure the values of your hidden parameters (...). [78, letter 1313]

But this was only a minimal concession to Bohm. In the absence of any new prediction the de Broglie-Bohm theory was accused of being not physics but “metaphysics” [77]. Heisenberg questioned whether the de Broglie-Bohm theory should be regarded as a new theory at all:

From the fundamentally *positivistic* (it would perhaps be better to say *purely physical*) standpoint, we are thus concerned not with counter-proposals to the Copenhagen interpretation, but with its exact repetition in a different language. (quoted after [73])

⁵In fact every now and then such an experiment is proposed nevertheless. The attempts to construct circumstances in which the predictions of the de Broglie-Bohm theory and quantum mechanics disagree are actually pointless since the de Broglie-Bohm theory reproduces all predictions of ordinary quantum mechanics by definition. Above all, the Schrödinger-equation is part of the de Broglie-Bohm theory and the individual trajectories can not be controlled beyond the quantum equilibrium.

Louis de Broglie presented similar ideas already in 1927 [15]. David Bohm’s work in 1952 was done independently.

The Heisenberg pupil von Weizsäcker reports on a course in the winter term 1953/54 in which they discussed also Bohm's work [91]:

Unsere Überzeugung, daß alle diese Versuche falsch seien, wurde durch das Seminar bestärkt. Aber wir konnten uns nicht verhehlen, dass der tiefste Grund unserer Überzeugung ein quasi ästhetischer war. Die Quantentheorie übertraf alle Konkurrenten in der für eine "abgeschlossene Theorie" kennzeichnenden einfachen Schönheit.⁶

However, the above quoted passages alone do not constitute any reason to reject the de Broglie-Bohm theory. In the absence of any "logical contradiction" (Pauli) and while objecting to the mere "repetition in a different language" (Heisenberg) one needs to specify why the ordinary quantum theory actually "surpasses all competitors" (v. Weizsäcker). Or to put it differently: additional criteria need to be formulated which help to distinguish these theories.

In what follows we collect and evaluate a number of these additional criteria which have been suggested by the above mentioned authors and others to underpin their rejection. We classify them as "meta-theoretical" since they are largely based on requirements which are supposed to apply to physics theories in general.

3.1. Ockham's razor

The obvious objection against the de Broglie-Bohm theory is that it does not make any new predictions while postulating the particle-position as a new entity. If two theories are equivalent the one should be preferred which needs fewer premises. Likewise additional premises which do not enrich the explanatory power should be removed by invoking "Ockham's razor". Given this widely accepted principle, it appears natural by some to discard the de Broglie-Bohm theory since the particle trajectories seem to be exactly such an extra premise. However, this conclusion can be challenged by the following consideration: the de Broglie-Bohm theory supplements ordinary quantum mechanics by an equation-of-motion for the quantum-particles, but eliminates the postulates which are related to the measurement process (not to mention how (un)compelling these postulates are). Furthermore

⁶This course strengthened our conviction that all this trials were false. But we could not conceal to ourselves that the deeper cause for this belief was quasi "aesthetical". Quantum mechanics surpassed all competitors by its simple beauty which characterizes a "complete theory". (translation by the author)

the de Broglie-Bohm theory provides a completely new interpretation of quantum phenomena in which e.g. probability plays no fundamental role. In other words: the descriptive content is identical but one can question whether these theories are equivalent at all. Hence, it is questionable whether the precondition for applying Ockham's razor is met.

3.2. Asymmetry in the de Broglie-Bohm theory

Pauli and Heisenberg based their rejection of the de Broglie-Bohm theory mainly on its asymmetry with respect to position and momentum [73]. In the absence of any new prediction they did not accept this sacrifice.

The reply to this objection is twofold: (i) The de Broglie-Bohm theory gives position a different ontological status than all other "observables" [29] in order to achieve a clear ontology and to solve conceptual problems of the ordinary formulation of quantum mechanics. After all symmetry is no end in itself. (ii) Moreover, the Hamiltonian in orthodox quantum theory is not invariant under general unitary transformations, though it is under the usual space time symmetries. Thus even there it is not the case that all observables are on the same footing.

However, in reply to this criticism Hiley and Brown [24,59] explore the possibility to formulate a Bohm-like theory in other than the position representation. Bohm himself took this objection very seriously and was lead to the following modest claim:

Heisenberg shows that he perhaps did not appreciate that the only purpose of this phase of the work was to show that an alternative to the Copenhagen interpretation is at least logically possible. (D. Bohm, quoted from [73])

In fact the de Broglie-Bohm theory shows another asymmetry, namely with respect to the wavefunction. While the wavefunction acts on the particle position, the particles do not react on the ψ -field. It is determined independently by the Schrödinger equation. It is true that this constitutes a peculiar feature of the de Broglie-Bohm theory. In reply to this objection Dürr et al. [38] have suggested that the role of the wavefunction within the de Broglie-Bohm theory should be regarded as analogous to the role of the Hamiltonian in classical mechanics. They state [38]:

We propose that the reason, on the universal level, that there is no action of configurations upon wavefunctions, as there seems to be between

all other elements of physical reality, is that the wavefunction of the universe is not an element of physical reality. We propose that the wavefunction belongs to an altogether different category of existence than that of substantive physical entities, and that its existence is nomological rather than material. We propose, in other words, that the wavefunction is a component of physical law rather than of the reality described by the law.

