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QUANTUM MECHANICAL BLACK HOLES:TOWARDS A UNIFICATION OF QUANTUM MECHANICS AND GENERAL RELATIVITY

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Dedicated to the memory of my parents

Abstract

In this paper, starting from vortices we are finally lead to a treatment of Fermions as Kerr-Newman type Black Holes wherein we identify the horizon at the particle's Compton wavelength periphery. A naked singularity is avoided and the singular processes inside the horizon of the Black Hole are identified with Quantum Mechanical effects within the Compton wavelength.

Inertial mass, gravitation, electromagnetism and even QCD type interactions emerge from such a description including relative strengths and also other features like the anomalous gyromagnetic ratio, the discreteness of the charge, the reason why the electron's field emerges from Newman's complex transformation in General Relativity, a rationale for the left handedness of neutrinos and the matter-antimatter imbalance.

This model describes the most fundamental stable Fermions viz., the electrons, neutrinos and approximately the quarks. It also harmoniously unifies the hydrodynamical, monopole and classical relativistic perspectives.

1 Introduction

Ordinary Quantum Mechanics works at distances much greater than the Compton wavelength of elementary particles or roughly $10^{-12}cm$. In the domain of Quantum Field Theory, particles are points, space-time is a continuum and special relativity holds, though very recently there has been a school of thought (the spirit of Effective Field Theories) that Field Theory itself is a low energy approximation. On the other hand in Quantum Gravity we attempt to deal with phenomena at distances of the order of the Planck length or $10^{-33}cm$, though there has as of now been no successful unification of Quantum Mechanics and General Relativity.

In this preliminary communication, we consider an alternative viewpoint, dealing with distances of the order of the Compton wavelengh. At this level Quantum Mechanical phenomena like zitterbewegung and negative energies and luminal velocities come in. Taking a route through relativistic vortices, monopoles and classical considerations, we are lead to the model of leptons (and also approximately quarks) as "Quantum Mechanical Black Holes" (QMBH in what follows), wherein features of Quantum Mechanics and General Relativity are inextricably inter-woven. At the same time, we can trace the origin of inertial mass, gravitation, electromagnetism and even QCD type interactions in such a picture.

In section 2 we invoke the DeBroglie-Bohm Hydrodynamical Formulation to picture an elementary particle as a relativistic vortex from which it is possible to recover its mass and quantized spin. Taking the cue from here in section 3 we argue that the inertial mass of an elementary particle is the energy of binding of nonlocal amplitudes in the zitterbewegung Compton wavelength region. In section 4 it is shown how the Dirac monopole theory is really identical to the picture of a particle as a relativistic vortex. In section 5 the zitterbewegung is examined in greater detail and it is argued that the usual positive energy states we encounter in the physical universe are at scales greater than the Compton wavelength. In section 6 it is shown how the preceding Quantum Mechanical considerations can be equally well described in classical terms, that is for a relativistic collection, or for a hydrodynamical flow. In section 7 it is suggested that an electron, for example could be described by the Kerr-Newman metric, while a full General Relativistic rationale for such an identification is given in section 8. Finally in section 9 a number of comments are made.

Ultimately there is a convergence of the various approaches and a harmonious unified picture appears to emerge.

2 The Bohm Hydrodynamical Formulation

In the Bohm hydrodynamical formulation[1, 2], we start with the Schrodinger equation

$$\imath\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \tag{1}$$

In (1), the substitution

$$\psi = Re^{\imath S} \tag{2}$$

where R and S are real functions of $\vec{r}$ and t leads to,

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}.(\rho\vec{v}) = 0 \tag{3}$$

$$\frac{1}{\hbar}\frac{\partial S}{\partial t} + \frac{1}{2m}(\vec{\nabla}S)^2 + \frac{V}{\hbar^2} - \frac{1}{2m}\frac{\nabla^2 R}{R} = 0 \tag{4}$$

where $\rho = R^2, \vec{v} = \frac{\hbar}{m}\vec{\nabla}S$ and $Q \equiv -\frac{\hbar^2}{2m}(\nabla^2 R/R)$.

Using the theory of fluid flow, it is well known that (3) and (4) lead to the Bohm alternative formulation of quantum mechanics. In this theory there is a hidden variable namely the definite value of position while the so called Bohm potential Q can be non local, two features which do not find favour with physicists.

Let us now consider the stationary solutions in the above formulation viz. equation (4), in the absence of external fields. As is known, the non local quantum potential is given by

$$Q = \text{constant} = E,$$

the energy of the system. Further the velocity field is solenoidal,

$$\vec{\nabla}.\vec{v} = 0 \tag{5}$$

Remembering that S the phase is undefined up to a term which is a multiple of π, we can now see from equation (5) that there is a circulation which is given by (cf. ref.[1]).

$$\Gamma = \int_c \vec{v}.\vec{dr} = (\hbar/m)\int_c \vec{\nabla}S.d\vec{r} = (\hbar/m)\oint dS = \frac{\pi\hbar n}{m}, n = 1, 2, ... \quad (6)$$

For reasons which will become clear below we consider the ultra relativistic case, $|\vec{v}| = c$ for all particles of the fluid. We now get from equation (6),

$$m\Gamma = \int_c m\vec{v}.\vec{dr} = \frac{nh}{2}, n = 1, 2, ... \quad (7)$$

where, an integration over all elements ρ, is implied. Here n is the number of nodes (or, in three dimensions, the end points of nodal lines). We can immediately identify (7) with the quantum mechanical spin $\frac{n}{2}$. Interestingly there are $2 \times \frac{n}{2} + 1 = n + 1$ multiply connected regions, both in hydro-dynamics and in the theory of spin.

It is also worth noting that in (7), if the radius of the vortex is taken to be l, then l turns out to be the Compton wavelength, which thus appears as a fundamental length. This will be commented upon later.

Further, considering for simplicity the vortex in (6) to be a thin ring of radius l, we get

$$E = \oint \rho c^2 ds = mc^2,$$

where ρ is the (line) density. Further from (7) we get

$$mc\oint ds = \frac{nh}{2},$$

whence, taking $n = 1$,

$$l = \frac{\hbar}{2mc}$$

The physical picture is now clear[3]. A particle can be pictured as a fluid vortex which is steadily circulating along a ring (or in three dimensions, a spherical shell) with radius equal to the Compton wavelength and with velocity equal to that of light. Its total energy is given by, as seen above

$$Q = E = mc^2 \quad (8)$$

and its angular momentum, which in quantum theory is quantized is given by (7).

