Popper's Experiment and Superluminal Communication

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We comment on Tabish Qureshi, "Understanding Popper's experiment," AJP **73**, 541 (June, 2005), in particular on the implications of its Section IV. We show, in the situation envisioned by Popper, that analysis solely with conventional non-relativistic quantum mechanics suffices to exclude the possibility of superluminal communication.

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I. INTRODUCTION.

In a recent AJP paper[1] (hereinafter referred to simply as "Q.") Tabish Qureshi has presented an analysis of an experiment proposed by Karl Popper to test the standard interpretation of quantum theory. In this Introduction we describe Popper's experiment and–because we believe Qureshi's analysis could leave the readers of this journal with some misconceptions–comment on Q. In Section II we show that in the situation envisioned by Popper even conventional solely non-relativistic quantum mechanics suffices to exclude the possibility of superluminal (faster than light speed) communication. Some brief closing remarks are presented in Section III.

Popper and Qureshi (see Q. Fig. 1) consider a source S emitting non-interacting pairs of non-identical particles 1 and 2 moving predominantly along the x direction (horizontal), but with some small components of momentum along the y direction (vertical) and with zero components along the z direction (perpendicular to x and y); the experimental situation pictured in Q. Fig. 1 is to be visualized as two dimensional, therefore, lying in the x, y plane only. The total momentum of each pair is zero; also, any distribution in the components of momentum along the x direction is inconsequential, so that we are concerned solely with the momenta $p_1 = -p_2$ of particles 1,2 along the y direction; We assume, as Qureshi does in effect, that: (i) the source, at $x = 0$, emits a negligible number of particles with vertical momenta outside the range $|p_1| \leq P_m$; (ii) $P_m > 0$ is much larger than any Heisenberg Uncertainty Principle momenta realistically required to limit the spread of the beam along y in the region between A and B (see Q. Fig. 1); (iii) the beam of particles 1, moving to the left, encounters at $x = -X$ a screen with a narrow slit centered at $y = 0$ (slit A of Q. Fig. 1); and (iv) this slit introduces a momentum spread along y that is much larger than P_m, with the result that after passing through slit A the particle 1 beam spreads much more broadly along y than it did before encountering slit A. The question discussed by Popper and Qureshi is: Does conventional quantum mechanics (what Qureshi calls the "Copenhagen interpretation") predict that the particle 2 beam, moving to the right but not encountering a slit, also will be spread much more broadly along y at horizontal distances $x > X$, by virtue of the entanglement between particles 1 and 2 embodied in the requirement that when emitted $p_1 = -p_2$?

The unequivocal answer to this question, without the need to do any calculating is "No". Indeed the observable effects of the beam on the screen behind B (e.g., darkening as a function of y) must in every respect be completely independent of the size of the slit encountered at A. Otherwise the observer at A (conventionally named Alice) could essentially instantaneously transmit messages to her counterpart observer (conventionally named Bob) viewing the screen behind B, now placed at a very long distance $x >> X$; in particular, if what Bob observes can depend on the size of the slit Alice, using a code on which she and Bob had previously agreed, can send Bob a message simply by widening and narrowing slit A. Such superluminal (faster than light speed) communication of information is impossible.[2] Furthermore as Peres[3] has emphasized, conventional quantum mechanics implies that it is impossible for Alice, by solely local operations, to transmit any information whatsoever to Bob. Alice's control of the slit size at A, without performing any operations whatsoever at any points between A and the screen behind B, is a "local operation" by definition.

Unfortunately Section IV of Q. reasonably can be read to imply that Alice, by detecting the passage of particles 1 through slit A as she controls the width of the slit, can affect the spread of the beam on the screen behind B (see in particular the text immediately following Eq. (13) of Q.). Qureshi has assured us[4] that this reading is not his intention; rather, his section IV is supposed to be concerned with coincidence measurements, on particle 1 at slit A and on particle 2 at the screen behind B, performed on a pair of particles that originally were simultaneously emitted from the source S. In fact Qureshi, in an analysis[5] of Popper's experiment written only a few months before Q. was submitted, explicitly states that in the absence of such coincidence measurements the observable effects of the particle 2 beam on the screen behind B will be independent of the width of slit A. The clarifications, in this

paragraph concerning the implications of Section IV of Q., and in the preceding paragraph concerning the predictions of conventional quantum mechanics respecting Popper's experiment, are among our reasons for writing this paper.