In [47] this idea is applied in the context of quantum gravity.

3.3. Return to classical physics?

A rather unspecific but never the less common objection against the de Broglie-Bohm theory is its supposed return to classical notions. For example Englert states in [43] about the purpose of the de Broglie-Bohm theory and its trajectories:

Mit Berufung auf diese Bahnen sind atomare Vorgänge dann deterministisch, und das erspart uns die Trauerarbeit, die uns der Verlust des deterministischen Newton-Maxwellschen Weltbildes abverlangt.⁷

This claim of “backwardness” is in itself no strong argument against the de Broglie-Bohm theory. One needs to add (and explain) at least why this “return” is supposed to be artificial or needless. However, this argument remains weak, since the de Broglie-Bohm theory possesses so many traits which are common to quantum mechanics and completely unclassical (e.g. wavefunction on the configuration space, nonlocality etc.pp.) that it does a disservice to anyone seeking for a “return to a Newton-Maxwell world view”. Agreed, the features of determinism and ‘objectivity’⁸ are ‘classical’, but in this respect the de Broglie-Bohm theory is as classical as the theory of relativity.

3.4. Departure from established principles

While Section 3.3 mentioned the objection of “backwardness” the de Broglie-Bohm theory meets also with the contrary criticism. Here the bizarre features of the de Broglie-Bohm theory are the subject of discomfort⁹.

According to the de Broglie-Bohm theory the wavefunction produces an actual physical effect

on the particle motion. In this respect it may be compared to other physical fields like electromagnetic or gravitational fields. This view was for example held by Bell [11][p.128]. This introduces peculiar notions into physics indeed. First of all the quantum mechanical wavefunction is defined on the configuration space of the system. This is in sharp contrast to any other physical field. The non-locality is closely related to this feature and will be the subject of Sec. 4.3. As mentioned above (see Sec. 3.2) Dürr et al. [38] have proposed that the role of the wavefunction within the de Broglie-Bohm theory should be rather compared to the role of the Hamiltonian in classical mechanics. The Hamiltonian is a function on the phase space, that is of greater dimension and even more abstract than configuration space. Following this suggestion certainly weakens this allegation. Interestingly, in this reading of the de Broglie-Bohm theory the role of ψ is similar to the widespread (“orthodox”) view of the wavefunction as a computational tool.

Viewing the wavefunction as “nomological” rather than “physical real” helps also to reply to the objection that the de Broglie-Bohm theory introduces myriads of “empty waves” into the picture of physical reality. These are the branches of the wavefunction which do not contain the particle on its trajectory hence do not correspond to the actual state of the system. Although one can argue that due to decoherence effects these empty branches do typically not affect the actual system any more¹⁰, this feature remains unaesthetic. However, as mentioned above, viewing the wavefunction as analog to the Hamiltonian invalidates this allegation.

Finally, no matter whether based on a physical real or nomological wavefunction, does the dynamics of the de Broglie-Bohm theory possess a very unclassical trait. The effect on the particle motion via the wavefunction is independent of its amplitude. This can be seen for example when the quantum-potential formulation is used. Since ψ appears in the numerator and the denominator of expression 3, ψ and $c \cdot \psi$ lead to the same effect. Bohm and Hiley have therefore compared the ψ -field to radio waves which guide an object like a ship on automatic pilot. Here too, the effect of the radio waves is independent of their intensity and depends on their form [22] only. Bohm and Hiley have coined the expression “active information” for this sort of influence and suggest that the quantum potential is a source of this kind of

⁷With appeal to these trajectories the atomic sequence of events gets deterministic and the mourning-labor about the loss of the Newton-Maxwell world view gets dispensable. (translation by the author)

⁸In the sense of ‘observer independence’

⁹Some of the objections which have been mentioned in Sec. 3.2 do fit into this category as well.

¹⁰It seems to be possible to construct circumstances in which empty waves do have subtle effects [55,88]. This discussion is closely related to the “surreal trajectory” debate and will be reviewed in Sec. 4.1.

information. Whether this radio-wave analogy is just a metaphor or leads to any deeper insight remains to be seen. A critical assessment of especially Bohm's metaphors can be found in [53].