3 The Origin of Inertial Mass

We will now compare the above conclusions with the results of[4, 5]. Our starting point is an equation deduced by Feynman[6] in a simple way,

$$\imath\hbar\frac{\partial C(x)}{\partial t} = \frac{-\hbar^2}{2m'}\frac{\partial^2 C(x)}{\partial x^2} \tag{9}$$

where $C(x) \equiv |\psi(x) >$ is the probability amplitude for the particle to be at the point x at some given moment of time.
To deduce equation (9) we follow the development of[6] and define a complete set of base states by the subscript $\imath$and$U(t_2, t_1)$ the time elapse operator that denotes the passage of time between instants t_1 and t_2, t_2 greater than t_1. We denote by, $C_\imath(t) \equiv < \imath|\psi(t) >$, the amplitude for the state $|\psi(t) >$ to be in the state $|\imath >$ at time t, and

$$< \imath|U|j > \equiv U\imath j, U\imath j(t + \Delta t, t) \equiv \delta_{\imath j} - \frac{\imath}{\hbar}H_{\imath j}(t)\Delta t.$$

We can now deduce from the super position of states principle that,

$$C_\imath(t + \Delta t) = \sum_j [\delta_{\imath j} - \frac{\imath}{\hbar}H_{\imath j}(t)\Delta t]C_j(t)$$

and finally, in the limit,

$$\imath\hbar\frac{dC_\imath(t)}{dt} = \sum_j H_{\imath j}(t)C_j(t) \tag{10}$$

where the matrix $H_{\imath j}(t)$ is identified with the Hamiltonian operator. (To facilitate comparison we stick to the notation and development as given in[6]. Before proceeding to derive the Schrodinger equation, we apply equation (10) to the simple case of a two state system ($\imath, j = 1, 2$ respectively; (cf.ref.[6]). This will provide a physical picture for the later work. For a two state system we have

$$\imath\hbar\frac{dC_1}{dt} = H_{11}C_1 + H_{12}C_2$$

$$\imath\hbar\frac{dC_2}{dt} = H_{21}C_1 + H_{22}C_2$$

leading to two stationary states of energies $E - A$ and $E + A$, where $E \equiv H_{11} = H_{22}, A = H_{12} = H_{21}$. We can choose our zero of energy such that $E = 2A$. Indeed as has been pointed out by Feynman, when this consideration is applied to the hydrogen molecular ion, the fact that the electron has amplitudes C_1 and C_2 of being with either of the hydrogen atoms, manifests itself as an attractive force which binds the ion together, with an energy of the order of magnitude $A = H_{12}$.

To proceed, we consider in (10), the $\imath$ to be the space point $x_\imath$ and we denote $C(x_n) \equiv C_n$ the probability amplitude for the particle to be at this space point. Further let $x_{n+1} - x_n = b$. Then considering only the point x_n and its neighbours $x_{n\pm1}$, the equation (10) goes over into

$$\imath\hbar\frac{\partial C(x_n)}{\partial t} = EC(x_n) - AC(x_n - b) - AC(x_n + b) \tag{11}$$

In the limit $b \to 0$, with our choice of the arbitrary zero of energy, (11) goes over into equation (9) where we have now dropped the subscript distinguishing the space point, and $m' = \hbar^2/2Ab^2$.

We now observe that while equation (9) resembles the free Schrodinger equation, as has been pointed out by Feynman, m' is not really the inertial mass, but an "effective mass" that emerges from the probability amplitude for the particle to be found at a neighbouring point. So (9) is not the Schrodinger equation.

The Schrodinger equation can be obtained from (9) if it can be shown that m' can somehow be replaced by m. This is what we propose to do.

To start with let us suppose that the particle has no mass other than the effective mass m', so that we can treat equation (9) as the Schrodinger type equation for such a particle which has only amplitude to be at neighbouring points. Let us now suppose that the particle acquires non zero probability amplitude to be present non locally at other than neighbouring points. We can then no longer work with equations (11) and (9). We will have to use the full equation (10) which explicitly exhibits this possibility. We rewrite equation (10) as

$$\imath\hbar\frac{dC_\imath(t)}{dt} = H_{\imath\imath}C_\imath(t) + H_{\imath,\imath-1}C_{\imath-1}(t) + H_{\imath,\imath+1}C_{\imath+1}(t) + \sum_j H_{\imath,\imath+j}(t)C_j(t), (j = \pm2, \pm3,)$$

or as in the transition of equation (11) to equation (9),

$$\imath\hbar\frac{\partial C(x)}{\partial t} = \frac{-\hbar^2}{2m'}\frac{\partial^2 C(x)}{\partial x^2} + \int H(x,x')C(x')dx' \tag{12}$$

where we have replaced $H_{\imath j}$ by $H(x,x')$ and the points $x_\imath$ are in the limit taken for the time being to be a continuum. This is as in the well known case of the non-local Schrodinger equation for a non-local potential[7] but for a particle having only an effective mass.
The matrix $H(x,x')$ gives the probability amplitude for the particle at x to be found at x', that is,

$$H(x,x') =< \psi(x')|\psi(x) > \tag{13}$$

where as is usual we write $C(x) \equiv \psi(x)(\equiv |\psi(x) >$, the state of a particle at the point x).
Usually the amplitude $H(x,x')$ is non-zero only for neighbouring points x and x', that is, $H(x,x') = f(x)\delta(x-x')$. But if $H(x,x')$ is not of this form, then there is a non-zero amplitude for the particle to "jump" to an other than neighbouring point. In this case $H(x,x')$ may be described as a non local amplitude. Indeed such non-local amplitudes are implicit in the Dirac equation also and this will be commented on later.
We now give a quick derivation of how the inertial mass emerges from equation (12). The non local Schrodinger equation (12), given only the effective mass m', can be written, with the help of (13), as,

$$\imath\hbar\frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m'}\frac{\partial^2 \psi}{\partial x^2} + \int \psi^*(x')\psi(x)\psi(x')U(x')dx', \tag{14}$$

where,
i)$U(x) = 1$ for $|x| < R, R$ arbitrarily large and also $U(x)$ falls off rapidly as $|x| \to \infty; U(x)$ has been introduced merely to ensure the convergence of the integral; and
ii)$H(x,x') =< \psi(x')\psi(x) >= \psi^*(x')\psi(x)$.
(14) is an integro-differential equation of degree three.
The presence of the, what at first sight may seem troublesome, non-linear and non-local term, viz., the last term on the right side of (14) will be satisfactorily explained in the sequel.

In (14),in the first approximation $\psi(x)$ can be taken to be the solution of the Schrodinger like equation (9), viz.,

$$\imath\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m'}\frac{\partial^2\psi}{\partial x^2} \tag{15}$$

In effect, we linearize (14), so that we get,

$$\imath\hbar\frac{\partial\psi}{\partial t} = [-\frac{\hbar^2}{2m'}\frac{\partial^2}{\partial x^2} + m_0]\psi \tag{16}$$

where,

$$m_0 = \int \psi^*(x')\psi(x')U(x')dx'$$

In operator language, (16) becomes,

$$\bar{H} = \frac{p^2}{2m'} + m_0 \tag{17}$$

where $\bar{H}$ is the Hamiltonian operator, p the momentum operator and where, what can now be anticipated as a rest mass like term m_0, appears for a particle assumed not to have any rest mass in the absence of the non-local amplitude term in (14). Also we have replaced the Hamiltonian matrix H by $\bar{H}$ to stress that, to start with, in (12) and (14), the particle has no inertial mass. To facilitate comparison with the usual theory, we next multiply both sides of (17) by the constant $\frac{m'}{m}$, where,

$$m = (m_0 m')^{\frac{1}{2}}/c,$$

c being the velocity of light. (The reason for the appearance of the velocity of light, c can be seen below (cf.equation (19)) and the constant could be absorbed into the state vector, whose direction is all that matters. We then get,

$$\hat{H} = \frac{p^2}{2m} + mc^2 \tag{18}$$

The physical meaning of (18) is now clear. In an expansion of the classical relativistic expression for energy,

$$E = (p^2c^2 + m^2c^4)^{1/2}$$

as is well known, if we keep terms up to the order $(p/mc)^2$, we get,

$$E = \frac{p^2}{2m} + mc^2 \tag{19}$$

We can now easily identify m in (18) with the rest mass on comparing this equation with (19). (Interestingly it is not accidental that equation (18) corresponds to the approximation (19) as will be seen below). If further, we denote

$$H = \hat{H} - mc^2,$$

where H can be easily identified with the usual kinetic energy operator (or energy operator in non-relativistic theory, remembering that we are considering a free particle only), (18) becomes

$$H = \frac{p^2}{2m} \tag{20}$$

In a strictly non relativistic context, where the rest energy of the particle is not considered, the Hamiltonian is given by (20); otherwise, it is given approximately by (18). We get from (20), the Schrodinger equation,

$$\imath\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \tag{21}$$

All these considerations can be considered in a postulative development [5] and also generalized in a simple way to three dimensions, but as there is no new physical insight, the details are not given.