II. QUANTUM MECHANICAL ANALYSIS.

We believe it will be useful to present a simple easily grasped (though admittedly non-rigorous) derivation demonstrating that application of conventional quantum mechanics to Popper's experiment predicts the observable effects of the beam on the screen behind B must be completely independent of the size of the slit encountered at A, or indeed of any other local operations at A; thus, in Popper's experiment at least, local operations cannot be employed to transmit information. No such derivation is to be found in Qureshi's earlier paper,[5] nor in any other source of which we are aware. It is important to note that manipulations of measurement equipment at A, e.g., switching on electromagnetic fields in the vicinity of A, are included in the "local" operations to which our derivation pertains (see Subsections II.A and II.B), as are the performances of actual measurements at A (see Subsection II.C). In our derivation, however, actual measurement performances, which collapse the wave function (Subsection II.C), require a different treatment than do all other local operations, which affect the wave function via force terms generated in the Schrodinger equation (Subsections II.A and II.B). Overall our derivation is confined to local operations in Popper's experiment, although the analysis in Subsection II.B does have wider application as will be seen. More general proofs, not restricted to Popper's experiment, that information cannot be transmitted by local operations can be found in the literature (cf., e.g., Peres[2] or Bruss[6]), but are difficult for non-experts.

It may seem surprising that a theorem of this nature can be established in non-relativistic quantum mechanics. One might think that a proper relativistic theory would be required to show the impossibility of superluminal communication. What we show is that in Popper's experiment "ordinary" quantum mechanics precludes any local operations on particles 1 from changing any probability distributions in the entire beam of particles 2, no matter how entangled the particles are. This showing prevents information about the manipulations of slit A, or about measurements peformed by Alice, from being transmitted to Bob via the beams at any speed, whether superluminally or relativistically allowed.

A. Freely Moving Particles.

For any given pair of particles 1 and 2 which simultaneously leave the source S, the unnormalized wave function expressing the aforesaid entanglement between them at the instant they leave the source is

$$\Psi(y_1, y_2) = \int_{-\infty}^{\infty} dK W(K) e^{-iKy_1} e^{iKy_2}, \tag{1}$$

wherein: the plane waves have momenta $p_2 = -\, p_1 = \hbar K$; $W(K)$ describes the particle momentum distribution along the y direction; $|W(K)|^2$ is negligible for $|\hbar \mathrm{K}| \geq P_m$; and the initial presumably random phase $e^{i\phi(K)}$ multiplying each plane wave pair $e^{-iKy_1} e^{iKy_2}$ has been absorbed in $W(K)$. Because every entangled particle pair moves independently of every other such pair, the time evolution of this $\Psi(y_1, y_2)$ predicts the probability distribution of all the particle 2 trajectories toward the screen behind B (see Q. Fig. 1) even though this Ψ depends on the coordinates of only a single pair of particles. Unhappily Ψ given by Eq. (1) is not normalizable. Instead

$$\begin{aligned}\Psi^\dagger \Psi &= \int dy_1 dy_2 \int dK W^*(K) e^{iKy_1} e^{-iKy_2} \int dK' W(K') e^{-iK'y_1} e^{iK'y_2} \\ &= (2\pi)^2 \int dK dK' W^*(K) W(K') [\delta(K - K')]^2 = (2\pi)^2 \delta(0) \int dK |W(K)|^2,\end{aligned} \tag{2}$$

where: all the integrals in Eq. (2) [and all integrals below] run from -∞ to ∞; herein and below the dagger † denotes the adjoint; and, as is customary, $\delta(K)$ denotes the Dirac delta function of K. If $W(K)$ itself is normalized so that $\int dK |W(K)|^2 = 1$, $|W(K)|^2 dK$ legitimately can be interpreted as the probability that, when the source emits a particle pair, the wave number of particle 2 will lie between K and $K + dK$ (still concentrating solely on motion along the y direction, of course); henceforth we will assume that $W(K)$ has been so normalized. The singularity on the right side of Eq. (2) can be avoided, therewith making Ψ normalizable, by confining the system vertically to the region between the two distant horizontal planes $y = \pm L$, at which planes the momentum eigenfunctions in the expansion of Ψ are required to satisfy periodic (or other suitable) boundary conditions. This procedure replaces the singular factor $(2\pi)^2 \delta(0)$ in Eq. (2) by a well behaved factor.

