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Emergent measure-dependent probabilities from modified quantum dynamics without state-vector reduction

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Counting outcomes is the obvious algorithm for generating probabilities in quantum mechanics without state-vector reduction (i.e. many-worlds). This procedure has usually been rejected because for purely linear dynamics it gives results in disagreement with experiment. Here it is shown that if non-linear decoherence effects (previously proposed by other authors) are combined with an exponential time dependence of the scale for the non-linear effects, the correct measure-dependent probabilities can emerge via outcome counting, without the addition of any stochastic fields or metaphysical hypotheses.

1 Introduction

The central question in the interpretation of quantum mechanics is how unambiguous macroscopic observations arise probabilistically from an underlying theory whose dynamical equation, in so far as it is known, obeys superposition and is deterministic. (See, e.g. [1].) In this paper I shall argue that the correct quantum probabilities may be obtained by outcome-counting [2, 3], the process which directly corresponds to our operational definition of probabilities, in a model without state-vector reduction, i.e. a type of many-worlds model. To obtain this result, it will be necessary to employ a modified state-vector dynamics, violating superposition. I shall illustrate the idea using a cartoon form of non-linear decoherence effects (previously proposed to arise in quantum gravity [4, 5]), along with a simplified non-random form of the effectively non-unitary time-dependence employed in recent explicit stochastic collapse theories [6, 7, 8, 9, 10]. However, it will be unnecessary to invoke the intrinsically stochastic constituents or metaphysical interpretive addenda required in other approaches.

I shall begin by briefly reviewing related approaches to clarify what problem this new approach is addressing. Then I shall present a toy model to illustrate how the standard quantum probabilities can emerge as the limits of ratios

of numbers of distinct non-interfering outcomes in a non-linear many-worlds picture, when non-linear decoherence processes produce decoherent branches whose measures become equal in the long-time limit. Some constraints on more complete and realistic models based on the same idea will then be presented.

2 Background: Probability with and without collapse

Many-worlds interpretations have pointed out that the branching of the state-vector into parts which represent macroscopically distinct outcomes arises directly without alteration of the standard linear time-dependence. [11, 12] Branching here means simply that the state evolves to one that can be written as the sum of components between which so many variables (e.g. particle coordinates) differ in complicated ways that the chance of subsequent interference between the components is negligible. When parts of a system become correlated with variables intrinsically outside the system we can refer to the branching as true decoherence, for which subsequent interference between branches is essentially undetectable.

The "consistent histories" program of explaining the states along which the branching preferentially occurs, within a linear dynamical equation lacking an "outside", [13] is far from complete. [14] In particular, if the time-dependence of the state is purely linear, the time dependences of the state components are entirely independent for *any* decomposition of the state into components. Reference to interference *assumes* physical significance of some non-linear function of the state (e.g. its square), a feature which is not present in the linear dynamics.

An open environment, however, can break the symmetry between states in a system's Hilbert space, giving a natural set of "pointer states" by a process dubbed "einselection"[15]. These pointer states are distinguished by having a collection of predictable quasi-classical observables.[15] Thus, despite the difficulties discussed above, it is possible to at least consider defining probabilities of some preferred outcomes within a linear theory without state-vector collapse.

It was realized long ago that, if one accepts the claim that state-components representing macroscopically distinct outcomes form non-interfering distinct "worlds", the many-worlds interpretation leads to an obvious prediction for the probabilities in simple experiments with a finite number of possible discrete outcomes. [2, 3] Since each world is equally real, so is the version of the experimenter's mind which is represented in that world and correlated with the other macroscopic outcomes represented in that world. In an experiment repeated many times, most worlds would show nearly equal numbers of each discrete outcome, regardless of the measures of the state components representing those outcomes. As Graham put it, "It is extremely difficult to see what significance measure can have when its implications are completely contradicted by a simple count of the worlds involved, worlds that Everett's own work assures us must be on the same footing." [3] The prediction that probabilities of each macro-

scopically distinct outcome of a quantum experiment would be equal, regardless of the measure of the components of the state-vector, obviously contradicts observation. [2, 3]

Thus if the standard many-worlds (no collapse) view can explain probabilities, it explains the wrong probabilities. As reviewed recently by Saunders, [16] attempts to fix this probability problem within the many-worlds interpretation of the linear time-dependence (e.g. [17, 18]) have required adding some metaphysical hypothesis beyond the evolving quantum state, thus losing the initial appeal of deriving the interpretation directly from the dynamical equation.

Recently, Zurek [15] has given a formal argument concerning the relation between outcome-counting and probabilities. In order to extend the argument to cases in which different outcomes have different measures, he makes a formal decomposition of the density matrix into parts which have correlations with various states of a hypothetical outside system. (See his equations 4.10 through 4.12.[15]) The components of the density matrix correlated with each individual outside state are implicitly assumed to have equal measure, and a limit is considered in which the number of outside states is so large that the number matched with any pointer state of the system becomes proportional to the measure along that pointer state.