The physical origin of the rest mass is clear from equation (18): in the two state hydrogen molecular ion case considered earlier, it was the amplitude for the single electron to be with one hydrogen atom or the other which showed up as a binding energy. Similarly the amplitude of a particle to be at x or x' viz. the second term on the right side of equation (14) manifests itself as an (attractive) energy, which may be called the mass energy of the particle or the self energy or the energy of self interaction. This can be seen to be the particle's inertial mass.

We now come to the non local term in equation (14), the term which gives the inertial mass. Non locality implies superluminal velocities and the breakdown of causality which is not permissible in general. However without any contradiction to the theory it is well known that Quantum Mechanics allows

such non locality, owing to the uncertainity principle [8], within the Compton wavelength of a particle. So there is no contradiction if the non local integral in (14) is taken within the region of the particle's Compton wavelength, that is, the inertial mass is a result of non local processes within the Compton wavelength of the particle.
Indeed the usual Dirac equation also has a non local character: The operator $c\vec{\alpha}.\vec{p}+\beta mc^2$ is equivalent to and replaces the non-local square-root operator, $(-\hbar^2\nabla^2 + m^2c^4)^{1/2}$. Here also, the non-local effects in the form of negative energies are encountered - again within the Compton wavelength region (cf.ref.[9]).
In the light of the preceding considerations, we can derive the Schrodinger equation from an alternative angle: It appears that the "point" particle is really spread over the non-locality region $\sim \bar{b} = \frac{\hbar}{mc}$, the Compton wavelength. Further, the energy of the particle i.e., the energy tied up within this region viz., 2A is the inertial mass energy mc^2. We could now, speak of the amplitude for the particle at x to be found (locally) at a neighbouring point $x + b$, except that in the limit, $b \rightarrow \bar{b}$ (and not as earlier 0). The effective mass m' in equation (9) is then given by,

$$m' = \frac{\hbar}{2Ab^2} = m,$$

that is the mass itself!
So, equation (9) can be interpreted as the Schrodinger equation.
It is worth re-emphasizing that it is the force of binding of non-local positions within the Compton wavelength, rather like the Hydrogen molecular ion binding, that manifests itself as inertial mass.
Finally we briefly comment on the appearance of the extra mass energy term in equations like (12), (14), (17), (18) or (19)[5, 10, 11].
The Schrodinger equation is really the limiting case of the Dirac equation in which process an inessential phase factor is dropped. Another way of looking at this is that the constant potential m_oc^2 does not affect the dynamics. That is the reason why the Schrodinger equation is not Galilean invariant, as a non relativistic theory should be, and infact exhibits the Sagnac effect, which a strictly Galilean invariant theory should not[12].

The convergence of the above formulation and the Bohm hydrodynamical formulation is evident once we restrict ourselves to the Compton wavelength

and luminal velocities. The particle is now a relativisitc fluid vortex circulating along a ring of radius equal to the Compton wavelength. The Q given by (8) is the energy of this system or particle and corresponds to the inertial mass term given by the integral in (14) or equivalently the constant potential term in (19).

4 Monopoles

It is well known that there is a close connection between the hydrodynamic theory discussed in Section 2 and Dirac's theory of monopoles [13]. The starting point in this latter case is precisely the decomposition of the wave function (2), but the focus is on the phase function S which need not be integrable: exactly as in the case of the vortex above, there can be nodal singularities. Infact the S in this theory defines the function $\vec{K}$ of Dirac, exactly as it does the momentum vector of section 2. But this time $(\vec{K}, K_0)$ is identified with the electromagnetic potential and an integral like (7) then consists of, in addition to the term $\frac{n\hbar}{2}$ the electromagnetic flux, again n being the number of nodal lines with end points inside the vortex or region of integration. Thus the well known equation of the magnetic monopole viz. $\mu = \frac{1}{2}n\hbar\frac{c}{e}$, on identifying

$$\vec{K} \equiv \frac{e}{\hbar c}\vec{H} \tag{22}$$

($\vec{H}$ being the magnetic field) with the momentum of section 2 gives back equation (7) for quantized spin. We will come back to this point later.

5 Zitterbewegung and the Compton Wavelength

We will now examine briefly the phenomenon of zitterbewegung in the context of the Compton wavelength. In the usual theory of the Dirac equation [14], it is well known that the eigen values of the velocity operator $c\vec{\alpha}$are $\pm\, c$, the velocity of light while the position operator is non Hermitian: It consists of a real part which is the usual position and a rapidly oscillating (or

zitterbewegung) imaginary part,

$$x = (c^2 p_1 H^{-1} t) + \frac{\imath}{2} c\hbar(\alpha_1 - c p_1 H^{-1}) H^{-1} \tag{23}$$

Both these puzzling facts are reconciled by the fact that our measurements are really averaged over time intervals of the order $\hbar/mc^2$ and correspondingly over the space intervals of the order of $\hbar/mc$, the Compton wavelength. In this case the imaginary part in (23) disappears (cf. ref.[14]). Hermiticity and Physics begins after such an averaging necessitated by our gross measurements.

One could say that (23) applies in a non local region bounded by the Compton wavelength as we saw in section 2. Within the region, we have to contend with unphysical phenomena like superluminal velocities and negative energies and in general non Hermitian operators. Outside the Compton wavelength, that is on averaging over space time intervals of this order, we are back in usual Physics.

We consider now, for simplicity, the free particle Dirac equation. The solutions are of the type,

$$\psi = \psi_A + \psi_S \tag{24}$$

where

$$\psi_A = e^{\frac{\imath}{\hbar}Et} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \text{ or } e^{\frac{\imath}{\hbar}Et} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \text{ and}$$

$$\psi_S = e^{-\frac{\imath}{\hbar}Et} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ or } e^{-\frac{\imath}{\hbar}Et} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \tag{25}$$

denote respectively the negative energy and positive energy solutions. From (24) the probability of finding the particle in a small volume about a given point is given by

$$|\psi_A + \psi_S|^2 = |\psi_A|^2 + |\psi_S|^2 + (\psi_A \psi_S^* + \psi_S \psi_A^*) \tag{26}$$

Equations (25) and (26) show that the negative energy and positive energy solutions form a coherent Hilbert space and so the possibility of transition to negative energy states exists. This difficulty however is overcome by the Hole theory which uses the Pauli exclusion principle.