Introducing such boundary conditions, however, with the concomitant requirement that integrals over all wave numbers be replaced by sums over the allowed values of those wave numbers, leads to equations, e.g., the analogs of Eqs. (1) and (2), which tend to obfuscate the transparency of our analysis. We have decided not to impose boundary conditions, therefore, believing it will be obvious that none of the inferences we draw from our analysis are obviated either by our employment of the unnormalized Ψ of Eq. (1) or by our retention of singular delta funtion factors as in Eq. (2). One might think this unnormalizability can be simply dealt with by appending a factor $\exp[-\gamma(y_1^2+y_2^2)]$ to the right side of Eq. (1), where γ is a small positive constant. But the inclusion of this factor means the initial wave function is not surely describing particle pairs leaving the source S with equal and opposite momenta; for such a source the dependence on y_1, y_2 of the initial wave function must be through the difference $y_1 - y_2$ only, as is evident from Eq. (1).[7] Once $W(K)$ has been normalized as described in the preceding paragraph, $\Psi(y_1, y_2)$ can be formally "normalized" by appending the singular factor $[(2\pi)\sqrt{\delta(0)}]^{-1}$ to the right side of Eq. (1). We will denote this "normalized" Ψ by Ψ_n, and will employ the subscript n to denote quantities calculated using Ψ_n.

The unit basis vectors $w(y,k)$ in wave number space (which by not requiring the repeated inclusion of $\hbar$ is more convenient than momentum space), satisfying $\int dk w^*(y,k)w(y',k) = \delta(y-y')$, are $\mathrm{w}(y,k) = (1/\sqrt{2\pi})e^{iky}$. Thus the components $\Phi(k_1,k_2)$ of Ψ in wave number space are

$$\Phi(k_1,k_2) = (1/2\pi)\int dy_1 dy_2 e^{-ik_1y_1}e^{-ik_2y_2}\Psi(y_1,y_2) = 2\pi\int dK W(K)\delta(K+k_1)\delta(K-k_2) = 2\pi W(k_2)\delta(k_1+k_2). \quad (3)$$

Then at the source the number of particles 2 with wave numbers between k_2 and $\mathrm{k}_2 + dk_2$ must be proportional to $D(k_2)dk_2$, where the particle 2 wave number distribution function

$$D(k_2) = \int dk_1|\Phi(k_1,k_2)|^2 = (2\pi)^2\int dk_1|W(k_2)|^2[\delta(k_1+k_2)]^2 = (2\pi)^2\delta(0)|W(k_2)|^2, \quad (4)$$

a result whose total independence of our initial random phases $e^{i\phi(K)}$ and proportionality to the probability $|W(k_2)|^2$ supports the validity of our analysis. Eqs. (2)-(4) imply that

$$\Phi^\dagger\Phi = \int dk_1 dk_2|\Phi(k_1,k_2)|^2 = \int dk_2 D(k_2) = (2\pi)^2\delta(0)\int dk_2|W(k_2)|^2 = \Psi^\dagger\Psi, \quad (5)$$

as consistency of the analysis requires. It is readily seen that when Ψ is replaced by Ψ_n in Eq. (3), and the corresponding Φ_n is employed in Eq. (4) to compute D_n, the factor $(2\pi)^2\delta(0)$ disappears from the right side of Eq. (4), yielding $D_n(k_2) = |W(k_2)|^2$. Thus $D_n(k_2)dk_2$ is interpretable as the probability at the source of finding particle 2 with wave number between k_2 and $k + dk_2$, consistent with the interpretation that ordinarily would be afforded a calculation of the particle 2 wave number distribution function starting from a properly normalized wave function. This additional consistency additionally supports our belief, which we will not argue any further, that the inferences we draw from our analysis in this Subsection are valid although we use unnormalizable and singular functions.

Because the particles are assumed to move freely (without external influences of any kind) in the space betwen A and B (see Q. Fig. 1), until particles 1 reach A the number of their paired particles 2 with wave numbers between k_2 and $\mathrm{k}_2 + dk_2$ surely continues to be proportional to $D(k_2)dk_2$. This obvious result can be derived formally by recognizing that as long as particles 1 and 2 are moving freely their motions can be thought to be described by the Schrodinger equation $i\hbar\partial\Psi/\partial t = \mathbf{H}\Psi$, whose formal solution when $\mathbf{H}$ is time-independent is[8] $\Psi(t) = e^{-i\mathbf{H}t/\hbar}\Psi(0)$. In the present free particles 1,2 case $\mathbf{H}$ is time-independent and $= p_1^2/2m_1 + p_2^2/2m_2 \equiv \mathbf{H}_1 + \mathbf{H}_2$. Then, since $\mathbf{H}$ does not involve K and $\mathbf{H}_1$,$\mathbf{H}_2$ act respectively on y_1, y_2 only,