Zurek's procedure may be illustrated by considering a case in which only two pointer outcomes (A and B) are possible for the system in state ϕ. The density matrix from Zurek's account is then:

$$\rho \;=\; \frac{\sum_{j=1}^{a} \mid A,j\rangle\langle A,j \mid \;+\; \sum_{j=a+1}^{a+b} \mid B,j\rangle\langle B,j \mid}{a+b} \tag{1}$$

where the j's denote orthogonal states of the outside system and it is postulated that $a/b \approx |< \phi \mid A >^2 \;/\; < \phi \mid B >^2|$. However, Zurek treats this decomposition as a purely formal procedure, within the context of purely linear dynamical equations. If the states "j" and the density matrix (1) are fictive, it is difficult to see how they can determine the operationally defined probabilities. If they represent a real outcome of dynamics starting with a system state uncorrelated with the outside, it is certain that those dynamics are not linear, since superposition forbids the dependence of a and b on $< \phi \mid A >^2$ and $< \phi \mid B >^2$. Linear dynamics, of course, instead produce a sum of *fixed numbers* of terms whose *coefficients* depend on $< \phi \mid A >^2$ and $< \phi \mid B >^2$.

In contrast, macro-realist theories such as that of Ghirardi, Rimini, and Weber (GRW) [19] or the more developed versions of Pearle and coworkers, e.g. [7, 8], propose that there are explicit modifications of the dynamics, which always directly or indirectly require non-linear time-dependence, causing the state to collapse stochastically along some pointer projection operator, rather than forming a superposition of macroscopically distinct outcomes. [7, 8, 19] There is no time-reversed analog of the collapse, so the dynamics do not obey CPT. The postulated set of pointer operators directly addresses the preferred-state question raised by linear theories. Whether these pointer operators must be introduced ad hoc or can in some way be deduced from a deeper theory,

perhaps including quantum gravity [4, 5, 8], remains to be seen. The key testable feature of such explicit collapse theories is that the collapse process would cause anomalous decoherence, so interference would be lost which would have been present if there were strictly linear time dependence of the state. [20]

One argument supporting the existence of such non-linear processes is based on the possibility that there are intrinsically unobservable variables (due to horizons) associated with quantum gravity, so that the correlations among the usual variables could only be represented by density matrix given by a trace over the unobserved variables. [4, 5] Within such genuine decoherence theories, in general, the time derivative of the state-vector is not representable by a linear operator and CPT symmetry need not be obeyed. [4, 5]

The better-developed versions of the explicit collapse theories require the inclusion of a non-quantized stochastic field. [7, 8, 9, 10] These stochastic fields possess non-local correlations to account for the results of Bohm-EPR-Bell experiments. [21] An attempt to quantize the stochastic field produced only decoherent superpositions, not actual collapse. [6, 10] A "moving finger" was then hypothesized to pick which decoherent solution actually occurred, [10] similar to the hypotheses outside the dynamical equations required in other approaches. Thus, if the explicit collapse theories are to dispense with metaphysics, they appear to require not only a non-linear dynamical equation but also explicit random elements and classical fields, leaving a theory with very diverse constituents.

3 A new approach

The idea I will present essentially amounts to a justification for how density matrices resembling that in equation (1) can arise within non-linear dynamics, thus justifying their use in predicting actual probabilities. In other words, the incorrect probabilities which have been predicted to arise from counting branches in the linear dynamical theory may be fixed by altering the dynamical equations rather than by giving up the compelling outcome-counting algorithm. Although I shall borrow the ideas of a non-linear loss of interference and an intrinsic time asymmetry from arguments employed in explicit collapse pictures, [4, 5, 6, 10, 19], by dropping the requirement of unique outcomes I shall remove the need for non-quantized stochastic fields. (Many-worlds interpretations have always pointed out that the consistency of observed macroscopic reality does not require macroscopic outcomes to be unique unless one makes the auxiliary assumption that the observer remains unique.[11])

The essential idea is that, as found in the attempt to fully quantize collapse theory by replacing the classical stochastic field with a quantum field representing new variables, [6, 10] each macroscopic branch of the state-vector has many sub-branches, with distinct values of some new quantum variables. However, rather than arbitrarily consign all but one of these sub-branches to "non-reality", we assume that they all actually persist. Thus probabilities are determined simply by their numbers, as in the obvious outcome-counting algo-

rithm which gave incorrect probabilities in purely linear collapse-free theories. In order for the newapproach to work, the number of sub-branches associated with each macroscopic outcome must become proportional to the measure of the component of the state representing that outcome on the time-scale of a measurement, i.e. on the same time-scales invoked in explicit collapse theories, for the sorts of macroscopic systems in which quantum probabilities have been recorded. [9, 19]

In the discussion to follow, "sub-branch" has a fairly precise meaning: a state component which cannot be divided into distinct parts between which subsequent interference is vanishingly small, when all the quantum variables are included. A "sub-branch" would be called a "world" in many-worlds terminology. (It will turn out not to be necessary to specify ahead of time whether two macroscopically distinct outcomes are always on distinct sub-branches, since if sub-branches containing macroscopic superpositions are allowed, they will become an exponentially rare fraction of the total number of sub-branches.) "Branch" will have a less precise meaning: a collection of sub-branches which comprise a macroscopically distinct outcome. When a continuum of macroscopic outcomes are possible, this grouping is a bit arbitrary, as usual, but none of our arguments will rely on how that grouping is made.

4 Illustration via a toy model

We may illustrate the basic workings of approach by using an initial toy model, closely analogous of GRW. [19] This toy model is not relativistic, and is not intended to give an accurate representation of the non-linear branching dynamics. Instead, I simply assume a particularly simple non-linear branching algorithm to illustrate the generic consequences of such processes, whose origins and details must be sought elsewhere.