However the last term on the right side of (26) is like the zitterbewegung term. When we remember that we really have to consider averages over space time intervals of the order of $\hbar/mc$ and $\hbar/mc^2$, this term disappears and effectively the negative energy solutions and positive energy solutions stand decoupled in what is now the physical universe.

A more precise way of looking at this is[15] that as is well known, for the homogeneous Lorentz group, $\frac{p_0}{|p_0|}$ commutes with all operators and yet it is not a multiple of the identity as one would expect according to Schur's lemma: The operator has the eigen values ± 1 corresponding to positive and negative energy solutions. This is a super selection principle pointing to the two incoherent Hilbert spaces or universes [16] now represented by states ψ_A and ψ_S which have been decoupled owing to the averaging over the Compton wavelength space- time intervals. But all this refers to energies such that our length scale is greater than the Compton wavelength. As we reach energies corresponding to the Compton wavelength scale, negative energy solutions show up as anti particles. Thus the super selection principle which comes into play on averaging over Compton wavelength scales dispenses with the Pauli exclusion principle.
Thus once again we see that outside the Compton wavelength region we recover the usual physics.

6 A Classical Viewpoint

Let us now try to understand from the standpoint of classical theory, why we encounter luminal (or superluminal) velocities and complex coordinates, corresponding to non-Hermitian operators, within the Compton wavelength region. From a classical point of view, we could say that if in the Lorentz transformation,

$$x = \gamma(x' - vt), \gamma = (1 - v^2/c^2)^{-1/2} \tag{27}$$

$v > c$ is allowed, then the coordinates become imaginary, this being true

within the Compton wavelength as in (23), in the sense that non locality is allowed there. So (23) can be understood as representing a coordinate which is imaginary within the Compton wavelength but becomes the usual position coordinate outside, that is after averaging over these intervals. One way of interpreting (23) would be that from our physical point of view using (27) there is a region where $v > c$, consisting of virtual or superluminal ghost particles bounded by a region, a sphere of radius equal to the Compton wavelength consisting of massless "particlets" (to distinguish them from partons, instantons and the like, or to make a clean break, "Ganeshas") with velocity of light. Only on averaging over this vortex like sphere or region, do we come to the domain of conventional physics and the usual particles moving with sub luminal velocities. It may be remarked that the De Broglie-Bohm picture of a particle is that of an average over an ensemble (cf. ref.[1]) but the above picture is different: It is an averaging over a physically inaccessible region.

Indeed it is known that for a collection of relativistic particles, the various mass centres form a two-dimensional disc perpendicular to the angular momentum vector $\vec{L}$ and with radius (ref.[17])

$$r = \frac{L}{mc} \tag{28}$$

Further if the system has positive energies, then it must have an extension greater than r, while at distances of the order of r we begin to encounter negative energies.

If we consider the system to be a particle of spin or angular momentum $\frac{\hbar}{2}$, then equation (28) gives, $r = \frac{\hbar}{2mc}$. That is we get back the Compton wavelength.

On the other hand it is known that (cf. ref.[9]), if a Dirac particle is represented by a Gausssian packet, then we begin to encounter negative energies precisely at the same Compton wavelength as above. Thus a particle can indeed be treated as a vortex or a spherical shell of relativistic sub constituents or "particlets" (or Ganeshas).
Another way of looking at this from a hydrodynamical perspective is to consider for example a one dimensional streamlined flow of a fluid, with velocity $\vec{v} \equiv v_x$, along the x axis. In this case $\vec{\nabla} \times \vec{v}$ vanishes everywhere, that is

there is no circulation (no vortices).
However all this applies to a singly connected space. As is well known one could still have a circulation about a point P (for example cf.ref.[18]) exactly as in the case of equation (6) because P would be a singularity and the space would no longer be singly connected, rather it would be doubly connected. In this case $\vec{\nabla} \times \vec{v}$ would vanish everywhere except on the boundary of a closed curve around the point P. We would now have a complex vector potential in the complex plane $x + \imath y$ (cf.ref.[18]). Away from the point P in what may be called the asymptotic region we would have the one dimensional flow, but as we approach the point P, that is the boundary of the curve enclosing P, we encounter circulatory motion in the $x + \imath y$ plane.
In our case the region bounded by the Compton wavelength plays the role of the closed curve around the point P. Outside this region we have the usual space (or space time) of Physics. But as we approach the Compton wavelength region we encounter a region where each of the space time axes becomes as it were a complex plane. We will return to this point.

7 Particles as Black Holes

The fact that, as we saw in sections 2 and 3, the mass generating non-local amplitudes are confined to a region of width $\sim \frac{\hbar}{mc}$ suggests that the particle could be a black hole, because in this case also, there is a width, the horizon, inside which such unphysical phenomena appear. The possibility that a particle could be a Schwarzchild black hole has been examined earlier by Markov, Motz and others[19, 20, 21, 22] and leads to a high particle mass of $10^{-5}gm$, without much insight into other properties.
So let us approach the problem from a different angle. We consider a charged Dirac (spin half) particle. If we treat this as a spinning black hole, there is an immediate problem:The horizon of the Kerr-Newman black hole becomes in this case, complex[23],

$$r_+ = \frac{GM}{c^2} + \imath b, b \equiv (\frac{G^2Q^2}{c^8} + a^2 - \frac{G^2M^2}{c^4})^{1/2} \tag{29}$$

where G is the gravitational constant, M the mass and $a \equiv L/Mc, L$ being the angular momentum. That is, we have a naked singularity apparently

contradicting the cosmic censorship conjecture. However, in the Quantum Mechanical domain, (29) can be seen to be meaningful.
Infact, the position coordinate for a Dirac particle as we have seen is given by Dirac[14]

$$x = (c^2 p_1 H^{-1} t + a_1) + \frac{\imath}{2} c\hbar(\alpha_1 - c p_1 H^{-1}) H^{-1}, \tag{30}$$

where a_1 is an arbitrary constant and $c\alpha_1$ is the velocity operator with eigen values $\pm c$. The real part in (30) is the usual position while the imaginary part arises from zitterbewegung. Interestingly, in both (29) and (30), the imaginary part is of the order of $\frac{\hbar}{mc}$, the Compton wavelength, and leads to an immediate identification of these two equations. We must remember that our physical measurements are gross as noted earlier - they are really measurements averaged over a width of the order $\frac{\hbar}{mc}$. Similarly, time measurements are imprecise to the tune $\sim \frac{\hbar}{mc^2}$. Very precise measurements if possible, would imply that all Dirac particles would have the velocity of light, or in the Quantum Field Theory atleast of Fermions, would lead to divergences. (This is closely related to the non-Hermiticity of position operators in relativistic theory as can be seen from equation (30) itself [15]. Physics as pointed out earlier begins after an averaging over the above unphysical space-time intervals. In the process as is known (cf.ref. [15]), the imaginary or non-Hermitian part of the position operator in (30) disappears. That is in the case of the QMBH (Quantum Mechanical Black Hole), obtained by identifying (29) and (30), the naked singularity is shielded by a Quantum Mechanical censor.
To continue, we first adhoc treat a Dirac particle as a Kerr-Newman black hole of mass m, charge e and spin $\frac{\hbar}{2}$. The gravitational and electromagnetic fields at a distance are given by (cf.ref.[24],