$$\begin{aligned}\Psi(t) \equiv \Psi(y_1,y_2;t) = e^{-i\mathbf{H}t/\hbar}\Psi(0) &= \int dK W(K)[e^{-i\mathbf{H}_1t/\hbar}e^{-iKy_1}][e^{-i\mathbf{H}_2t/\hbar}e^{iKy_2}] \\ &= \int dK W(K)[e^{-i\hbar\mathbf{K}^2t/2m_1}e^{-iKy_1}][e^{-i\hbar\mathbf{K}^2t/2m_2}e^{iKy_2}] = \int dK W(K;t)e^{-iKy_1}e^{iKy_2}, \quad (6)\end{aligned}$$

where $\Psi(0)$ is $\Psi(y_1,y_2)$ from Eq. (1) and $W(K;t) = e^{-i\hbar\mathbf{K}^2t/2\mu}W(K)$ with $\mu = m_1m_2/(m_1+m_2)$. The right side of Eq. (6) has the same form as the right side of Eq. (1) except that $W(K;t)$ has replaced $W(K)$; the components $\Phi(t) \equiv \Phi(k_1,k_2;t)$ of the wave function in momentum space are defined as was $\Phi(0) \equiv \Phi(k_1,k_2)$ in Eq. (3) except that $\Psi(t)$ replaces $\Psi(0)$; and $D(k_2;t)$, the particle 2 wave number distribution function at time t, is defined by Eq. (4) except that $\Phi(t)$ replaces $\Phi(0)$. It follows that $D(k_2;t)$ equals the right side of Eq. (3), provided $W(k_2;t)$ replaces $W(k_2)$. Therefore, since $|W(k_2;t)|^2 = |W(k_2)|^2$, it has been shown that $D(k_2;t) = D(k_2)$, i.e., it has been shown (as just asserted) that as long as the particles can be assumed to move freely the number of particles 2 with wave numbers between k_2 and $\mathrm{k}_2 + dk_2$ continues to be proportional to $D(k_2)dk_2$. Moreover as Peres[9] has proved, when

the individual particles are represented by wave packets, as they should be, the paired particles 1 and 2 emitted with opposite momenta do move in opposite directions along the same straight line. Consequently, remembering the paired particles are emitted with opposite momenta along x as well as along y, until any burst of particles 1 reaches A the distribution function $D(k_2)$ of Eq. (4) determines the distribution–as a function of y–of any darkening or other observable effects produced by the corresponding burst of particles 2 on any screen intercepting the particle 2 beam.

B. Motion Under Forces.

Now suppose that, as a consequence of some local operation at A other than the performance of an actual measurement, the particles 1 no longer move freely once they reach the vicinity of A. Then the Hamiltonian $\mathbf{H}$ governing the particle motions still can be written in the form $\mathbf{H} = \mathbf{H}_1 + p_2^2/2m_2$, but $\mathbf{H}_1$ (though of course still independent of y_2 since the operation is local) now differs from $p_1^2/2m_1$ by terms depending on the particular local operation adopted. If we justifiably are to visualize particles 1 as moving freely until they reach the vicinity of A, these local operation terms should be negligible unless x is very close to the location $x = -X$ of A (as we have been assuming to this juncture in this paper). But, once no longer negligible, these local operation terms in $\mathbf{H}_1$ may be expected to mix particle momenta along x and y, as well as to deflect particles 1 out of the x, y plane in which we have assumed they move; certainly this is what is likely to occur if the local operation involves electromagnetic forces. It follows that for the purpose of determining the time dependence of the particle motions when particles 1 are subject to local operations in the vicinity of A, Eq. (1)–with its neglect of all particles 1,2 coordinates other than y_1 and y_2–generally is no longer useful. Instead it is necessary to start from

$$\Psi(0) \equiv \Psi(\mathbf{r}_1, \mathbf{r}_2) = \int d\mathbf{K} W(\mathbf{K}) e^{-i\mathbf{K}\cdot\mathbf{r}_1} e^{i\mathbf{K}\cdot\mathbf{r}_2}, \tag{7}$$

wherein $d\mathbf{K} = dK_x dK_y dK_z$ and the notation should otherwise be obvious.