Another intriguing property of Bohmian-trajectories gave rise to a specific objection from Einstein. Since he was one of the famous antagonists of the Copenhagen interpretation it is interesting to note that he did not endorse the de Broglie-Bohm theory likewise. In a *Festschrift* in honor of Max Born in the year 1953, Einstein discussed a system for which the de Broglie-Bohm theory predicts a vanishing velocity. Einstein discussed a particle in a box as a specific example but the same behavior appears in any system which is described by a real wavefunction like e.g. the energy eigenstates of the harmonic oscillator. According to Einstein this vanishing velocity “contradicts the well-founded requirement, that in case of a macrosystem (i.e. for highly excited states) the motion should agree approximately with the motion following from classical mechanics” [73].

However, any measurement on the particle would need a change in the arrangement (e.g. one side of the box would have to be removed). The predicted outcome of any such *measurement* of e.g. the particle-momentum would be the same as in ordinary quantum mechanics. More generally, the Einstein-objection illustrates, that within the de Broglie-Bohm theory the representation of any system is provided by the *pair* of wavefunction and position, (ψ, Q_i) . To focus on properties of one element only can be misleading¹¹.

3.5. Underdetermination in the de Broglie-Bohm theory

The de Broglie-Bohm theory reproduces the statistical predictions of ordinary quantum mechanics and underpins them with deterministic and continuous particle-trajectories. It is tempting to regard the Bohm-trajectories as the actual motion of the quantum-objects.

However, Deotto and Ghirardi [30] have shown, that the de Broglie-Bohm theory is underdeter-

mined i.e. the quantum mechanical current can be “gauged” by a divergenceless vector field $\mathbf{j}' = \mathbf{j} + \mathbf{a}$ with $\nabla \mathbf{a} = 0$. The corresponding guidance condition $\mathbf{v}' = \mathbf{j}'/|\psi|^2$ yields the same statistical predictions, while the individual trajectories differ from the standard de Broglie-Bohm theory. Hence it is problematic to regard the Bohm-trajectories derived from Equ. 1 as representing the “actual motion” of the quantum particles.

In order to sustain the “ontological status” of the Bohm trajectories one has to formulate additional criteria which restrict the possible value of the vector field \mathbf{a} . E.g. Dürr et al. [36] motivate the guidance equation 1 from symmetry and simplicity constraints. The argument in [62] is based on the assumption that the corresponding problem for relativistic spin $\frac{1}{2}$ particles has been solved uniquely [61]. Holland and Philippidis then derive the guidance equation for the non-relativistic limit and result in a equation which contains an additional spin-dependent term. Hence, for spin 0 particles the original form is recovered.

However, this ambiguity of the de Broglie-Bohm theory does not undermine its conceptual value. If the above motivations for the specific guidance condition (Equ. 1) are felt unconvincing, the de Broglie-Bohm theory still provides a proof of principle that the deterministic interpretation of quantum mechanics is possible. Since the measurement of individual trajectories is beyond the principle reach of experiments one should not put too much emphasize on their particular form anyway.

3.6. The status of the quantum equilibrium hypothesis

As mentioned in Sec. 2, the de Broglie-Bohm theory reproduces all predictions of ordinary quantum theory provided that the initial positions of particles described by the wavefunction ψ are $|\psi|^2$ distributed. One may include this assumption in the very definition of the de Broglie-Bohm theory. Equation 2 ensures that this postulate is consistent i.e. any system will stay $|\psi|^2$ distributed when the quantum equilibrium hypothesis holds initially.

However, introducing the quantum equilibrium hypothesis as a postulate provokes the objection that thereby the wavefunction gets two distinct and logically independent meanings: (i) as the guiding field and (ii) as a probability distribution for the particle position. This double role for the wavefunction looks suspicious and unaesthetic. Further more it would remain obscure how random behavior enters into the deterministic de Broglie-Bohm theory. Finally the very

¹¹Einstein's rejection of the de Broglie-Bohm theory is clearly not only related to the problem discussed above. By now famous is his remark in a letter to Born in 1952 about the de Broglie-Bohm theory being “too cheap” [41, letter from 12.5.1952]. Squires writes in the same context, that Einstein “was not interested in attempts to ‘cure’ the theory; rather he wanted to look elsewhere, to start again” [81]. Squires makes an other insightful remark about the de Broglie-Bohm theory and Einstein's probable reason to reject it: “And it is certainly true that we would not have discovered statistical mechanics by adding small corrections to thermodynamics, or by adding hidden variables that were in some way ‘guided’ by the free energy, or some other thermodynamic quantity” [81].

meaning of such a postulate would be not clear at all.