$$\begin{aligned} \Phi(r) = -\frac{Gm}{r} + 0(\frac{1}{r^3}) E_{\hat{r}} = \frac{e}{r^2} + 0(\frac{1}{r^3}), E_{\hat{\theta}} = 0(\frac{1}{r^4}), E_{\hat{\phi}} = 0, \\ B_{\hat{r}} = \frac{2ea}{r^3} cos\theta + 0(\frac{1}{r^4}), B_{\hat{\theta}} = \frac{easin\theta}{r^3} + 0(\frac{1}{r^4}), B_{\hat{\phi}} = 0, \end{aligned} \tag{31}$$

exactly as required. Infact, as is well known, (31) also exhibits the electron's anomalous gyromagnetic ratio $g = 2$. So we are on the right track!
We next examine more closely, this identification of a Dirac particle with a Kerr-Newman black hole. We reverse the arguments after equation (30)

which lead from the complex or non-Hermitian coordinate operators to Hermitian ones: We consider instead the displacement,

$$x^\mu \rightarrow x^\mu + \imath a^\mu \tag{32}$$

and first consider the temporal part, $t \rightarrow t + \imath a^0$, where $a^0 \approx \frac{\hbar}{2mc^2}$, as before. That is, we probe into the QMBH or the zitterbewegung region inside the Compton wavelength as suggested by (29) and (30). Remembering that $|a^\mu| << 1$, we have, for the wave function,

$$\psi(t) \rightarrow \psi(t + \imath a^0) = \frac{a^0}{\hbar}[\imath\hbar\frac{\partial}{\partial t} + \frac{\hbar}{a^0}]\psi(t)$$

As $\imath\hbar\frac{\partial}{\partial t} \equiv p^0$, the usual fourth component of the energy momentum operator, we identify, by comparison with the well known electromagnetism-momentum coupling, $p^0 - e\phi$, the usual electrostatic charge as,

$$\Phi e = \frac{\hbar}{a^0} = mc^2 \tag{33}$$

In the case of the electon, we can verify that the equality (33) is satisfied:
We follow the classical picture of a particle as a rotating shell with velocity c, as encountered in sections 2 and 6, and which will be further justified in the sequel. The electrostatic potential inside a spherical shell of radius $'a'$ is,

$$\Phi = \frac{e}{a} \tag{34}$$

As is well known, the balance of the centrifugal and Coulomb forces gives, for an electron orbiting another at the distance a,

$$a = \frac{e^2}{mc^2},$$

which is the classical electron radius.
So, (34) now gives,

$$e\Phi = mc^2,$$

which is (33).
If we now use the usual value of $'a'$ viz., $2.8 \times 10^{-13} cm.$, in (34) and substitute

in (33), while rewriting the right side as $\hbar c/(\hbar/mc)$ and substitute the value of the electron Compton wavelength, $\frac{\hbar}{mc} = 3.8 \times 10^{-11} cm.$, we get

$$\hbar c \approx 136 e^2$$

That is, we get the rationale for this fundamental relation, which no longer turns out to be accidental. In any case, equation (33) throws up the connection between the charge, mass and the velocity of light.

It may be noted in passing that in the usual displacement operator theory ([14]) the operators like $\frac{d}{dx}$ or $\frac{d}{dt}$ are indeterminate to the extent of a purely imaginary additive constant which is adjusted against the Hermiticity of the operators concerned.

We next consider the spatial part of (32), viz.,

$$\vec{x} \rightarrow \vec{x} + \imath \vec{a}, \text{where} |\vec{a}| = \frac{\hbar}{2mc},$$

given the fact that the particle is now seen to have the charge e (and mass m). As is well known[25], this leads in General Relativity from the static Kerr metric to the Kerr-Newman metric where the gravitational and electromagnetic field of the particle is given by (31), including the anomalous factor $g = 2$. In General Relativity, the complex transformation (32) and the subsequent emergence of the Kerr-Newman metric has no clear explanation. Nor the fact that, as noted by Newman[26] spin is the orbital angular momentum with an imaginary shift of origin. But in the Quantum Mechanical context and in view of the considerations of section 2 and 6, we can see the rationale: the origin of (32) lies in the QMBH. We started with a massless particle. Then we saw the emergence of mass and also the origin of gravitation and electromagnetism in the processes inside the Compton wavelength represented by an imaginary displacement - the nonlocal QMBH region.

More specifically, the temporal part of the transformation (32) lead to the appearance of charge in (33). The space part then, as is known leads to the Kerr-Newman metric.

There is another way to see the emergence of electromagnetism. It is well known that for the Dirac four spinor,(θ_χ) where θ denotes the positive energy two spinor and χ the negative energy two spinor, at and within the Compton wavelength, it is χ that dominates. Further, under reflections, while $\theta \rightarrow \theta, \chi$ behaves like a psuedo- spinor[9]

$$\chi \rightarrow -\chi$$

Hence the operator $\frac{\partial}{\partial x^\mu}$ acting on χ, a density of weight $N = 1$, has the following behaviour[27],

$$\frac{\partial \chi}{\partial x^\mu} \rightarrow \frac{1}{\hbar}[\hbar \frac{\partial}{\partial x^\mu} - NA^\mu]\chi \tag{35}$$

where,

$$A^\mu = \hbar \Gamma^{\mu\sigma}_{\sigma} = \hbar \frac{\partial}{\partial x^\mu} log(\sqrt{|g|}) \equiv \nabla^\mu \Omega \tag{36}$$

As before we can identify NA^μ in (35) with the electro-magnetic four potential. That $N = 1$, explains the fact that charge is discrete. It will be shown in the next section that,

$$A^\mu \sim const. \frac{e^2}{r} \tag{37}$$

in agreement with (33). That is, electromagnetism is the result of the covariant derivative that arises due to the Quantum Mechanical behaviour of the negative energy components within the Compton wavelength region.
We observe, that in case the mass $m \rightarrow 0$, the considerations of section 3 imply that there are no negative energy components while (33) and (37) show that such a particle has no charge. The massless neutrino fits this description exactly: it has a two component wave function and is chargeless.
There is also the muon which satisfies (37) in the order of magnitude sense. But it is unstable and disintegrates into an electron (or positron) and two neutrinos anyway.
Thus these considerations describe the stable leptons, viz., electrons and neutrinos, and approximately the remaining unstable lepton.
It is worth noting that equation (36) strongly resembles Weyl's formulation for the unification of electromagnetism and gravity. But there is an important difference [28]: Weyl's Christoffel symbol contains two independant entities - the metric tensor $g^{\mu v}$ and the electromagnetic potential Φ. So there is no unification of electromagnetism and gravity. Our formulation uses only the Quantum Mechanical pseudo spinor property.
It is interesting that, in the light of the above considerations an application of Maxwell's equations in this Compton wavelength region of "charged matter" leads to meaningful results: In this case the fact that A^μ in (36) is a four gradient poses no problem. We have, using Maxwell's equations [29],

$$\phi \equiv A^0 = \frac{\partial \Omega}{\partial t}, \vec{B} = \vec{\nabla} X \vec{A} = \vec{\nabla} X (\vec{\nabla} \Omega) = 0,$$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi = -2\vec{\nabla}\Omega, \vec{E} = -2\vec{\nabla}\phi \tag{38}$$