We return to Eq. (7) below. For the moment, however, let us assume that the local operation permits us to concentrate solely on motions along y as we have been doing, and as would be the case if the local operation were the interruption of the particle 1 beam by a narrow horizontal slit at A. In this event Eq. (6), giving the time dependence of the wave function in the purely free particle case, legitimately can be replaced by

$$\Psi(t) \equiv \Psi(y_1, y_2; t) = e^{-i\mathbf{H}t/\hbar}\Psi(0) = \int dK W(K) u(y_1, K; t)[e^{-i\hbar K^2 t/2m_2} e^{iKy_2}], \tag{8}$$

where we are defining $u(y_1, K; t)$ as the y_1 component of the function $e^{-i\mathbf{H}_1 t/\hbar} e^{-iKy_1}$, i.e.,

$$u(y_1, K; t) \equiv [e^{-i\mathbf{H}_1 t/\hbar} e^{-iKy_1}]_{y_1} = \int dy \mathbf{U}(y_1, y; t) e^{-iKy}, \tag{9}$$

with $\mathbf{U}$ the unitary operator $e^{-i\mathbf{H}_1 t/\hbar}$. Eqs. (8) and (9) still assume the Hamiltonian operator $\mathbf{H}_1$ is time-independent. When $\mathbf{H}_1$ is time-dependent, however, as for instance it would be if Alice were to change the slit width at A while the beam of particles 1 is impinging on A, it merely is necessary[8] to replace $\mathbf{H}_1 t$ in $e^{-i\mathbf{H}_1 t/\hbar}$ by the appropriately time ordered integral $\int_0^t dt' \mathbf{H}_1(t')$. The key point is that the right side of Eq. (9) remains a valid formula for $u(y_1, K; t)$ in Eq. (8), with $\mathbf{U}$ still a unitary operator. Thus whatever the local operations, time-independent or time-dependent, the components $\Phi(t) \equiv \Phi(k_1, k_2, t)$ of the wave function in momentum space now are, recalling Eq. (3) and using Eq. (9),

$$\Phi(t) = (1/2\pi) \int dy_1 dy_2 e^{-ik_1 y_1} e^{-ik_2 y_2} \Psi(t) = W(k_2)^{-i\hbar k_2^2 t/2m_2} \int dy_1 e^{-ik_1 y_1} u(y_1, k_2; t). \tag{10}$$

Consequently, recalling Eq. (4), once particles 1 have reached A the number of particles 2 with wave numbers between k_2 and $k_2 + dk_2$ becomes proportional to $D(k_2; t)dk_2$, where the particle 2 wave number distribution function now is

$$\begin{aligned} D(k_2; t) &= \int dk_1 |\Phi(t)|^2 = |W(k_2)|^2 \int dk_1 \int dy_1 e^{-ik_1 y_1} u(y_1, k_2; t) \int dy_1' e^{ik_1 y_1'} u^*(y_1', k_2; t) \\ &= (2\pi)|W(k_2)|^2 \int dy_1 \int dy_1' \delta(y_1 - y_1') u(y_1, k_2) u^*(y_1', k_2) = (2\pi)|W(k_2)|^2 \int dy_1 |u(y_1, k_2, t)|^2. \end{aligned} \tag{11}$$

In Eq. (11), furthermore, remembering Eq. (9) and the fact that $\mathbf{U}$ is unitary,

$$\int dy_1 |u(y_1,k_2,t)|^2 = u(t)^\dagger u(t) = [\mathbf{U}e^{-ik_2y_1}]^\dagger \mathbf{U}e^{-ik_2y_1} = [e^{-ik_2y_1}]^\dagger \mathbf{U}^\dagger \mathbf{U}e^{-ik_2y_1} = \int dy_1 |e^{-ik_2y_1}|^2 = (2\pi)\delta(0), \quad (12)$$

where we have employed standard matrix manipulations. Eqs. (11) and (12), taken together, make $D(k_2,t)$ identical with $D(k_2)$ from Eq. (4). It again is readily seen that if the normalized $\Psi_n(0)$ replaces $\Psi(0)$ in Eq. (8), the resultant $D_n(k_2,t) = |W(k_2)|^2 = D_n(k_2)$. In other words despite the local operations at A that we have been discussing, the observable effects of the particle 2 beam on the screen behind B remain precisely what they would have been had the particle 1 beam moved totally freely after leaving source S. In short, such local operations do not permit Alice to send messages to Bob

The preceding key result that $\mathrm{D}(k_2,t) = D(k_2)$ has been proved only for those special local operations which do not affect the motions of particles 1 along the x or z directions; for more general local operations it is necessary to start from Eq. (7) rather than Eq. (1), as we have explained. But it now is readily seen that the above derivation of $D(k_2,t) = D(k_2)$ starting from Eq. (1) is directly parallelled by a derivation, starting from Eq. (7), which–whether or not the paticles 1 move freely–yields $D(\mathbf{k}_2,t) = D(\mathbf{k}_2)$, where $D(\mathbf{k}_2)$ is the particle 2 wave number distribution function at the source S for arbitrary wave number vectors $\mathbf{k}_2$. Of course, when starting from the unnormalizable three dimensional $\Psi(\mathbf{r}_1,\mathbf{r}_2)$ of Eq. (7), the equations corresponding to Eqs. (2)-(6) and (8)-(12) contain three dimensional delta functions rather than one dimensional delta functions. For instance the equation corresponding to Eq. (4) is