It was therefore among the early efforts of Bohm to clarify the status of the quantum equilibrium hypothesis and to possibly derive rather than postulate it. The paper [20] from 1953 was devoted to this question but could derive the quantum equilibrium hypothesis only for a limited class of systems [26,90]. This problem gave rise to the development of a modified version of the theory in 1954 including the effect of a stochastic disturbance [21]. A dynamical explanation of the quantum equilibrium hypothesis within the original version of the de Broglie-Bohm theory was also attempted by Valentini [89].

A different approach was developed by Dürr et al. [36]. Their analysis is an elaboration of work of John Bell and is ultimately rooted in the approach of Ludwig Boltzmann to statistical mechanics. The starting point is that regarding the de Broglie-Bohm theory as a fundamental theory implies that the behavior of subsystems is determined by the “wavefunction of the universe”, $\Psi(q)$, and the corresponding configuration. One is therefore not free to simply postulate that subsystems have wavefunctions and are governed by the de Broglie-Bohm theory. However, applying the quantum equilibrium hypothesis to $\Psi(q)$ seems to be physically meaningless since we do not have a sample of universes. Thus the following two questions need to be addressed: (i) how to assign a wavefunction to a subsystem and (ii) what is the meaning of the quantum equilibrium hypothesis when applied to $\Psi(q)$. Finally one can ask how to relate these points i.e. how to justify the quantum equilibrium hypothesis for empirical distributions¹².

Question (i) leads Dürr et al. to the introduction of the *effective* wavefunction. Let $q = (x, y)$ be a decomposition of the configuration of the universe into the variable x of a subsystem and y for the rest. Dürr et al. define the *effective* wavefunction, ψ , of the subsystem as part of the following decomposition:

$$\Psi(x, y) = \psi(x)\Phi(y) + \Psi^\perp(x, y) \quad (4)$$

The wavefunction $\psi(x)$ represents the subsystem provided that the y -support of $\Phi(y)$ and $\Psi^\perp(x, y)$ is macroscopically distinct and that the actual value of y lies in the support of $\Phi(y)$. A typical situation of this kind occurs during a measurement on the system described by x with a measuring device that has, at the end of the measurement, a definite value in the support of $\Phi(y)$.

¹²By them we mean relative frequencies obtained from repeated experiments on subsystems.

Regarding (ii) Dürr et al. argue, that the meaning of the quantum equilibrium distribution $|\Psi(q)|^2$ on the universal level is *not* probabilistic since we do not have a sample of universes. Instead, it provides a so-called measure of *typicality*. The notion of *typicality* (though not the word) was introduced by Boltzmann in justifying the second law of thermodynamics. This statement holds because an “overwhelming majority” of initial conditions leads to a behavior in accordance with the second law [68]. However, the meaning of “overwhelming majority” i.e. a measure on the corresponding set, needs to be specified. One important requirement for this measure is that it should be “equivariant” i.e. the notion of typicality should be independent of time. And in fact, the continuity equation 2 ensures that the measure $|\Psi(q)|^2$ is equivariant.

Finally, and that is the central result of [36], Dürr et al. can prove that within a “typical” universe the quantum equilibrium hypothesis holds for all subsystems. Hence the typical Bohmian universe – although deterministic – gives the *appearance* of randomness in agreement with quantum mechanics.

This justification of the quantum equilibrium hypothesis has been questioned e.g. by Dickson [33]. He notes that Dürr et al. have not shown that $|\Psi(q)|^2$ provides the *only* equivariant measure. Further more Dickson questions that equivariance is a preferred property of measures over the initial distributions at all. He states [33][p. 123]:

Equivariance is a *dynamical* property of a measure, whereas the question ‘Which initial distribution is the correct one?’ involves no dynamics, nor it is clear why dynamical properties of a measure are relevant.

This objection challenges the claim that the quantum equilibrium hypothesis can be derived rather than postulated. However, it should be noted that for the justification of classical thermodynamic the question of how to derive apparent randomness from deterministic laws is just as controversial.

4. The theory immanent debate

Until now we were mainly concerned with meta-theoretical objections which might be viewed as partially subjective. Consequently, some of the above mentioned feature of the de Broglie-Bohm theory have been either used to reject this theory or to praise its radical novelty. An other strategy to disclaim the de Broglie-

Bohm theory has been to seek for a more textual debate, e.g. challenging its consistency or its ability to be generalized. One might say that these arguments try to refute the de Broglie-Bohm theory from “inside”, hence we have classified them as “theory-immanent”. Most important is the question whether a trajectory-interpretation is sustainable in the relativistic domain.

A clear-cut disproof of the de Broglie-Bohm theory would be an experiment in which the predictions of the de Broglie-Bohm theory and ordinary quantum mechanics differ while the latter is confirmed. In fact every now and then such an experiment is proposed. The attempts to construct circumstances in which the predictions of the de Broglie-Bohm theory and quantum mechanics disagree are actually pointless since the de Broglie-Bohm theory reproduces all predictions of ordinary quantum mechanics by definition. Above all, the Schrödinger-equation is part of the de Broglie-Bohm theory and the individual trajectories can not be controlled beyond the quantum equilibrium. This attempts will not be considered further and the interested reader may consult [49,51,84].