We assume that the Hilbert space in which the statevector $\mid \Phi >$ resides is the direct product of the space in which the conventional state $\mid \phi >$ resides and a space which describes new variables. The state of these new variables will be denoted by a label L, whose form will be discussed below. Then, a basis set for the whole space can be formed from states of the form $\mid \Phi >=\mid \phi >\mid L >$. We assume that there is set of pointer projection operators $\{P_\Gamma\}$, where each Γ denotes both a sub-space (denoted γ) of the standard Hilbert space and a particular sub-space of the new-variable space. These pointer operators will not be be assumed to be normalized, i.e. $P_\Gamma^2 = pP_\Gamma$ with $p \neq 1$ in general. In fact (for reasons soon to be explained) they will be assumed to have a time-dependent normalization and thus be written as $P_\Gamma(t)$.

For the non-linear branching process, we employ the simplest cartoon of such a process we can come up with. When any $M_\Gamma(t) \equiv < \Phi \mid P_\Gamma(t) \mid \Phi >$, reaches a definite value (which we can arbitrarily define to be unity), a new sub-branch emerges, on which the unspecified new variables acquire a new state orthogonal to their old state as described by the following algorithm

$$
\mid \Phi\rangle \rightarrow \frac{Z^{1/2}\langle\Phi \mid P_\Gamma \mid \Phi\rangle}{\langle\Phi \mid P_\Gamma^2 \mid \Phi\rangle} \; C_\Gamma(t) P_\Gamma \mid \Phi\rangle \; +
$$

$$
\left\{ \mid \Phi\rangle \; - \; \frac{(1-(1-Z)^{1/2})\langle\Phi \mid P_\Gamma \mid \Phi\rangle}{\langle\Phi \mid P_r^2 \mid \Phi\rangle} \; P_\Gamma \mid \Phi\rangle \right\} \tag{2}
$$

Here we have described the development of an orthogonal state of the new quantum numbers on the new sub-branch by the unitary operator $C_\Gamma(t)$ which acts on the Hilbert space of the new variables. Z is a real number, $0 < Z < 1$, which specifies the fraction of the measure in the Γ sub-space which is assigned to the new sub-branch. The expression in curly brackets represents the old state with some of its measure in the Γ pointer sub-space removed. The coefficient of the second term within the brackets has been picked to keep the norm $< \Phi \mid \Phi >$ of the total state unchanged. We shall refer to the dynamics represented by algorithm (2) as anomalous branching. Algorithm (2) is written in a form independent of the normalization of P_Γ and also independent of the dimensionality of the subspace to which P_Γ projects. If two distinct M_Γ's reach 1 at the same time, it is easy to check that the state resulting from the bifurcation algorithm is independent of the order in which the two branchings occur, so long as the product of the two projection operators is zero. The parameter Z in algorithm (2) unfortunately remains arbitrary in the absence of a deeper theory. For certain illustrative purposes, we shall employ Z=1/2, but shall argue that, although no fine-tuning of Z is required, the simple choice Z=1/2 is not in general suitable. Most details of the splitting process (even our choice of bifurcations rather than some more general multifurcation) will not, however, be essential to the workings of the model, so long as it preserves $< \Phi \mid \Phi >$.

Although I have nothing new to add to previous ideas concerning the origins of non-linear decoherence, [4, 5] a cartoon account of the operator $C_\Gamma(t)$ might be helpful. One could imagine the Hilbert space of the new variables as something associated with the as yet unrealized quantum theory of gravity and $C_\Gamma(t)$ as the creation operator for a particle propagating endlessly into an open "outside", starting at time t. Permanent maintenance of orthogonality between states created at different times might require either some non-linearity to avoid dispersion or horizons to make dispersion irrelevant.

We assume that there is an initial unique state for all the new variables (of which there may be an uncountable number) so all that needs to be expressed to distinguish their current state is a countable list of the changes in that initial state induced by the anomalous branching process. One may then regard $C_\Gamma(t)$ as adding a label (γ,t) specifying the pointer operator involved and the time at which the anomalous branching occurred:

$$
C_\Gamma(t) \mid \Phi_\Gamma\rangle \; \equiv \; C_\Gamma(t) \mid \phi_\gamma\rangle \mid L_O\rangle \; = \mid \phi_\gamma\rangle \mid (\gamma, t), L_O\rangle \tag{3}
$$

where L_O is a list of previously acquired labels. Each sub-branch here acquires a unique list of labels recording the previous anomalous branching times for each pointer operator. The labeling (although not the branching algorithm)

closely resembles ones employed both in linear decoherence theories [15] and in non-linear collapse pictures. [9, 10]

Processes like algorithm (2) would not suffice to maintain non-linear branching if each P_Γ maintained a fixed normalization. The reason is that once the measures of the separately labeled (permanently decoherent) sub-branches fell to less than $1/p$ there would be no way for any M_Γ to reach one again. Therefore it is necessary, in order to obtain a theory in which non-linear branching persists, to invoke a non-unitary time-dependence of the pointer operators:

$$P_\Gamma(t) \;=\; P_\Gamma(0)e^{t/\tau} \tag{4}$$

Although there will be one particular time (before the first anomalous branching) for which $P_\Gamma(t)$ is a normalized (p=1) projection operator, that need not be chosen as t=0. At this point we shall not discuss how to assign τ, except that the same τ will be used for each pointer operator within any family related by a symmetry such as spatial translation.