$$D(\mathbf{k}_2) = (2\pi)^6 \delta(\mathbf{0}) |W(\mathbf{k}_2)|^2, \quad (13)$$

where $\delta(\mathbf{K}) = \delta(K_x)\delta(K_y)\delta(K_z)$ is the three-dimensional Dirac delta function. Note in particular that $D(\mathbf{k}_2)$ is proportional to $|W(\mathbf{k}_2)|^2$, just as $D(k_2)$ is proportional to $|W(k_2)|^2$, so that regarding $D(\mathbf{k}_2)d\mathbf{k}$ as proportional to the number of particles 2 at the source with wave number vectors lying in the range $\mathbf{k}_2$ to $\mathbf{k}_2 + d\mathbf{k}_2$ is no less reasonable than was our corresponding interpretion of $D(k_2)dk_2$. As when starting from the one-dimensional Eq. (1), the three-dimensional Ψ of Eq. (7) can be made normalizable by confining the system to the interior of the volume formed by the planes $x = \pm L$, $y = \pm L$, $z = \pm L$ and introducing suitable boundary conditions; in this fashion the singular factor in Eq. (13) can be avoided, at the expense of having to replace integrals over all wave numbers with sums over allowed wave numbers only.

C. Measurements by Alice.

To this point, however, our derivation has not pertained to local operations at A involving the performance of actual measurements. To see that measurements at A also cannot enable Alice to send messages to Bob, let us examine the consequences of a decision by Alice to make wave number measurements of her own on the particle 1 beam, before Bob has a chance to make his measurements. We want to see whether these measurements of Alice's can alter our previous conclusions, e.g., our conclusion in Subsection II.A that $D(k_2)$ of Eq. (4) determines the distribution of wave numbers k_2 Bob observes irrespective of the local operations–of the sort, e.g., considered in Subsection II.B–performed on particles 1. For this purpose it is desirable to examine first an experimental situation which is not complicated by the facts: (i) that Ψ of Eqs. (1) or (3) is unnormalizable; and (ii) that the unit basis vectors $w(y,k)$ in wave number space [defined immediately preceding Eq. (3)] lie in the continuum. So suppose that: (i) we again have entangled pairs of particles 1,2, with Alice and Bob capable respectively of making local measurement observations on particles 1 at A and on particles 2 at B; but (ii) at some instant the wave function describing the state of a representative entangled pair 1,2 now is

$$\Psi = \sum_{i,j} a_{ij}\alpha_i\beta_j. \quad (14)$$

In Eq. (14): the α_i are an orthonormal set of eigenstates for the measurement operation Alice plans to make; the β_j are similarly defined for Bob; and Ψ is normalized, implying the numbers a_{ij} satisfy $\sum_{i,j}|a_{ij}|^2 = 1$. Then at this instant, for any given a_{ij}, the quantity $|a_{ij}|^2$ customarily is regarded as the probability that measurements on the particle pair will find particle 1 in the eigenstate α_i and particle 2 in the eigenstate β_j. Correspondingly, summing $|a_{ij}|^2$ over all possible states i in which the particle 1 paired with this particle 2 might have been found, one obtains the actual probability ρ_{2j} of finding particle 2 in the eigenstate β_j, namely $\rho_{2j} = \sum_i |a_{ij}|^2$. If many independently moving entangled pairs 1,2 are being observed, the numbers of particles 2 in the various different states β_j actually observed by Bob cannot but be proportional to their respective probabilities ρ_{2j}.

The preceding paragraph has not specified whether or not Alice actually has performed measurement observations on particle 1. Since nothing has been said about any collapses of Ψ induced by Alice's measurements, we might infer that

the preceding paragraph presumed Alice had not made any actual measurements before Bob made his measurements. The important point, which we are about to demonstrate, is that whether or not Alice did her measuring before Bob is irrelevant to the validity of the above interpretations of $|a_{ij}|^2$ and ρ_{2j}. In particular suppose Alice, before Bob makes any measurements on particle 2, observes that the paired particle 1 is in the state α_i. According to the conventional understanding of measurements in quantum mechanics, this measurement immediately collapses Ψ of Eq. (14) into the new wave function[10]

$$\Psi_{ci} = \alpha_i \{[\sum_k |a_{ik}|^2]^{-1/2}\} \sum_j a_{ij}\beta_j. \tag{15}$$