4.1. The “surreal trajectory” objection

In 1992 Englert, Scully, Süssmann and Walther (ESSW) challenged the de Broglie-Bohm theory. They claimed that Bohm trajectories are not realistic, but “surrealistic”.

The corresponding authors analyze the famous delayed-choice double-slit experiment invented by Wheeler [92] and discussed in the context of the de Broglie-Bohm theory by Bell [11]. Before we turn to the actual ESSW argument we will first discuss the original set-up.

The delayed-choice double-slit experiment (see Fig. 1) consists of a double slit arrangement in which one can freely choose to detect either interference patterns in the region I or particles in the detectors C_1 or C_2 ¹³. The whole arrangement is set up in such a way that by symmetry arguments the trajectories of the de Broglie-Bohm theory are not allowed to cross the midplane behind the two-slit screen. They show the “unclassical” behavior, that the Bohm-trajectories of the particles hitting the upper part behind the screen have traversed the upper slit and vice versa.

One may modify the arrangement by supplying it with additional detectors directly behind the two-slit screen in order to investigate which slit has been traversed. In such a modified version

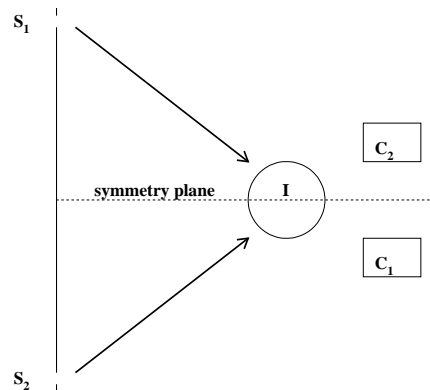


Figure 1. *Delayed-choice double slit experiment.* One may either detect particles in the detectors C_i or observe interference in region I by insertion of a screen.

the interference pattern would not occur. Additionally the de Broglie-Bohm trajectories would be allowed to cross the midplane since, given the degrees of freedom related to the detector, it is no symmetry plane anymore (see [11][p.111] for the details of this argument.).

The above mentioned extra-detectors directly behind the screen were assumed to be “ordinary detectors”, i.e. devices which show a macroscopic change of state (e.g. pointer positions). We now turn to the actual ESSW argument. According to these authors a problem for the de Broglie-Bohm theory emerges when these extra detectors are chosen to be advanced quantum optical devices, so-called “which-way detectors”. These respond on the transition of single particles without affecting the translational part of the wavefunction¹⁴. Again, in the presence of these devices, we expect the interference pattern to be destroyed. The special feature of these “one-bit detectors” is that even their excitation does not alter the symmetry (i.e. $\psi_1(x, y, z) = \psi_2(x, y, -z)$, with $z = 0$ being the midplane). Hence the de Broglie-Bohm trajectories are still forced to “bounce off” the midplane. However, the probabilities $|\psi_1|^2$ or $|\psi_2|^2$ are in general not confined to one half of the screen. According to ESSW one arrives at the paradoxical situation that the upper which-way detector fires while the screen is hit below the midplane. ESSW conclude:

The Bohm trajectory is here macroscopically at variance with the actual, that is: observed track. Tersely:

¹³i.e. one can insert a screen in region I in order to detect the interference pattern. This choice can be made after the “particles” have passed the slits already, hence the name *delayed-choice*.

¹⁴E.g. within a micromaser excited Rydberg atoms can radiate off one photon without any other significant change of state.

Bohm trajectories are not realistic,
they are surrealist.

This paper has created a lively debate on the “surreal trajectory problem” [7,31,37,58,85,82] and we do not aim at a complete review. One objection against the conclusion of ESSW has been their use of the term “actual track” in connection with quantum mechanics. ESSW try to defend the orthodox interpretation – but the notion of a “particle path” is denied within this interpretation. What is meant by “actual track” is not obvious here. However, ESSW claim that even the observed tracks in a bubble-chamber are at variance with the Bohm-trajectories. This would be a serious objection against the de Broglie-Bohm theory indeed.

The essential flaw in the reasoning of ESSW is that they consider devices which are not linked to any macroscopic change of state. This feature is crucial because it ensures that their symmetry argument applies. But given that within the de Broglie-Bohm theory only a change in position (or of the wavefunction) constitutes a physical fact, such a which-way detector is not regarded as a reliable detector for the actual position of the particle on its Bohmian path. The additional claim of ESSW, that even the tracks in a bubble-chamber differ from the predicted Bohm-trajectories, is therefore unfounded, since a bubble-chamber does convert the excitation into a macroscopic displacement.