Let us initially consider a particle for which we can ignore the ordinary quantum dynamics, e.g. because the particle is very massive. We shall also assume that the projection operators are one-dimensional, i.e. they project to states. Let us also initially assume for convenience that the parameter Z=1/2.

We shall follow what becomes of a state component (with a single initial label, L_O) whose state ϕ happens to have been split by a "measurement" into two widely separated pieces A and B, each conveniently lying along one pointer state. In other words, there are pointer operators $P_{A,L} \;=$ $ge^{t/\tau} \mid A >\mid L >< L \mid< A \mid$ and $P_{B,L} \;=\; ge^{t/\tau} \mid B >\mid L >< L \mid< B \mid$ where g is a constant whose value depends exponentially on the choice of time origin.

We first consider the case in which $M_{A,Lo}(t) = M_{B,Lo}(t)$. For convenience we assign t=0 to the time of the first branching, which is equivalent to setting the prefactor g in $P_{A,L}$ to g=2 if we choose to express this state component in normalized form. Applying algorithm (2) gives simultaneous branching for A and B, with the resulting state independent of the order in which we perform the two branching operations. We define T$\equiv \tau ln(2)$, the time between successive paired branching events. We obtain the following sequence of states after these events:

$$\begin{aligned}
&1/2^{1/2})(\mid A\rangle + \mid B\rangle) \mid L_O\rangle \quad \rightarrow \\
&(1/2)(\mid A\rangle \mid (A,0), L_O\rangle \;+\; (\mid A\rangle + \mid B\rangle) \mid L_O\rangle + \mid B\rangle \mid (B,0), L_0\rangle) \quad \rightarrow \\
&(1/2^{3/2})(\mid A\rangle(\mid (A,T),(A,0),L_O\rangle + \mid (A,T),L_O\rangle + \mid (A,O),L_O\rangle) \;+ \\
&(\mid A\rangle + \mid B\rangle) \mid L_O\rangle + \mid B\rangle(\mid (B,T),(B,O),L_O\rangle + \mid (B,T),L_O\rangle + \mid (B,O),L_O\rangle))
\end{aligned} \tag{5}$$

It is easy to check that after J doubling times, there will be $2^J - 1$ each of the distinctly labeled A and B sub-branches as well as a component of the original form $(\mid A > + \mid B >) \mid L_o >$. Whether this latter component is to be considered one or two sub-branches, i.e. whether subsequent interference between $\mid A >$

and $\mid B >$ is possible, becomes irrelevant as it becomes exponentially outnumbered by macroscopically distinct sub-branches. Since the fundamental postulate is that each sub-branch has equal probability, the probability of such a macroscopic superposition falls to zero exponentially in time, while the two distinct macroscopic outcomes remain equally likely.

Now let the initial state have 2/3 of its measure along A and 1/3 along B. We choose g=1.5 to set t=0 as the time of the first branching, which occurs only for A. Subsequent branching is again simultaneous for A and B. We obtain the following sequence of normalized states, in which the notation L_0 (shared by all these sub-branches) has been suppressed in all the new branches, for brevity:

$$\begin{aligned}
&(1/3^{1/2})(2^{1/2} \mid A\rangle + \mid B\rangle) \mid L_0\rangle \quad \rightarrow \\
&(1/3^{1/2})(\mid A\rangle \mid (A,0)\rangle \; + \; (\mid A\rangle + \mid B\rangle) \mid L_0\rangle) \quad \rightarrow \\
&(1/6^{1/2})(\mid A\rangle(\mid (A,T),(A,0)\rangle + \mid (A,T)\rangle + \mid (A,0)\rangle) \; + \\
&(\mid A\rangle + \mid B\rangle) \mid L_0\rangle + \mid B\rangle \mid (B,T)\rangle)
\end{aligned} \tag{6}$$

It is easy to check that after another (J-1) doublings, there will be $(2^{J+1}-1)$ pure-A sub-branches with distinct labels, $(2^J - 1)$ pure-B sub-branches with distinct labels, and one state component of the form $(\mid A > + \mid B >) \mid L_o >$. If one considers this component as consisting of one A and one B sub-branch, the ratio of the number of A to B sub-branches becomes exactly two. Again, even if one does not *a priori* rule out macroscopic superpositions as possible outcomes, the probability of such a superposition falls to zero, as discussed above, and the ratio of the numbers of A to B sub-branches approaches two, with the difference from that limit decreasing exponentially in time.

The two examples above were contrived so that the branching along the two pointer states occurred synchronously, by making the initial ratio of the measures along those pointer states a power of two, matching the factor by which the measure along a pointer state changes in the branching algorithm with Z=1/2. For different initial ratios of the measures of those components, the ratios of numbers of A andB sub-branches would oscillate as first the A sub-branches, then the B sub-branches, bifurcated. The definition of the time-averaged limiting ratio of the numbers of sub-branches of each type would then be slightly arbitrary. That problem is an artifact of the choice Z=1/2, which was employed only for illustrative simplicity, not on the basis of any real dynamical theory. For a more general bifurcating case, in which ln(Z)/ln(1-Z) is irrational, synchronization becomes impossible since bifurcations can give rise to new values of M_Γ's, distinct from other sub-branches. In the more general discussion which appears in a later section, I shall argue (on the basis of simulations and of analytic arguments a bit short of a proof) for a conjecture that in this case initial synchronization becomes unnecessary because the distribution of M_Γ's, of descendant sub-branches will spread and acquire a branch-independent mean. Since the M_Γ's, are simply standard measures scaled by a common time-dependent factor, one obtains a branch independent ratio of number of sub-branches on a branch to the standard measure of the branch, with vanishing

fluctuations around that limiting ratio.