Evidently, except for the factor $[\sum_k |a_{ik}|^2]^{-1/2}$, Ψ_{ci} has plucked from Ψ of Eq. (14) all the terms containing α_i and only those terms, as one expects for the collapsed wave function after observing particle 1 in the state α_i. The factor $[\sum_k |a_{ik}|^2]^{-1/2}$, which is consistent with the so-called Born rule,[10] is required in order that Ψ_{ci} be normalized, i.e., in order that $\Psi_{ci}^\dagger \Psi_{ci} = 1$, as any wave function describing an actual physical situation should be. According to Eq. (14) the probability $\rho_{2j/i}$ of observing particle 2 in the state β_j, *knowing* that particle 1 has been observed in the state α_i, is $\rho_{2j/i} = |a_{ij}|^2[\sum_k |a_{ik}|^2]^{-1}$. But, consistent with the preceding paragraph, the probability ρ_{1i} that Alice has observed particle 1 in the state α_i must be $\rho_{1i} = \sum_j |a_{ij}|^2$.

Thus the probability of *Alice first* observing particle 1 in the state α_i and *Bob only then* observing the paired particle 2 in the state β_j must be $\rho_{1i}\rho_{2j/i} = |a_{ij}|^2$, exactly the probability, quoted in the penultimate paragraph, for finding particle 1 in the eigenstate α_i and the paired particle 2 in the eigenstate β_j without any specified temporal order in making the measurements on the two particles. Correspondingly, since Alice had to find her particle 1 in some α_i, the actual probability that Bob will find the paired particle 2 in the state β_j after Alice made her measurement again will be the probability $\rho_{2j} = \sum_i |a_{ij}|^2$ obtained in the penultimate paragraph. We conclude that when many independently moving entangled pairs are being observed (as in Popper's experiment), the numbers of particles 2 in the various different states β_j actually observed by Bob will be proportional to the same respective probabilities ρ_{2j} whether or not Bob has made his observations after measurements by Alice. Note that this conclusion does not in any way depend on the nature of the states α_i and β_j, i.e., does not depend on the kinds of measurements Alice (on particles 1 only) and Bob (on particles 2 only) have chosen to perform; it is assumed of course that the measurements are performed independently, meaning that Bob receives no communications from Alice which could enable him to modify his measurements depending on Alice's measurement results. Therefore we have proved that when the experimental situation involves many pairs of independently moving pairs of entangled particles 1,2, and when the state of any representative entangled pair is described by the wave function Ψ of Eq. (14), Alice cannot employ her local measurement observations on particles 1 at A to send messages to Bob at B, because the nature of her measurements, and whether or not she performs them, will not in any way alter Bob's observations of the particles 2 at B.

The proof in the preceding paragraph, which we henceforth will term the "foregoing" proof, is quite generally valid for particle pair systems described by Eq. (14), wherein Ψ is normalized and is defined by a discrete sum; for instance the foregoing proof is valid for the very commonly discussed case of observations on a large number of similarly entangled qubit pairs. Unfortunately, primarily because we have relied so importantly on our ability to interpret the quantities $\rho_{2j} = \sum_i |a_{ij}|^2$ arising from Eq. (14) as probabilities, we have not been able to convincingly generalize the foregoing proof to this paper's analysis of Potter's experiment, wherein the analogs of ρ_{2j} are the singular $D(k_2)$ of Eq. (4) [in the one-dimensional case starting from Eq. (1)] or the even more singular $D(\mathbf{k}_2)$ of Eq. (13) [in the full three-dimensional case starting from Eq. (7)]; the facts that Eqs. (1) or (7), and their respective succeeding Eqs. (4) or (13), involve integrals rather than sums is a further complication. We argue as follows, however, now confining our attention to the full three dimensional case starting from Eq. (7): We already have pointed out that, via the device of confining the system to the interior of the volume formed by the planes $x = \pm L$, $y = \pm L$, $z = \pm L$ and imposing suitable boundary conditions, our analysis of Potter's experiment could have been made to start with a wave function which was normalized and involved a discrete sum, which wave function analog of Ψ defined by Eq. (7) we will call Ψ_d. The foregoing proof unquestionably applies to this alternative formulation of Potter's experiment starting with Ψ_d. Moreover it is reasonable that with arbitrarily large L it should be possible to represent the actual physics of a spatially confined experiment like Potter's with arbitrarily high precision, even though the allowed particle wave numbers are limited to a discrete set; certainly physicists have not hesitated to use discretized wave expansions and box normalization ever since the dawn of quantum mechanics.[11] To put it differently, since the allowed discrete wave numbers are very close to each other for large L and change as L changes, it is unreasonable to think our rigorous proof–that Alice's observations on particles 1 cannot affect Bob's observations on particles 2 when the source emits only allowed wave numbers for some particular specified L–does not carry over to all wave numbers. We conclude that our foregoing proof applies to Potter's experiment, whether Alice chooses to make measurements on particles 1 when they still are moving freely (as in Subsection II.A), or defers her measurements until particles 1 have reached

the slit A (as in Subsection II.B).