But the situation which has been considered by ESSW is a bit more subtle: The authors assume that a macroscopic read-out could be connected *after* the particle has been finally detected. However, it remains true that within the de Broglie-Bohm theory the which-way device is not regarded as a detector. A delayed read-out can not turn it into a more trustworthy device. The arrangement which has been considered by Englert et al. can be viewed as a special case in which “empty waves” [55] show an effect if they are still coherent. In fact, the non-locality of the de Broglie-Bohm theory makes it possible to explain how the which-way detector can be excited even without any trajectory passing through it [31,58]. A detailed discussion of how to resolve the “surreal trajectory problem” within the de Broglie-Bohm theory can also be found in [7].

Along similar lines also other arguments have been advanced in order to show that “the Bohmian position does not help to understand the result of a measurement” [2]. Especially Aharonov et al. [2–4] have explored Bohm trajectories in the case of “weak” and “protective”

measurements¹⁵ in order to challenge any “realistic interpretations of Bohm trajectories” [3]. Similar to the original ESSW argument these authors construct circumstances in which non-local effects are exerted i.e. alleged measuring devices are triggered while the Bohm trajectories do not pass through them. They conclude that their analysis

(...) implies that the Bohm trajectories are forever hidden. If you cannot rely on local interactions to determine the ‘actual position’ of the particle, then you cannot determine it at all. The concept of position itself becomes shaky. [3]

However, Aharonov et al. do not claim the inconsistency of the de Broglie-Bohm theory:

The examples considered in this work do not show that the Bohm’s causal interpretation is inconsistent. It shows that Bohmian trajectories behave not as we would expect from a classical type model. [2]

Furthermore Aharonov and Vaidman admit, that “these difficulties follow from our particular approach to the Bohm theory in which the wave is not considered to be a ‘reality’.”

Recapitulating, we note that these investigations have given fascinating insight into detailed aspects of quantum mechanics in general and the de Broglie-Bohm theory in particular. They clearly demonstrate that (especially given the exotic measuring devices considered above) the trajectories behave completely unclassical and that the de Broglie-Bohm theory is as unintuitive as the usual quantum theory. However, most adherents of the de Broglie-Bohm theory never argued that point.

4.2. Fractal wavefunctions

A recent argument against trajectory-based interpretations of quantum mechanics in general and the de Broglie-Bohm theory in particular was advanced by Hall [54]. He considers so-called fractal wavefunctions for which the expression $\mathcal{H}\psi$ is divergent¹⁶ while the equation $[\mathcal{H} - i\hbar\partial_t]\psi = 0$ is satisfied still¹⁷. Given that the usual Schrödinger equation does not hold for these states, Hall argues that the modified Hamilton-Jacobi equation

¹⁵A “weak measurement” [1] is designed to change the corresponding system only minimally. A “protective measurement” is both, weak and adiabatic [3].

¹⁶There are examples for which the expectation value $\langle \mathcal{H} \rangle$ is finite nevertheless [54].

¹⁷The corresponding states are said to be solutions of the Schrödinger equation in the “weak” sense [94].

can not be derived. Further more, $\nabla\psi$ is not defined and the guidance equation of the de Broglie-Bohm theory fails to provide trajectories for these states.

Given that the corresponding states and their unitary evolution are well defined Hall claims that trajectory-based interpretations are at least *formally* incomplete. Provided that these states could be actually prepared they may even demonstrate the *physical* incompleteness.

In reply to this criticism one may note that the wavefunctions considered by Hall are unphysical. More relevant in this context is the question of global existence of Bohmian trajectories. This issue was settled in [13]. Only recently Tumulka and Teufel [86] have simplified and extended this proof to the Bohm-Dirac theory. If a wavefunction satisfies the conditions for global existence and uniqueness, then it is ensured that it can not evolve into e.g. a fractal state [48]. Hence, Hall's claim about a possible "physical incompleteness" seems to be unfounded and his claim of "formal incompleteness" amount to no more than a specific definition of "formal".

4.3. Non-locality and relativistic generalization

The by far most common objection against the de Broglie-Bohm theory is based on its non-locality and its apparent conflict with relativity. We will try to disentangle these questions in turn.

The de Broglie-Bohm theory is explicitly non-local, i.e. the motion of each particle is in general a function of the coordinates of the whole system, no matter whether they are space-like separated. This non-locality vanishes only if the wavefunction factorizes in the contributions of the different quantum objects. Whether this is viewed as an unacceptable feature depends on the attitude towards the problem of non-locality in quantum mechanics in general. In ordinary quantum mechanics the problem of non-locality appears in at least two places: (i) violation of Bell-inequalities and (ii) reduction of the wavefunction.

