The opacity of the Earth to incident ultra high energy neutrinos is directly connected with the behaviour of the neutrino - nucleon ($\sigma_{\nu N}$) cross sections in a kinematic range utterly unexplored. In this work we investigate how the uncertainties in $\sigma_{\nu N}$ due the different QCD dynamic models modify the neutrino absorption while they travel across the Earth. In particular, we compare the predictions of two extreme scenarios for the high energy behaviour of the cross section, which are consistent with the current experimental data. The first scenario considered is based on the solution of the linear DGLAP equations at small-$x$ and large-$Q^2$, while the second one take into account the unitarity effects in the neutrino - nucleon cross section by the imposition of the Froissart bound behaviour in the nucleon structure functions at large energies. Our results indicate that probability of absorption and the angular distribution of neutrino events are sensitive to the the QCD dynamics at ultra high energies.

The observation of ultra high energy (UHE) neutrino events at PeV by the IceCube Collaboration marks the birth of neutrino astronomy [1, 2]. Astrophysical neutrinos are good messengers from sky. They have small cross-sections even at ultra high energies and hence they are weakly absorbed by the medium that they travel. This property allows neutrinos to travel large distances trough the universe basically unperturbed, bringing to us information about the nature of the medium in which they are produced. Also neutrinos are not deflected by any magnetic field, and hence, when UHE neutrinos are detected in the Earth, the muon tracks produced into detector points to their source. In this way, these astrophysical neutrinos would help to solve the puzzles of what are their source. In this way, these astrophysical neutrinos are good messengers from sky.

In order to interpret the experimental results is fundamental to take into account that the attenuation of the neutrino beam in route to a detector is strongly dependent on the high energy behaviour of the neutrino - nucleon cross section ($\sigma_{\nu N}$), which determines the opacity of the Earth to incident neutrinos (For a review see, e.g. Ref. [3]). As discussed by several authors in the last years [4, 22], at ultra high energies, the neutrino-nucleon cross section provides a probe of Quantum Chromodynamics (QCD) in the kinematic region of very small values of Bjorken-$x$ and large virtualities $Q^2$, which was not explored by the HERA measurements of the structure functions [23]. These studies demonstrated that the uncertainties present in the extrapolations of $\sigma_{\nu N}$ for this new kinematic range has direct impact in the event rate in high energy neutrino telescopes [15, 19, 22]. In particular, the results from Ref. [15] shown that the solution of the linear Dokshitzer - Gribov - Lipatov - Altarelli - Parisi (DGLAP) equation [24] at small - $x$ and large $Q^2$ obtained in Ref. [8], denoted FJKPP hereafter, provides an upper bound for the behaviour of $\sigma_{\nu N}$ at ultra high energies. In contrast, the solution proposed in Ref. [9] denoted BBMT hereafter, which imposes that $\sigma_{\nu N}$ satisfies the Froissart bound at high energies, can be considered a lower bound. As demonstrated in Ref. [15], models that taken into account the non - linear effects to the QCD dynamics predict high energy behaviours between these extreme scenarios.

Our goal in this paper is to extend these previous studies for the analysis of the probability of neutrino absorption by the Earth’s interior at ultra high energies and determine the theoretical uncertainty present in this quantity. In particular, we compare our predictions with those obtained using the standard approach proposed in Refs. [4, 23], denoted GQRS hereafter. For completeness, we also present the results for the absorption due to the Glashow resonance in the anti neutrino - electron scattering [25]. Our analysis is motivated by the fact that the IceCube [2] and Antares [26] observatories are sensitive to neutrinos below the horizon line. However, depending of the magnitude of the charged current neutrino interactions and the Glashow resonance, the Earth’s can become fully opaque to neutrinos with very high energies, which implies that e.g. the IceCube can becomes blind to neutrinos coming from north hemisphere [27]. Moreover, as neutrinos coming from different direc-
nucleon cross section is given by \[30\] charged current (CC) interactions. The total neutrino - nucleon cross section is expressed in terms of the nucleon structure function \(F^N_{i,CC}\) as follows \[30\]. In this work we use the density profile of the Earth. In this work we use the density profile from \[28\] and, following \[28\], we define \(N_e = N_A(\rho_{tot}/g)(Z/A)\). The factor \(\langle Z/A \rangle\) is the average ratio between electrons \((Z = e)\) and nucleons \((A = p+n)\). We have that \(\langle Z/A \rangle = 0.475\) for \(r \leq 3480\) \(\text{km}\) and \(\langle Z/A \rangle = 0.495\) for \(r > 3480\) \(\text{km}\) [See Fig. 10.26 from Ref. \[29\] for details]. In this way we can write