Also,

$$\vec{\nabla}.\vec{E} = 2\nabla^2\phi = 4\pi\sigma, \vec{\nabla}.\vec{B} = 0, \tag{39}$$

while

$$\vec{\nabla} \times \vec{B} = 0 = 4\pi s + \frac{\partial \vec{E}}{\partial t} \tag{40}$$

and,

$$\vec{\nabla} \times \vec{E} = -\sigma\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (2\vec{\nabla}\phi) = 0$$

Equations (38), (39), and (40) show that effectively this is a steady field with potential ϕ that is, we would work as if we have a steady field of potential ϕ except that there is an anamolous doubling of the charge and current. Now, as is well known the usual orbital magnetic moment is given by [30]

$$\mu = \frac{e}{2mc}p_\phi \tag{41}$$

where p_ϕ is the angular momentum and e is the charge. In our case, e in (41) is effectively replaced by $2e$, so that in the usual units of $e/2mc$, we now have for the Dirac particle, instead of (41),

$$g = \frac{\mu}{p_\phi} = 2$$

This is the anomalous gyromagnetic ratio which arises because as noted earlier spin is the orbital angular momentum with an imaginary shift of origin, or equivalently within the Compton wavelength region. We come to this point now, in greater detail.

8 A General Relativistic Approach: Origin of QCD Interactions

Thus far it appears that the QMBH description applies to electrons and more generally Leptons. In the light of the preceding considerations, we will now approach the problem from a General Relativistic point of view. This will

also reveal the origin of QCD type interactions. Taking the cue from the foregoing considerations, we now treat the particle as a relativistic fluid of "particlets" (or Ganeshas). Our starting point is the linearized theory [24]:

$$g_{\mu v} = \eta_{\mu v} + h_{\mu v}, h_{\mu v} = \int \frac{4T_{\mu v}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|} d^3x' \tag{42}$$

(A bar on T has been dropped.)
In (42), velocities comparable to the velocity of light c are allowed and also the stresses T^{jk} and momentum densities T^{0j} can be comparable to the energy momentum density T^{00}. As in ref.[24], we can easily deduce that, when $\frac{|\vec{x}'|}{r} << 1$, where $r \equiv |\vec{x}|$, and in a frame with origin at the centre of mass and at rest with respect to the particle,

$$Gm = \int T^{00} d^3x \tag{43}$$

$$S_k = \int \epsilon_{klm} x^l T^{m0} d^3x \tag{44}$$

where m is the mass (or approximate mass because of the linear approximation), and S_k is the angular momentum. We next observe that,

$$T^{\mu v} = \rho u^{\mu} u^{v} \tag{45}$$

If we now work in the Compton wavelength region of the QMBH, we have, while $u^0 = 1$,

$$|u^l| = c \tag{46}$$

(This is the Quantum Mechanical input)
Substitution of (45) and (46) in (44) gives on using the Mean Value Theorem,

$$S_k = c < x^l > \int \rho d^3x$$

As $< x^l > \sim \frac{\hbar}{2mc}$, using (43), we get, $S_k \approx \frac{\hbar}{2}$, as required for a spin half particle. Infact this relation becomes exact if we treat the QMBH as effectively a rotating shell distribution of radius $\hbar/2mc$ as noticed earlier and, keeping in mind the fact that the interior region is in any case unphysical as seen in section 6, and is described by complex space-time coordinates. Once again we can see why orbital angular momentum with a complex shift gives spin,

as noticed earlier by Newman but without a rationale (cf.ref.[26]).
The gravitational potential can similarly be obtained from (42) and (43) (cf. ref.[24]),

$$\Phi = -\frac{1}{2}(g^{00} - \eta^{00}) = -\frac{Gm}{r} + 0(\frac{1}{r^3}) \tag{47}$$

We saw in section 7 that the electromagnetic potential is given by,

$$A^{\mu} = \hbar\Gamma^{\mu\sigma}_{\sigma}$$

Using the expression for the Christoffel symbols, we have,

$$A_{\sigma} = \frac{1}{2}(\eta^{\mu v}\hbar_{\mu v}), \sigma,$$

so that, from (42),

$$A_0 = 2\int \eta^{\mu v}\frac{\partial}{\partial t}[\frac{T_{\mu v}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|}]d^3x'$$

Remembering that $|\vec{x} - \vec{x}'| \approx r$ for the distant region we are considering, we have,

$$A_0 \approx \frac{2}{r}\int \eta^{\mu v}[\frac{\partial}{\partial \tau}T_{\mu v}(\tau, \vec{x}').\frac{d}{dt}(t - |\vec{x} - \vec{x}'|)]d^3x' \approx \frac{2}{r}\int \eta^{\mu v}\frac{d}{d\tau}T_{\mu v}.(1 + c)d^3x',$$

or finally

$$A_0 \approx \frac{2c}{r}\int \eta^{\mu v}\frac{d}{d\tau}T_{\mu v}d^3x' \tag{48}$$

as $c >> 1$, and where we have used the fact that in the Compton wavelength region, $|u_v| = c$.
It has already been observed that QMBH can be treated as a rotating shell distribution with radius $R \equiv \frac{\hbar}{2mc}$. So we have,

$$|\frac{du_v}{dt}| = |u_v|\omega \tag{49}$$

where ω, the angular velocity is given by,

$$\omega = \frac{|u_v|}{R} = \frac{2mc^2}{\hbar} \tag{50}$$

We get the same relation in the theory of the Dirac equation, remembering that in (43) and (44) the centre of mass is at rest:

$$\imath\hbar\frac{d}{dt}(c\alpha_\imath) = -2mc^2(c\alpha_\imath),$$

where $c\alpha_\imath$ is the velocity operator (cf.ref.[14]). Finally, on using (45), (49) and (50) in (48), we get,

$$\frac{e'e}{r} = A_0 \sim \frac{\hbar c^3}{r}\int \rho\omega d^3x' \sim (Gmc^3)\frac{mc^2}{r} \tag{51}$$

where $e' = 1esu$ corresponds to the charge $N = 1$ and e is the test charge. Because of the approximations taken in deducing (51), a dimensional constant $(\frac{L}{T})^5$ has to be multiplied on the left side, which then becomes, in units, $c = G = 1$,

$$e'e.(\text{dimensional constant}) \approx 1.6 \times 10^{-111}cm^2$$

The right side is,

$$Gm^2c^5 \approx 4.5 \times 10^{-111}cm^2,$$

in broad agreement with the left side.
Alternatively, using the values of G, m and c in (51), we get,

$$e \sim 10^{-10}esu,$$

which is correct.
Yet another way of looking at (51) is, that we get, as $e' = 1esu \sim 10^{10}$

$$\frac{e^2}{Gm^2} \sim 10^{40},$$

which is well known empirically. But equation (51) gives the reason for this relation.
In any case, equations (33) and (51) show the inter-relation between $e, m, c$and$G$.
So far we have been considering distances far from the particle: $|\vec{x}' - \vec{x}| >> |\vec{x}'|$. This is the approximation invoked in a transition from (42) to equations (43), (44) etc. Let us now see what happens when $|\vec{x}| \sim |\vec{x}'|$. In this case, we have from (42), expanding in a Taylor series about t,

$$h_{\mu v} = 4\int \frac{T_{\mu v}(t,\vec{x}')}{|\vec{x}-\vec{x}'|}d^3x' + (\text{terms independent of}\vec{x}) + 2$$
$$\int \frac{d^2}{dt^2}T_{\mu v}(t,\vec{x}').|\vec{x}-\vec{x}'|d^3x' + 0(|\vec{x}-\vec{x}'|^2) \tag{52}$$