Following the work of Bell [9] and the experimental confirmation of quantum mechanics in EPR-Bell experiments [5] it became widely (but not universally [70,72]) accepted that quantum mechanics itself is "non-local". Following this opinion the non-locality allegation against the de Broglie-Bohm theory seems to be even completely groundless. However, the precise meaning of the term "non-local" is far from being unique and there exists a vast literature on that topic (see e.g. [25]). A thorough discussion of that issue is far beyond the scope of the present paper. However, one can reasonably state, that the

"non-locality" of the de Broglie-Bohm theory is more explicit (i.e. dynamical) than the "non-separability" of ordinary quantum mechanics¹⁸. Anyhow, for both, ordinary quantum mechanics and the de Broglie-Bohm theory, it is ensured that the "non-locality" or "non-separability" can not be used for superluminal signalling. But whether this is enough for full compatibility between quantum mechanics and special relativity has been challenged e.g. by Ballentine [6]:

However it is not clear that the requirements of special relativity are exhausted by excluding superluminal signals. Nor is it clear how one can have superluminal influences (so as to violate Bell's inequality and satisfy quantum mechanics) that in principle can not be used as signals. (...) Whether or not there is a deeper incompatibility between quantum mechanics and relativity is not certain.

Another indication for "non-locality" in quantum mechanics is given if one adopts the collapse of the wavefunction to be a real physical process¹⁹. After all the collapse is supposed to reduce the wavefunction instantaneously and requires thereby a preferred frame-of-reference²⁰ [26,69]. Maudlin argues that the collapse postulate in combination with entangled states leads necessarily to a preferred foliation of space-time [69, p.297]. While the Dirac equation provides a Lorentz covariant generalization of the Schrödinger equation the satisfactory generalization of the measurement theory into the relativistic domain is still wanting. Ironically this specific source of non-locality does not arise in the de Broglie-Bohm theory since here the collapse of the wavefunction becomes dispensable. However, as mentioned above, non-locality figures prominently in the de Broglie-Bohm theory which makes the reconciliation with relativity challenging as well.

4.3.1. The Bohm-Dirac theory

Non-locality clearly provides a challenge for a satisfactory relativistic generalization of quantum

¹⁸According to fairly common usage, "separability" means that the *state* of an extended system can be written as a product of local states while "locality" expresses that no *interaction* propagates faster than light.

¹⁹In fact, in the context of the "measurement problem" the collapse of the wavefunction gives rise to other problems as well.

²⁰An obvious solution to this problem is to suppose that the collapse occurs only along the backward light cone of the measurement interaction [17,56]. See e.g. [8] for a discussion of the problems one faces in this approach.

mechanics or the de Broglie-Bohm theory. However, relativistic generalizations of the de Broglie-Bohm theory do exist. E.g. for a Dirac particles Bohm [23] has proposed the following guiding equation (the corresponding framework may be called “Bohm-Dirac theory”):

$$\mathbf{v} = \frac{\psi^\dagger \boldsymbol{\alpha} \psi}{|\psi|^2} \quad (5)$$

Here ψ is a solution of the Dirac equation, ψ^\dagger its conjugate and $\boldsymbol{\alpha}$ a 3-vector with components that are build from the Pauli matrices:

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \quad (6)$$

The generalization to the many-particle case is straightforward [23][p.274]. Thus, the generalization itself is not problematic. But it is an essential property of the many-particle generalization that it requires a preferred reference-frame i.e. the many-particle analogue of Equ. 5 considers all particles at the *same* time. The predictions do nonetheless agree with the standard theory and most important the preferred reference-frame can be made unobservable.

In fact, as shown in [39], it is even possible to restore Lorentz invariance for the Bohm-Dirac theory by introducing additional structure. Dürr et al. introduce an arbitrary space-like preferred slicing of space-time, determined by a Lorentz invariant law. An other strategy is pursued by Berndl et al. [16] who suggest a preferred joint parameterization (i.e. synchronization). This works provide an important step towards a Lorentz invariant de Broglie-Bohm theory and a counter example to the common claim that non-locality and Lorentz invariance are in strict opposition. However, these authors admit that they have not reached yet what Bell called “serious Lorentz invariance” [11][p.179f]. Furthermore the corresponding models consider only entangled but *noninteracting* Dirac particles. The relativistic generalization of the de Broglie-Bohm theory is also addressed in [32,46,80]. A thorough discussion of the relation between non-locality and relativity can be found in [33].

Summing up, we have seen that non-locality and the relativistic generalization provide a challenge not only for the de Broglie-Bohm theory but also for ordinary quantum mechanics²¹. The violation of the Bell-inequality implies that the relation between quantum mechanics and special relativity is more subtle than customarily assumed.