We next give an example of the emergence of standard probabilities when the macroscopic branches are spread out over different numbers of pointer states. For example, let us suppose that the pointer states ϕ_γ are minimum-uncertainty Gaussian states of a single particle spread over regions with width w, in analogy with the GRW collapse picture. [19] Now let piece A initially have 2/3 of the measure and have a Gaussian spread with a width of $400w$, and piece B have of 1/3 of the measure with a width of $100w$. The pointer states here are uniformly distributed in three-dimensional real-space, so (with the above choices of widths) A is spread over 64 times as many pointer states, and hence (with those choices of measure) has 2^{-5} as much measure on each pointer state. Therefore its anomalous branching will begin later than the anomalous branching of the B components by a time 5T. Thus at a particular part of the Gaussian distribution (e.g. at the peak, or at the shell one standard deviation from the peak) the components of B along each pointer state will split into 32 sub-branches before the corresponding components of A also start to branch. However, A is distributed along 64 times as many pointer states. Therefore the total number of A sub-branches becomes just twice the number of B sub-branches, as expected from their measures, despite the asymmetry between their spatial distributions.

We have seen examples in which the standard probabilities emerge when ordinary quantum rates of spreading between pointer states are negligible compared to the anomalous branching rates. Except for the issue that the ratio of the numbers of sub-branches on two branches can oscillate, the basic procedure did not depend on special features of the initial states. However, *those probabilities are not just built-in by fiat, but rather arise by dynamics.* In fact, immediately after a "measurement" process, i.e the development by ordinary quantum dynamics of a state with substantial projections on distinct pointer sub-spaces, non-standard probabilities would appear transiently before the anomalous branching has occurred many times.

Now we can consider the effect of the standard quantum dynamics on the probabilities. The examples will show that the branching algorithm is not guaranteed to produce standard quantum probabilities when applied to an isolated microscopic system (for which the rate at which quantum dynamics spreads the state out over different pointer sub-spaces is not negligible compared to the anomalous branching rate), but that once enough particles are coupled to form a macroscopic system, ordinary probabilities emerge.

Let us say that the state representing a particle with mass m, with pointer-state growth-rate parameter $1/\tau_1$, has just split at t=0 into (among others) a piece which initially lies along one of the minimum uncertainty pointer states, with $M_\Gamma = 1/2$. The velocity spread $\hbar/(2w\mathrm{m})$ will give an increase with time of the number of pointer states over which the resulting state is spread, with the maximum measure along the initial state decreasing as a result. Then the maximum M_Γ will be $M_\Gamma(t) = exp(t/\tau_1)(1+(t/t_0)^2)^{-3/2}/2$, where $t_0 \approx w^2 m/\hbar$. If $\tau_1 \gg t_0, M_\Gamma(t)$ will not reach one again until $t = \tau_1(ln(2) + 3ln(t/t_0)) \approx 3\tau_1 ln(\tau_1/\tau_0)$. The normal dynamics then delay the time for the anomalous branching by a factor of approximately $3ln(\tau_1/t_0)$. If we use GRW parameters

for a proton, i.e. rate $1/\tau_1 = 10^{-16}s^{-1}$ and width w= 10^{-5} cm we find $t_0 \approx 10^{-7}s$, and the logarithmic factor $3ln(\tau_1/t_0)$ would be about 150.

This example illustrates several points. First, a sufficiently small exponential growth rate for single-particle pointer operators, like the small GRW single-particle decoherence rate [19], allows the standard quantum dynamics to proceed uninterrupted for a very long time. In fact, in this model (unlike the GRW model, which has Poisson collapse statistics [19]) the reduction of the sub-branch measure by a branching event actually shuts down the anomalous process for a long time subsequently, so that the branching events are separated by intervals longer than τ_1. However, the spreading from the pointer states due to the uncertainty-principle (or more generally, the non-commutivity of the pointer operators with the Hamiltonian, H) means that each sub-branch spreads out over many pointer states before the next anomalous branching. As a result, the anomalous branching process would not fix the mean sub-branch measure independently of the quantum dynamics, which can differ among branches.

In this single-particle example, the measure which starts on a single pointer state after an anomalous branching spreads out over a very large number, about $(\tau_1/t_0)^3 \approx 10^{70}$, of pointer states before the next round of anomalous branching starts. Therefore, if one were to assign probabilities of the particle being in different locations by counting sub-branches in this isolated microscopic system, those probabilities would not be proportional to the square of the wave-function. For example, after about half of this state has undergone another splitting, half the measure would be represented by some 10^{70} sub-branches covering the spatial middle part of the state, while the half of the measure contained in the outer parts would be represented by only one sub-branch.

In the example above the deviations from the standard probabilities are quite striking in an isolated microscopic system, although any assignment of probabilities would make little sense on a short time scale, for which there is no stable ratio of numbers of sub-branches representing different outcomes. As most discussions of quantum measurement routinely point out, measurements can never be made on *isolated* microscopic systems, because any measurement for which we can possibly have tabulated probabilities requires that correlations be established with macroscopic variables, at least including ones describing our brains. Thus the requirement for a successful theory (assuming no special role for consciousness) is that it predict the standard probabilities for a large class of macroscopic systems, to which we now turn for further examples.