It is noteworthy that our derivation clearly implies any measurement observations by Alice capable of collapsing the wave function may decrease the range of $\mathbf{k}_2$ available to particles 2 on their way to the screen behind B, but surely cannot increase this range.

III. CONCLUDING REMARKS.

The immediately preceding completes our demonstration that application of conventional quantum mechanics to Popper's experiment predicts the observable effects of the beam on the screen behind B must be completely independent of the size of the slit encountered at A, or indeed of any other local operations at A. We recognize that our derivation is not rigorous, but believe it captures, in a fashion accessible to non-experts, the essence of the physics involved in Popper's experiment when the particles involved are not photons. We acknowledge that our demonstration is not convincing for a Popper's experiment with pairs of photons, which do not obey the usual Schrodinger equation and can be destroyed in the course of detection, a possibility our derivation does not contemplate. That for photons it must be possible to establish (though not necessarily to prove simply) theorems similar to those derived in this paper follows from our general remarks in the third paragraph of this paper.

Before closing we remark that the Schrodinger equation for freely moving particles impinging upon a slit screen normally would be solved by imposing some appropriate boundary condition at the screen (not at the slit of course), in which event the equation $\Psi(t) = e^{-i\mathbf{H}t/\hbar}\Psi(0)$ will not yield the correct solution if $\mathbf{H}$ is the usual free particle Hamiltonian. This paper assumes the relevant physics of particles impinging on a screen can be adequately reproduced via replacement of the boundary condition with suitable forces; since such forces must exist, because otherwise the particles simply would penetrate the screen, we do not doubt the validity of our assumption, but believe we should make it explicit. Note that merely postulating the existence of such forces describable by a Hamiltonian is sufficient for our purpose; our proof in (Subsection II.B) that $D(\mathbf{k}_2, t) = D(\mathbf{k}_2)$ depends only on the existence of an $\mathbf{H}$ (whose details we need not know) which, when substituted for the free particle Hamiltonian, will yield a $\Psi(t) = e^{-i\mathbf{H}t/\hbar}\Psi(0)$ that correctly predicts the motion of the particles 1 in the vicinity of the slit screen. Obviously a similar assumption must be made for any other conceivable local operation at A that Alice might impose and which at first sight is describable by a boundary condition, not by forces.

Finally we note that if $|W(\mathbf{k}_2)|^2$ in Eq. (13) is zero for $\mathrm{k}_2 = |\ \mathbf{k}_2| > P_m/\hbar$, no wave function collapsing measurements on particles 1, or any other local operations on particles 1 for that matter, can result in any particles 2 arriving at screen B along trajectories corresponding to $\mathrm{k}_2 > P_m/\hbar$. From this feature ALONE we can conclude that in Popper's experiment inserting a narrow slit in the path of particles 1 will NOT cause an increased spread in the angular trajectories of particles 2.

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1. Tabish Qureshi, "Understanding Popper's experiment." Am. J. Phys. **73**, 541-544 (June, 2005).

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5. Tabish Qureshi, "Popper's experiment. Copenhagen Interpretation and Nonlocality," quant-ph/0301123, (March 30 (2004). This paper is reference 18 of Q.

6. D. Bruss *et al.*, "Approximate quantum cloning and the impossibility of superluminal information transfer," Phys. Rev. A **62**, 062302-1 to 4 (2000).

7. Appending such a Gaussian factor is precisely how Qureshi deals with the unnormalizability [see Q. Eq. (1)]. Hence it is not evident that the results in his section IV pertain to actual coincidence measurements in Popper's experiment.

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9. Asher Peres, "Opposite momenta lead to oppposite directions," Am. J. Phys. **68**, 991-992 (2000).

10. N.D. Mermin, "From cbits to qubits: teaching computer scientists quantum mechanics," Am. J. Phys. **71**, 23-30 (2003), esp. his Section VI discussion of the Born rule. See also the corresponding discussion in E. Gerjuoy, "Shor's factoring algorithm and modern cryptography. an illustration of the capabilities inherent in quantum computers," Am. J. Phys. **73**, 521-540 (2005), Section III.B.2.

11. See, e.g., W. Heitler, *The Quantum Theory of Radiation* (Oxford, 1954), pp. 38 ff.