The first term gives a Coulombic $\frac{\alpha}{r}$ type interaction except that the coefficient α is of much greater magnitude as compared to the gravitational or electromagnetic case, because in this approximation, in an expansion of $(1/|\vec{x}-\vec{x}'|)$, all terms are of comparable order. To proceed further, using (49), we have,

$$\frac{d}{dt}T^{\mu v} = \rho u^v \frac{du^\mu}{dt} + \rho u^\mu \frac{du^v}{dt} = 2\rho u^\mu u^v \omega,$$

so that,

$$\frac{d^2}{dt^2}T^{\mu v} = 4\rho u^\mu u^v \omega^2 = 4\omega^2 T^{\mu v}$$

where ω is given by (50). Substitution in (52) gives,

$$h_{\mu v} \approx -\frac{\beta M}{r} + 8\beta M(\frac{Mc^2}{\hbar})^2.r \tag{53}$$

β being a constant.
This resembles the QCD quark potential [31], with both the Coulombic and confining parts. Taking for M the mass of a typical C quark $\sim 1.8Gev$ (cf.ref.[31]), the ratio of the coefficients of the r term and the $\frac{1}{r}$ term as obtained from (53) is $\sim \frac{1}{\hbar^2}(Gev)^2$ as in the case of QCD (ref.[31]). In any case these considerations show that we can get different interactions at different distances in a unified picture, which can approximately atleast represent quarks also.
In this picture, how do we accommodate anti-particles, for example positrons? While treating the negative energy spinor as a density in section 7 we had assumed that $N = 1$. Equally well, we could have chosen $N = -1$. This reverses the sign of the charge, all else remaining the same. So with $N = -1$, we get a positron. Similarly for quarks, N can be taken to be fractional. But this apart it must be remembered that whereas for electrons we took the asymptotic expansions of equations like (42), in the case of quarks we had to consider the region near the Compton wavelength itself.
Thus, it appears that the treatment of Leptons and approximately quarks as QMBH leads to meaningful results. On the other hand, these are the most fundamental constituents of matter, according to current thinking. An alternative is suggested in the next section.

9 Discussion and Miscellaneous Comments

1) We have seen that a particle could be treated as a relativistic vortex, that is a vortex where the velocity of circulation equals that of light or a spherical shell, whose constituents are again rotating with the velocity of light or as a black hole described by the Kerr-Newman metric for a spin $\frac{1}{2}$ particle.
The fact that we get the gravitational potential $\frac{m}{r}$ in equation (47) again confirms that mass comes from the Compton wavelength region.
2) The equation (6) emerges on using the fact that S is defined only up to a multiple of π, whence we get equation (7) giving quantized spin. As pointed out from equation (7) the Compton wavelength emerges. On the other hand equation (28) shows that given the spin $\frac{\hbar}{2}$, we get the Compton wavelength. It is also to be noted that equation (44) gives the spin $\frac{\hbar}{2}$ if we use the Compton wavelength. The Compton wavelength itself appears in quantum mechanics due to the Heisenberg uncertainity principle. So it appears that the quantum mechanical quantized spin and Compton wavelength can be obtained from classical considerations like relativistic vortices. In any case the remarkable universality of the Compton wavelengh was pointed out by Wigner[32] - the above considerations show why it emerges in a natural way. It is interesting to note that Wignall[33] has pointed out that it is the Compton wavelength which is primary, from which even the mass follows. The foregoing lends support to this viewpoint.
It may be pointed out, that interestingly, from a different viewpoint, using considerations of self symmetry, if we assume that the scale of the universe is broken at some stage, that is there is an ultimate micro-level for our measurements, then the Compton wavelength again appears as a fundamental length [34].
3) The fact that the spin of the particle is directly connected to the number of end points of the nodal lines, as seen in section 2 appears to indicate that Fermions are primary and that Bosons can be treated as bound states of Fermions. As pointed out, quarks also could be approximately treated as Quantum Mechanical Black Holes in the foregoing sense, and as it is known pions and other hadrons are indeed treated as bound states of a quark and an anti quark. (Indeed from considerations of the symmetry between leptonic and hadronic currents, leptons and hadrons appear to be the same [35].)
4) In ordinary Quantum Mechanics, ψ being the wave function, $\psi\psi^*$ is proportional to the <u>probability</u> density. On the other hand, we saw in section

3 that the mass density is produced by the non-linear amplitude $\psi\psi^*$ in the Compton wavelength region. More specifically we saw in sections 7 and 8 that it is χ, the negative energy part of the Dirac four spinor (which dominates in this region), that is relevant. That is, ρ being the <u>material</u> density,

$$\rho\alpha\chi\chi^* \tag{54}$$

As observed in section 7, for the two component neutrino, $\chi = 0$, and the neutrinos are massless.
It was shown in an earlier communication [36, 37], how Gravitation can emerge from the Schrodinger equation self-consistently. Again, it is the identification of the material density in (54) which gives substance to that result.
5) Treating the particle as a vortex as in section 2, arguments for a monopole in section 4 then show that there would be the Bohm-Ahranov like effect [38] at the Compton wavelength scale.
6) If in the position formula (30), we consider the real part and also a time interval of $\sim \frac{\hbar}{mc^2}$, we get for $p = mc$,

$$x = \frac{\hbar}{mc}$$

This result can also follow from the Heisenberg Uncertainity Principle.
We see the emergence of the Planck constant $\hbar$ in the extreme situation of the maximum velocity and minimum physical space-time intervals. Moreover, h, m and c are inter-related. Taking the cue from here, we pick up the result in section 3 viz.,

$$E\alpha m \quad \text{or} \quad E = my,$$

where y is the constant of proportionality which was identified adhoc earlier with c^2. Using now Heisenberg's Uncertainity relations, and considering the extreme case, we have, firstly,

$$t \sim \frac{\hbar}{my},$$

so that $x = ct \sim \frac{\hbar c}{my}$, where c is the maximum possible velocity. Further, in this case,

$$p = mc \sim \frac{\hbar}{x} = \frac{my}{c},$$

so that $y = c^2$.
This provides a Quantum Mechanical justification for the formula $E = mc^2$, without taking recourse to special relativity. Indeed as suggested by section 3, e.g. equation (18) and the discussion of the Sagnac effect, the origin of special relativity could be traced to these Quantum Mechanical considerations.
7) It is interesting to note that the above model of a particle could give a rationale for the left handedness of the neutrino in the light of sections 5 and 6. In the case of the neutrino, as the mass is vanishingly small, the Compton wavelength tends to infinity or turns out to be very large. On the other hand we encounter the negative energy solutions within this region. That is we encounter negative energy neutrinoes only. The equation for a negative energy neutrino is (cf. ref.[15]).