First, consider a particle of larger mass. The spreading time t_0 for the simple Gaussian pointer states scales linearly with m. Thus, even if τ were m-independent, a more massive particle would have $\tau < t_0$ for $m > 0.1g = (\tau_1/t_0)10^{-24}g$. Within the explicit collapse framework, it has been noted that experimental constraints indicate that the anomalous decoherence rate should increase with rest mass. [8, 9, 10] If, for example, $1/\tau$ were to scale as m, the massive particle would have $\tau < t_0$ for $m > 10^{-12}g$.

Thus for sufficiently massive particles, regardless of how the quantum process had dispersed the wave-function, measure-proportional probabilities would emerge. If $1/1/\tau$ were proportional to m, branching times would fall to less

than one microsecond for $m > 0.1g$.

5 Multi-particle systems

Now we consider a system of N particles, which for simplicity we make distinguishable. The model should allow branching to occur for any one particle without affecting the others, so that for uncorrelated particles the branching will not create spurious correlations. Therefore each pointer operator should be a product of a pointer operator for one particle by identity operators for the other N-1, e.g. $P_{1\Gamma} I_2 \ldots I_N$. Each pointer operator then projects not to a state of the overall system, but instead to a higher-dimensional sub-space. The identity operators for the other particles apply to both the standard Hilbert space and to the space represented by the corresponding labels. We shall assume that these pointer operators are also appropriate for correlated multi-particle states. The formalism is close enough to that of GRW [19] to not require repetition in detail.

Let us consider a crystal in its internal ground state, so that only the center-of-mass coordinate remains free. Assume that the masses of the (distinguishable) particles are all still m, and the pointer operators each still have an exponential growth rate $1/\tau_1$. The spreading time t_{0N} would be the same as for any other particle of mass Nm, i.e. Nt_0. Now as soon as any particle's state branches, the center of mass coordinate of the crystal is localized to within distance w on each new sub-branch, just as in the GRW theory. [19] After N_B repeated branching events most sub-branches are localized to within $w/N_B^{1/2}$, as is the unique state in the analogous collapse process in GRW.

The crucial question then becomes how long it takes for anomalous branching to occur in this multi-particle collective state. *The key point is that since the N particles arrived in the crystal through partially independent histories, the timings of the cyclings of their M_Γ between branching events are not the same. For long enough and complicated enough histories, one expects these timings to be uncorrelated.* Therefore the logarithms of those M_Γ (on any one of the allowed product states) are randomly distributed, e.g. from -ln(2) to 0 if we follow algorithm (2) with Z=1/2 and if $Nt_0 >> \tau_1/N$. Branching events can occur at any time, with the typical time delay between branching events reduced by a factor of N compared to the single-particle rate.

Here our procedure is different from GRW [19] in an important way (in addition, of course, to the interpretation that only branching, not collapse, is occurring). *The randomness in the timing of the branching events is not put in as a separate fundamental ingredient here.* Rather, it arises from the complicated but deterministic histories of the constituent particles. In other words, *it is an ordinary statistical mechanical effect, not an essential randomness in the constituent dynamics.*

6 More General Considerations

I have illustrated how non-linear decoherence effects can give standard quantum probabilities via outcome counting using a toy version of the non-linear dynamics, applied to simple cases. Although I am unable to develop a full theory of respectable non-linear dynamics, it is nonetheless possible to discuss some of the constraints on such a theory and to clarify why the non-linear procedure above produced correct probabilities in the examples given. The key points in the dynamics leading to the measure-proportional probabilities are that:
I. The anomalous sub-branching does not change the net measure on any branch.
II. A steady-state is approached in which the mean value of the measures of the sub-branches of a given branch is independent of the branch.

If these two conditions are met, then the sub-branch numbers must be proportional to branch measure.

The condition (I) is assumed to apply exactly to the anomalous branching, as in algorithm (2). The general form of the anomalous branching rate, a monotonically increasing function of M_Γ, will tend to cause condition (II) to be met approximately, but it will be met precisely only under further constraints.

Let us first consider the case in which the ordinary quantum dynamics can be ignored. The measures of the sub-branches of the different branches became precisely equal at all times in our simplest (Z=1/2) examples only because of the specially contrived initial measures. In general, to insure that the average over equally weighted sub-branches of M_Γ (denoted $< M_\Gamma >_C$) approach the same limiting values on different branches, one needs that $< M_\Gamma >_C$ approaches the mean of a limiting steady-state distribution. I conjecture that that will occur whenever ln(Z)/ln(1-Z) is irrational.

There is an informal argument for the conjecture above. The branching algorithm together with the pointer-operator growth implies that after time t, the log of the measure of each sub-branch will have to be reduced by t/τ to within an accuracy of max $(\mid ln(Z) \mid, \mid ln(1-Z) \mid)$. The reduction occurs in steps of ln(Z) and ln(1-Z) The number of ways that t/τ can be put together (to the specified accuracy) out of combinations of the form $(J_Z ln(Z) + J_{1-Z} ln(1-Z))$ with positive integer J's is of order t/τ. Each such combination leads to a different sub-branch measure, a set of pseudo-random numbers. Although at finite time the probability density function $\rho(M_\Gamma)$ of the set of M_Γ's is always a finite collection of delta functions, it seems likely that its moments will approach those of a distribution which is time-independent under the branching algorithm. (It is not too hard to demonstrate that for rational ln(Z)/ln(1-Z)= m/n, in reduced rational form, the distribution of the number of M_Γ's in n uniformly spaced logarithmic bins from ln(Z) to 0 approaches a unique limit, implying that $< M_\Gamma >_C$ becomes fixed to within a factor of $Z^{1/n}$. Formally taking a suitable limit to obtain the fixed-mean result for irrational ln(Z)/ln(1-Z) does not seem to be trivial.)