\[
\rho_{tot}(r) = \frac{N_e}{N_A} \frac{1}{\langle Z/A \rangle} \left[\text{g/cm}^3\right],
\]

\[
\rho_e(r) = \frac{N_e}{N_A} \left[\text{g/cm}^3\right],
\]

where \(N_A = 6.022 \times 10^{23}/\text{mol} = 6.022 \times 10^{-23}\) CMWE (centimeters of water equivalent) is the Avogadro’s number. For \(\cos \theta_z = -1\) neutrinos crosses all the Earth, and Eq. \[30\] results in \(10\text{kT/cm}^2\), or \(1 \times 10^{10}\) CMWE. Consequently, we can write

\[
P^3_{Shad}(E_\nu) = \exp \left\{ -\kappa_j \sigma_{\nu j}(E_\nu) \int_0^{r(\theta_z)} \rho_j(r) dr \right\},
\]

where \(\kappa_N = N_A\) and \(\kappa_e = \langle Z/A \rangle \cdot N_A\). Finally, we can define the absorption function for the neutrinos while it crosses the Earth as

\[
S^j(E_\nu) = \int_{-1}^0 d\cos(\theta_z) P^j_{Shad}(E_\nu) = \int_{-1}^0 d\cos(\theta_z) \exp \left\{ -\kappa_j \sigma_{\nu j}(E_\nu) \int_0^{r(\theta_z)} \rho_j(r) dr \right\}.
\]

In what follows we will estimate \(P^j_{Shad}(E_\nu)\) and \(S^j(E_\nu)\) considering different models for the (anti) neutrino - nucleon cross section and, for comparison, we also present the results for (anti) neutrino - lepton interactions. Deep inelastic neutrino - nucleon scattering is described in terms of charged current (CC) and neutral current (NC) interactions, which proceed through \(W^{\pm}\) and \(Z^0\) exchanges, respectively. As the neutral current (NC) interactions are sub-dominant, we will consider in what follows, for simplicity, only charged current (CC) interactions. The total neutrino - nucleon cross section is given by \[30\]

\[
\sigma^\text{NC}_{\nu N}(E_\nu) = \int_{Q^2_{\text{min}}}^s dQ^2 \int_{Q^2/s}^1 dx \frac{1}{xs} \frac{\partial^2 \sigma^\text{NC}}{\partial x \partial y},
\]
where $G_F$ is the Fermi constant and $M_W$ denotes the mass of the charged gauge boson. The calculation of $\sigma_{\nu N}$ involves integrations over $x$ and $Q^2$, with the integral being dominated by the interaction with partons of lower $x$ and $Q^2$ values of the order of the electroweak boson mass squared. In the QCD improved parton model the structure functions $F_2$, $F_L$ and $F_3$ are calculated in terms of quark and gluon distribution functions. In this case the neutrino - nucleon cross section for charged current interactions on an isoscalar target is given by (See, e.g. Ref. [50]):

$$\frac{\partial^2 \sigma_{\nu N}^{CC}}{\partial x \partial y} = \frac{G_F^2 M E_{\nu}}{\pi} \left( \frac{M_W^2}{M_W^2 + Q^2} \right)^2 \left[ 1 + \frac{(1-y)^2}{2} F_{2,CC}^N(x, Q^2) - \frac{y^2}{2} F_{L,CC}^N(x, Q^2) + y(1-y)x F_{3,CC}^N(x, Q^2) \right]$$

(8)

FIG. 1: (Color online) Comparison between the energy dependence predicted for the neutrino - nucleon cross-section and for the anti neutrino - electron cross-section.

with the quark and anti-quark densities given by $q_N = (d + u)/2 + s + b$ and $\bar{q}_N = (\bar{d} + \bar{u})/2 + c + t$.