$$(-p_o)v(p) = +\vec{\sigma}.\vec{p}v(p)$$

This is the equation for a left handed neutrino in the physical world of positive energy solutions.
8)There is a close connection between the complex shift of section 7, equation (32) (and so, ultimately the Kerr-Newman metric), the hydrodynamical formulation of section 2 and the monopole theory of section 4.
Infact, we could identify K^μ of section 4 and the momentum vector p^μ from section 2 with $\frac{1}{a^\mu}$ of (32). If further $a^\imath$ is taken to be of the order of the Compton wavelength, $\frac{\hbar}{mc}$ and similarly a^o to be of the order of $\frac{\hbar}{mc^2}$ we get immideately

$$|\vec{p}||\vec{a}| \sim mc\frac{\hbar}{mc} = \hbar,$$

which can also be obtained from the Heisenberg uncertainity principle.
9) It may be remarked that we started in section 3 with purely Quantum Mechanical postulates and deduced mechanical effects. We came a full circle in section 8 wherein, from purely classical considerations, we deduced Quantum Mechanical phenomena.
10) The fact that the magnetic field which arises in the monopole formulation as given by equation (22) and the quantized spin angular momentum which arises in the hydrodynamical formulation as given by equation (7) appear to be one and the same is remarkable. This is caused by the fact that the $\vec{K}$ and the momentum vector as given in the two formulations are really one and the same, as pointed out in section 4. Indeed the Coriolis and other effects

of rotating frames [39] bear a strong resemblence to the magnetic effects. As pointed out, electromagnetism and gravitation can be unified in a general relativistic version of quantum mechanics as symbolised by the complete description of the electron in terms of the Kerr-Newman metric. This has been indicated in section 8. Thus in this picture monopoles disappear. Indeed they have not been found todate and Dirac himself expressed his conviction that they do not exist [40]. (They appear again in the theory of non Abelian guages.)

11) The double valuedness that arises from a nodal singularity on the one hand and half integral spin on the other finds an immediate echo in the Kerr-Newman metric. This can be seen as follows.

In natural units the metric is given by (cf. ref.[24]).

$$ds^2 = -\frac{\Delta}{\rho^2}[dt - a\sin^2\theta d\phi]^2 + \frac{\sin^2\theta}{\rho^2}[(r^2+a^2)d\phi - adt]^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2,$$

where, a is the Compton wavelength and,

$$\Delta = r^2 - 2mr + a^2 + m^2 + e^2, \rho^2 \equiv r^2 + a^2\cos^2\theta$$

At $r = a$ and $\theta = \pi/2$, $\Delta = 2a^2$ as both e and $m << a$, and $\rho^2 = a^2$.

If further, we take $a\frac{d\phi}{dt} = \lambda$, we get,

$$ds^2 = (2\lambda^2 - 1)dt^2 + \frac{1}{2}dr^2$$

The choice $\lambda = \frac{1}{2}$ leads to,

$$ds^2 = -\frac{1}{2}dt^2 + \frac{1}{2}dr^2,$$

which is Minkowski like, except for the scale factor $\frac{1}{\sqrt{2}}$. In the foregoing model, $a\frac{d\phi}{dt}$ = velocity of light = 1. The choice $\lambda = \frac{1}{2}$ can be understood as follows: If the azimuthal angle measured by an observer at rest far away, is ϕ', then we get back the velocity of light at $r = a$ for this observer, if $\phi' = 2\phi$, which is precisely spinorial behaviour.

In other words, special relativity for the spin $\frac{1}{2}$ electron can be seen to emerge from the Kerr-Newman metric.

12) More general than the radius of the horizon given in (28) is the static limit of the Kerr-Newman Black Hole, wherein $'a'$ is replaced by acos θ, θ being the usual polar coordinate. However in the QMBH, as we approach the Compton wavelength $\sim a$, we encounter the unphysical zitterbewegung region where θ ceases to have any physical meaning. In other words, the spin in insensitive to θ unlike in the classical case. This is ofcourse well known in Quantum Mechanics.
13) As has been pointed out in the introduction, QFT works with point particles and a space-time continuum in a special relativistic context. Divergences appear when we go right upto $r = 0$. However, it appears that for Fermionic Fields atleast, this picture may be valid only for distances greater than the Compton wavelength: This is in the spirit of Effective Field Theories [41, 42]. (In the words of Weinberg, QFT could be a "low" energy approximation). The above model forbids such a limiting process for Fermions and sets a cut off. Once we enter the QMBH region, a very high energy phenomenon, space-time in the conventional sense becomes unphysical. The appearance of complex coordinates or non- Hermitian operators is a manifestation of this unphysical feature.
14) In the usual formulation of the Hole Theory, the Dirac sea is filled with negative energy electrons, and by invoking the Pauli exclusion principle, transitions to negative energy states are forbidden. In the present formulation, in effect, the Dirac sea of negative energy states is squeezed into the QMBH and the Quantum Mechanical censor of section 7 forbids transitions.
15) It may be remarked that there have been somewhat similar approaches, but these do not explain enough or they assume too much, being still somewhat tentative and preliminary. We discuss some of these very briefly.
Barut and Bracken[43] treat the zitterbewegung effects as a harmonic oscillator in the Compton wavelength region while spin appears as the orbital angular momentum associated with the internal system which is taken to be circulating with velocity c and whose space has a curved geometry. The rest mass is the internal energy in the rest frame of the centre of mass of the system. However this model has a number of shortcomings [44].
Hestenes [45] takes a slightly different view treating the zitterbewegung as arising from self interaction, there being an electro magnetic wave particle duality, though electron spin is again the orbital angular momentum with respect to an instantaneous rest system of radius equalling the Compton wavelength. But a number of assumptions are made for getting consistency

with the Dirac equation.
Chacko [46, 47] following a cue of John A. Wheeler models in a somewhat adhoc scheme, elementary particles as superdense geometrodynamical (that is General Relativisitc) entities confined to travel with the velocity of light in circular paths, again of radius equalling the Compton wavelength. Unfortunately properties like spin, magnetic moment, charge etc. are not incorporated in this scheme.
16) A question that arises is, can we arrive at a mass spectrum? A preliminary indication has been given in [48]. The point is that there are a few schemes which give the mass spectrum of a large number of elementary particles as composites of pions [49] or leptons like the electron positron and neutrino [50]. Indeed as the discussion following equation (33) shows, a pion can be considered as an electron positron composite because its Compton wavelength equals the classical electron radius which resembles the fact that the pion is a quark anti- quark composite. These could be incorporated into the above QMBH considerations to get a mass spectrum[48]. In particular it is interesting to note that in the above considerations it is possible to think of a proton as a composite of two positrons and an electron consistent with deep inelastic scattering data. Such a scheme gives a rationale for the matter anti-matter imbalance.
17) In recent years the problem of inertial mass has received some attention, apart from its usual Machian characterisation [51, 52].
18) Four final comments:
In the above considerations regarding the Compton wavelength we have considered free particles. However as equation (33) and the subsequent brief discussion indicates the scales could decrease by a few orders of magnitude in the presence of interaction.
For the record it may also be mentioned that the stability and indivisibility of hydrodynamical vortices had lead to pre-Quantum Mechanical speculation that atoms may be represented by such vortices[18].
Effects like the Lamb shift could be explained in the present model by arguments similar to those in ref.[44].
The inertial mass of section 3 could also be thought to arise for a free particle with effective mass m', from the known non-local Schrodinger equation (cf.ref.[7]).

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