The time-independent distribution for the bifurcation scheme can fairly easily be shown to be:

$$\rho(M_\Gamma) \;=\; \frac{Z'}{M_\Gamma^2} \quad \text{for } Z' \leq M_\Gamma < 1 - Z'$$
$$\rho(M_\Gamma) = \frac{1}{M_\Gamma^2} \quad \text{for } 1 - Z' \leq M_\Gamma \leq 0 \tag{7}$$

where $Z' \equiv$ min(Z, 1-Z). For this limiting distribution $< M_\Gamma >_C$ will be Zln(1/Z)+(1-Z)ln(1/(1-Z)). For some more general branching scheme, so long as the moments approach those of any well-defined distribution, condition II will be met.

One can run simulations of the branching algorithm to see if the moments do actually converge to the ones predicted from distribution (7). Simulations with ln(Z)/ln(1-Z)= $(1+5^{1/2})/2$ were run to times of t=8000τ. $< M_\Gamma >_C$ showed irregular fluctuations as a function of t, around the limit calculated from distribution (7). The envelope of the fluctuations was a decreasing function of t, with $\mid \Delta ln < M_\Gamma >_C \mid$ staying under 0.03 for $25\tau < t <$8000τ, under 0.015 for $150\tau < t <$8000τ, and under 0.003 for $4300\tau < t <$8000τ. (Slightly quicker simulations of $|< \Delta ln M_\Gamma >_C|$, nearly identical to $\mid \Delta ln < M_\Gamma >_C \mid$ for $t > 10\tau$, extended these limits to 13500τ.) The simulation results are consistent with the plausible hypothesis that the deviations of $< M_\Gamma >_C$ from its limiting mean scale as $(t/\tau)^{-1/2}$.

The key issue determining whether condition II (branch-independent $< M_\Gamma >_C$) is met is not the mathematical exercise required to show a limiting $< M_\Gamma >_C$ in a given branching algorithm, but rather what the effects are of including ordinary quantum dynamics. Given that events happen, the pointer operators cannot commute with H, so that even if a sub-branch starts out with a well-defined value of a pointer operator, it will not keep it.

I have argued that a pure branching dynamics of the type described will give a distribution of M_Γ whose mean is independent of the initial state. The mean of the sub-branch measures on each branch will approach $e^{-(t/\tau)} < M_\Gamma >_C$ so long as the ordinary quantum processes have little effect on the anomalous branching. That requires that a sub-branch which starts off as an eigenstate of some P_Γ spreads very little outside the Γ subspace before the next anomalous branching. Starting in an eigenstate of P_Γ, the first derivative of the measure of the projection of the state onto the Γ subspace will be zero. The second derivative will be negative and will depend on the commutator $[P_\Gamma, H]$ of the pointer operator P_Γ with H. We then require:

$$\tau_{eff}^2 \;\; \frac{\langle \Phi_\Gamma \mid [P_\Gamma, H] H \mid \Phi_\Gamma \rangle}{\hbar^2 \langle \Phi_\Gamma \mid P_\Gamma \mid \Phi_\Gamma \rangle} \quad \ll 1 \tag{8}$$

where $1/\tau_{eff}$ is the effective anomalous branching rate for the system. As discussed in the toy example, there is reason to expect that for macroscopic objects $1/\tau_{eff}$ would scale linearly with the size of the system. [7, 19] An essentially similar condition was found by GRW [19] as a requirement for the existence of classical trajectories in collapse pictures with stochastic ingredients

in the dynamics. Although we are far from having a suitable general choice of pointer operators, quantum commutators of operators representing measurables, e.g. [P,H], generally become relatively unimportant for large systems. It is then plausible that condition (8) will be met by macroscopic systems in general.

7 Remaining problems and prospects

We have seen that in a toy model correct quantum behavior is retained for small enough objects, while for large objects the probabilities obtained from outcome-counting approach values proportional to quantum measure. The essential features of that toy model seem likely to generalize. Nevertheless, this proposal is obviously in an early stage, with some elements directly borrowed from the GRW stochastic collapse proposal. [19] Most importantly, nothing here contributes anything new to the understanding of the pointer operators and the branching rates, or to making a continuous-time description of the branching process itself.

The pointer operators for multi-particle systems employed here were constructed to avoid having the anomalous branching induce any spurious correlations on independent particles. In simple examples with the quantum state of the system in the opposite limit, i.e. the zero-temperature crystal, the branching algorithm then also gave a reasonable results. I have not provided an analysis of the behavior of partially correlated particles. A proper description for collections of identical particles also has not been given here.

In the artificial model constructed here to show that a no-collapse picture can produce correct measure-proportional macroscopic probabilities, I have built up pointer operators out of single-particle operators. That procedure is easy, but it lacks a strong physical motivation and is very unlikely to be suitable in general. A more developed theory obviously should not be expressed in terms of individual particle states or any other decomposition which becomes arbitrary in a general case. Perhaps a more suitable set of pointer operators would depend on four-momentum density without reference to constituents, along lines being pursued in collapse pictures. [8] Construction of such a model is well beyond my capabilities.