The current estimates of the neutrino nucleon cross sections constrain the structure functions and/or parton distributions using the HERA data and are based on linear QCD dynamics (DGLAP or an unified DGLAP/BFKL evolution) [4, 5, 10] or by models that taken into account the non-linear effects of QCD dynamics that area expected to be present at high energies [4, 10, 17, 18, 20, 22]. In particular, the neutrino - nucleon cross section was originally calculated at leading order in Ref. [4], with the resulting parametrization being a benchmark for the evaluation of UHE cosmic neutrinos. In Refs. [14, 16] a next-to-leading order analysis was performed, and the uncertainties on high energy $\sigma_{\nu N}$ which are compatible with the conventional DGLAP formalism [24] were estimated. Moreover, in Ref. [8] it was estimated considering an analytical solution of the DGLAP equation, valid at twist-2 and small-$x$, which implies a power-like increasing of the cross section at ultra high energies. In contrast, in Ref. [9] the HERA data were successfully fitted assuming that the proton structure function saturates the Froissart bound, which implies $F_2^p \propto \ln^2(1/x)$ and, consequently, that the increasing of $\sigma_{\nu N}^{CC}$ is smaller in comparison to the DGLAP predictions. In Fig. 1 we present a comparison between the predictions of the linear approaches (GQRS and FJKPP) and the Froissart-inspired model (BBMT) for the energy dependence of the neutrino nucleon CC cross section. Moreover, for completeness of our analysis, we also present the predictions obtained the CT10 parametrization [31] for the parton distributions (PDFs), derived using the DGLAP evolution equations, which allows to estimate the uncertainty present in the global fits as well as those associated to the extrapolation of the PDFs in a kinematical range beyond that probed by HERA, represented by the shaded band in the Fig. As expected from the solution of the DGLAP equation at small-$x$, the GQRS, FJKPP and CT10 models predict a strong increase of the cross section at ultra high energies, with the CT10 predictions being consistent with the GQRS and FJKPP results. Morever, the uncertainties
we also present for comparison the predictions for the energy dependence of the Glashow resonance, in such way that it dislocates the effects of absorption due to CC neutrino-nucleon scattering, in such way that it dislocates the probability of neutrino absorption to lower values of neutrino energy. We have found that the peak of resonant \( \nu\bar{\nu}N \) cross section in the energy range characterized by \( E_{\nu} \approx 6.3 \times 10^6 \) GeV, for all values of \( \cos \theta_z \). Basically, the angular effect in the resonant absorption is to enlarge the width of the resonance for \( \cos \theta_z \rightarrow -1 \), when neutrinos travels greater distances inside Earth and experiment higher values of electron density. As expected, increasing the angle of the neutrino incidence, the higher density crossed by neutrinos amplify the effects of absorption due to CC neutrino-nucleon scattering, in such way that it dislocates the curves of \( \nu N \) absorption to lower values of neutrino energy. We have that the Earth becomes fully opaque to neutrinos for \( \cos \theta_z = -0.1 \) is \( E_{\nu} \approx 10^{10} \) GeV, while for \( \cos \theta_z = -1.0 \) it is \( E_{\nu} \approx 10^6 \) GeV. Consequently, the relative importance of neutrino - nucleon absorption to the Glashow resonance depends of the angle of incidence of the neutrinos. Our results indicate that for \( \cos \theta_z = -1.0 \), the attenuation due CC neutrino interactions becomes more important than the Glashow resonance even at IceCube energy range. Moreover, the comparison between the distinct \( \nu N \)
predictions demonstrate that they can differ by 30% (55%) at $E_\nu = 80 (300)$ TeV, with the Earth not being fully opaque to neutrinos in this energy range even at $\cos \theta_z \to -1$, as indicated in Fig. 2(c). On the other hand, for $\cos \theta_z = -0.1$ and ultra high energies, the distinct CC $\nu N$ predictions can be differ by $\approx 100%$. Basically, we obtain that the difference between these predictions is dependent on the zenith angle and the neutrino energy. Such uncertainty is not negligible and should be considered in the determination of the angular distribution of events in the IceCube and/or future observatories.

In Fig. 3 we present our predictions for the absorption function $S^j(E_\nu)$. We have that the integration over the zenith angle tends to reduce the energy range impacted by the Glashow resonance absorption. Moreover, we have that the distinct predictions for $\nu N$ interactions are very similar for $E_\nu \leq 10^8$ GeV, with the difference between the predictions reaching 10% at 80 TeV. On the other hand, at larger energies we have that the difference between the FJKPP (CT10) and BBMT predictions increases and becomes a factor 2 at $E_\nu \approx 10^{10}$ GeV, with the BBMT one being an upper bound. It is important to emphasize that considering the current estimates for the neutrino spectrum, which predict that the neutrino flux decreases with the energy with a power like behaviour, we have that the number of expected events at IceCube and/or future observatories should be small for these energies. Therefore, the difference of a factor two between the predictions has a strong impact in the analysis and interpretation of the possible few events that should be observed.

Finally, let's summarize our main conclusions. In this paper we have estimated the impact of the current uncertainty in the description of $\nu N$ interactions at ultra high energies in the absorption of neutrinos crossing the Earth until the detectors. Moreover, for comparison, the predictions considering $\bar{\nu}_e e$ were also presented. Our results indicated that the angular distribution of the neutrino events and the probability of absorption are sensitive to the treatment of the QCD dynamics at ultra high energies. Such results have direct implication in the determination of sources of UHE neutrinos below the horizon of IceCube neutrino observatory and in the analysis of the neutrino events in future observatories.

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