²¹Since the ultimate cause for non-locality is that the wavefunction of a N -particle system is defined on the configuration space, \mathbb{R}^{3N} , it is not surprising that this “non-locality” is not a particular problem of the de Broglie-Bohm theory but for quantum mechanics in general.

The concept of wavefunction collapse points at similar problems. However, the de Broglie-Bohm theory does allow for a relativistic generalization when either the requirement of Lorentz invariance is relaxed to apply only to the observations or by introducing additional structure into the theory.

4.4. The de Broglie-Bohm theory and quantum field theory

Finally (and related to the last paragraph) there is the widespread suspicion that the concepts of the de Broglie-Bohm theory can not be sustained in the realm of quantum field theories (see e.g. the letter to the editor in [52][p. 1227] together with the reply). However, several works on that issue have shown that there seems to be no principle problem to incorporate the concepts and reproduce the predictions of quantum field theories. In what follows we only sketch some of the corresponding results.

Similar to the situation of relativistic generalizations there are several different approaches to this question. The work on that issue can roughly be divided into two camps. The first (e.g. Bohm, Hiley and Holland [23,60]) introduces the notion of (bosonic-)field variables as being fundamental together with the particle position for fermions. These models provide laws for the evolution of these fields. However, boson like e.g. the photon do not possess a trajectory.

The other camp (e.g. Bell [11][p.173] and Dürr et al. [34,35]) sustains the particle-ontology also within quantum field theoretical extensions of the de Broglie-Bohm theory. To this end Dürr et al. associate the interaction part of the Hamiltonian with jump-processes like the creation of particle-antiparticle pairs.

While important questions remain open (see for example the discussion at the end of [34]) it seems premature to reject the de Broglie-Bohm theory on this basis.

5. Summary

We have collected common criticism against the de Broglie-Bohm theory. Most of them have the merit to illustrate the peculiar features of this theory but they do not provide a rigorous disproof.

One strategy has been to formulate additional requirements²² which are not met by the de Broglie-Bohm theory. It remains subjective whether this is viewed as a profound shortcoming or the radical novelty of this theory. After all,

²²The requirements are “additional” to the basic demand that the theory is in accordance with the experimental results.

quantum mechanics has likewise introduced many bizarre notions into physics. However, while it is subjective how desirable these additional requirements are, they are clearly not irrational.

A different strategy is to address the consistency of the de Broglie-Bohm theory and its ability to be generalized. The most substantial concern is the question of its relativistic and quantum field theoretical generalization. However, several models for such generalizations do exist in which either the preferred foliation of space-time is unobservable or even Lorentz-invariance can be (at least formally) sustained. Although important questions remain open it seems premature to reject the de Broglie-Bohm theory on this account. Above all, these objections should be compared to those which have been advanced against other interpretations of quantum mechanics, in particular against the orthodox view.

The merit of this discussion is to reveal that even in science a theory can not only be judged by its empirical confirmation²³. In the absence of any experimental test that can distinguish between standard quantum mechanics and the de Broglie-Bohm theory one may either leave this question undecidable or has to invoke e.g. “meta-theoretical” criteria like the one presented in Sec. 3. This is completely sound but should be stated explicitly. We fully agree with Hiley who states:

Unfortunately there is a great deal of unnecessary emotion generated when “alternative interpretations” to quantum mechanics are discussed. By now we have so many interpretations, that it must be clear to all that there is some basic ambiguity as to what the formalism is telling us about the nature of quantum processes and their detailed relation to those occurring in the classical domain. [57]

This “unnecessary emotions” (in part on both sides) complicate a sober discussion.

It would be highly desirable to have an open minded discussion in the spirit of appreciation for the different interpretations. Examples for this can be found e.g. in the camp of the de Broglie-Bohm theory, like Goldstein’s work about decoherent histories [44] or Tumulka’s contribution to the GRW program [87]. Similar the “many-worlder” Vaidman has made illuminative contributions to implications of the de Broglie-Bohm theory in [88], to pick just a few examples. How-

ever, also the “orthodox” view deserves a fair discussion as expressed by Bell in the following less known quote:

I am not like many people I meet at conferences on the foundation of quantum mechanics (...) who have not really studied the orthodox theory [and] devote their lives criticizing it (...) I think that means they have not really appreciated the strength of the ordinary theory. I have a very healthy respect for it. (quoted from [50])

Acknowledgement

I am particularly indebted to Prof. Sheldon Goldstein for his very helpful comments and suggestions. The paper benefited greatly from them. Thanks also to Travis Norsen, Raymond Mackintosh, Itamar Pitowsky, Ned Floyd, Alan Forrester, Gerhard Grössing, Hans Dieter Zeh, Hrvoje Nikolic, Matthew Donald, Francesco Cannata, Stephan Tzenov, Giorgio Kaniadakis, Marek Czachor, Josiph Rangelov and Abel Miranda.

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