The growth rates of the pointer operators ought to be expressed by some operator which transforms with other physical rates if a relativistic theory is to be obtained. The indications of rest-mass dependence of the collapse rate in collapse interpretations [8, 9, 10] suggest that the growth rate $1/\tau$ for a given P_Γ could be replaced by an operator $\epsilon H/\hbar$ where ϵ is a small positive number and H is understood to be the full Hamiltonian (i.e. whatever appears in the gravitational source term, typically dominated by the rest-mass term) for the particular subsystem on which P_Γ acts. If the anomalous branching rate for single particles is to fall within the rather broad range of postulated rates in collapse theories [8, 19], we would need $10^{-50} < \epsilon < 10^{-40}$.

A non-uniform τ of this type would have some interesting, problematic consequences. The branching rate would be higher on high-energy components

than on low-energy ones. Although that difference would in itself have no direct effect on Tr(ρH) where ρ is the global density matrix, it would lead to an increase in the relative numbers of high-energy sub-branches, giving a term $\epsilon < (\delta H)^2 >_C /\hbar$ in the rate of increase of $< H >_C$, where $<>_C$ again denotes averaging over equal-weighted sub-branches. This effect is smaller than the direct energy non-conservation predicted to arise from explicit collapse processes [9, 19] (an effect perhaps shared with the branching processes), and would lead to no significant anomalies over many times the current lifetime of the universe. For any collection of independent subsystems, the rate of energy increase would come out to be the sum of the subsystem rates. Intriguing problems would arise for the description of subsystems in a closed universe whose net energy was identically zero.

8 Conclusion: comparison with related ideas and observational constraints

The proposal is clearly mathematically and observationally distinct from prior many-worlds pictures, which lack the prediction of anomalous loss of interference. Ultimately, the parameters (e.g. ϵ) fixing the rate of anomalous interference loss should be measurable in mesoscopic experiments if either the collapse approach or this approach is correct. I shall argue that the non-linear many-worlds approach may have subtle observational differences from the family of explicit collapse pictures which share the prediction of anomalous decoherence.

The standard quantum probabilities here *emerge* under some limiting conditions. There is a regime, illustrated in our toy examples, in which it is meaningful to use outcome-counting to assign probabilities which are not proportional to measure. However, the standard quantum mechanical probabilities would result for observations on a time scale large compared to the effective anomalous branching rate, so long as the commutators of the pointer operators and the Hamiltonian are small enough, i.e so long as condition (8) is met.

Given that as observers we are intrinsically limited to the size-time scale on which experiments in the past have always given Born probabilities, the most reasonable hope for finding different predictions would be to find some experiment in which the commutators of the relevant pointer operators and the Hamiltonian were large, so that condition (8) would be violated. That would require some way of arranging for the state to maintain a rapid decrease in the log of the maximum measure along any pointer sub-space simply due to ordinary quantum dynamics. That decrease would have to be unequal between different macro-outcomes and would have to be maintained over the entire course of the experiment. That leaves (slightly) open both the unpleasant possibility that in a more developed theory some case could show probabilities already so far from known observations as to rule out this approach and the pleasant possibility that some case might predict probabilities subtly but measurably distinct not only from standard quantum mechanics (which lacks the anomalous interference

loss) but also from the explicit collapse pictures.

It may seem that an approach which invokes non-linearity without getting rid of many worlds has combined the worst features of two types of theory. However, the acceptance of non-linearity seems to be a necessity if one is to obtain the correct probabilities from the dynamical equations. The choice between many-worlds and collapse is not dictated by any known observation. By not insisting on unique outcomes, we avoid some of the ingredients of the collapse models which seem most distinct from ordinary quantum mechanics, partially compensating for the currently less-developed state of the no-collapse approach. Non-quantized fields, hypotheses about constructs outside the dynamical equations, and explicit stochastic constituents are all avoided, as hoped for in early versions of many-worlds pictures. [11] As Squires noted, [22] dropping the collapse hypothesis also avoids the problem of requiring prior non-local correlations in random collapse-generating fields. Any no-collapse picture, including this one, avoids postulating unobserved state-reduction, although at the obvious cost of postulating state-components unobserved by a given macroscopic observer.

Of course, if one or the other non-linear approach is found to flow in a natural way from a deeper understanding of constituent physics (e.g. gravity), no such mere postulates will be required. On the other hand, if a full, consistent theory of all observed interactions were found to have the form of a linear quantum field theory, then both approaches might be left with the need for some highly arbitrary assumptions, without prospects of confirmation from another line of reasoning.

Proposals which simply add the usual probability rule to a purely linear dynamics cannot distinguish *in principle* whether a single-world [23] or multi-world [16] interpretation is correct. Likewise, the Bohm interpretation [24] is unable in principle to distinguish if there is one "real" coordinate point guided by the wave or an ensemble of such points, since the actual coordinates have no effect on the linear evolution of the wave. In contrast, I have pointed to a route by which the non-standard collapse and non-standard no-collapse pictures might conceivably be distinguished.

The point has been made before that proposed experimental tests designed to distinguish explicit collapse models from standard quantum mechanics would determine only whether an additional source of interference loss exists, not whether all but one of the resulting components of the state disappear. [10] This point is not a mere formality, because a non-linear no-collapse account may give a straightforward mechanism for directly explaining quantum probabilities in terms of simple numbers of non-interfering outcomes arising directly from the dynamics, without postulating any stochastic non-quantized fields.

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