Three in one go: consequential angular momentum loss can solve major problems of CV evolution

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ABSTRACT
The average white dwarf (WD) masses in cataclysmic variables (CVs) have been measured to significantly exceed those of single WDs, which is the opposite of what is theoretically expected. We present the results of binary population synthesis models taking into account consequential angular momentum loss (CAML) that is assumed to increase with decreasing WD mass. This approach can not only solve the WD mass problem, but also brings in agreement theoretical predictions and observations of the orbital period distribution and the space density of CVs. We speculate that frictional angular momentum loss following nova eruptions might cause such CAML and could thus be the missing ingredient of CV evolution.

Key words: binaries: close – novae, cataclysmic variables – white dwarfs.

1 INTRODUCTION
The first theoretical binary population synthesis (BPS) models of Cataclysmic Variables (CVs) revealed dramatic disagreements between observations and theoretical predictions: 1. CVs below the period gap should make up ~99 per cent of the population (Kolb 1993), while roughly the same number of systems is observed below and above the gap (Knigge 2006). 2. The observed orbital period minimum is at 76 min (Knigge 2006), while the predicted orbital period minimum is 65-70 min (Kolb & Baraffe 1999). 3. A strong accumulation of systems at the orbital period minimum is predicted, but not observed (Patterson 1998; Knigge 2006). 4. The predicted space density (e.g. Ritter & Burkert 1986; Kolb 1993) exceeds the observed one by 1-2 orders of magnitude (e.g. Patterson 1998; Pretorius & Knigge 2012).

Within the last decade, two of these long standing problems of CV evolution seem to have been solved. First, the problem of the missing spike at the orbital period minimum was solved as recent deep surveys identified a previously hidden population of faint CVs (Gänsicke et al. 2009). Secondly, the orbital period minimum problem can be solved if additional angular momentum loss, apart from gravitational radiation, is assumed to be present in systems with fully convective donor stars (Knigge, Baraffe & Patterson 2012). However, recent observational advances not only solved previous problems but also generated a new one, which is currently perhaps the most severe. BPS models predict an average white dwarf (WD) mass of \( \lesssim 0.6 M_\odot \) (e.g. Politano 1996) and large numbers of CVs containing helium core (He-core) WDs. Yet, the observed average mass is \( \sim 0.83 M_\odot \) and not a single system with a definite He-core WD primary has been identified so far (Zorotovic, Schreiber & Gänsicke 2011, hereafter ZSG11). As shown by ZSG11, this absence of CVs with low-mass primaries can no longer be explained as an observational bias as suggested earlier by Ritter & Burkert (1986). Moreover, the problem is also clearly not related to our limited understanding of common envelope (CE) evolution as speculated recently by Ge et al. (2015). This is because low-mass WDs are found in large numbers in samples of detached post common envelope binaries (Rebassa-Mansergas et al. 2011) and the observed WD mass distribution of these objects is in good agreement with the model predictions (Toonen & Nelemans 2013; Zorotovic et al. 2014; Camacho et al. 2014). Finally, in Wijnen, Zorotovic & Schreiber (2015) we have shown recently that the two scenarios suggested by ZSG11, i.e. mass growth in CVs or a preceding phase of thermal time scale mass transfer for many CVs cannot solve the WD mass problem in CVs either.

Here we present BPS models of CVs taking into account angular momentum loss generated by the mass transfer in CVs. This consequential angular momentum loss (CAML) can drive CVs with low-mass WDs into dynamically unstable mass transfer which solves the WD mass problem. Interestingly, this approach simultaneously solves the space
density problem and brings into agreement the predicted and observed orbital period distributions.

2 GENERAL MODEL ASSUMPTIONS

The aim of this work is to evaluate whether assuming CAML can explain the observed WD masses in CVs. To that end we simulate CV evolution with different CAML approaches combined with standard assumptions for CV evolution.

We generate a population of $10^7$ initial mass sequence (MS) binaries. The star masses ($M_1, M_2$) are distributed according to the initial-mass function of Kroupa, Tout & Gilmore (1993) for the primary and a flat initial mass-ratio distribution (Sana, Gosset & Evans 2009). $M_1$ ranges from 0.8 to 9 $M_\odot$ and $M_2$ from 0.05 to 9 $M_\odot$ (requesting of course $M_2 < M_1$). The initial orbital separation is assumed to be flat in log $a$ (Popova, Tutukov & Yungelson 1982; Kouwenhoven et al. 2009), ranging from 3 to $10^4 R_\odot$.

The sample of generated MS binary stars is then evolved until the end of the common envelope (CE) phase using the binary-star evolution (BSE) code from Hurley, Tout & Pols (2002) assuming a CE efficiency of $\alpha$. By evolving $\alpha$ in the BSE code from Hurley, Tout & Pols (2002) we simulate CV evolution with different CAML approaches combined with standard assumptions for CV evolution.

Finally, we ignore CVs that may descend from thermal time scales, i.e. we assume these systems make up at most a few per cent of the current CV population which is probably correct (Gänsicke et al. 2003). While all the above model assumptions are quite uncertain, we did run several tests varying the uncertain parameters and find that the conclusions of this paper are not affected.

3 STABILITY OF MASS TRANSFER AND CAML

The mass ratio ($M_2/M_{\text{WD}}$) required for stability against dynamical time scale mass transfer is a crucial part of BPS models because it separates CVs from systems that evolve through a second CE and most likely merge. The dividing line can be obtained by equating the adiabatic mass radius exponent and the mass radius exponent of the secondaries Roche-lobe:

$$\zeta_{ad} \equiv \frac{d\ln(R_L)}{d\ln M_{ad}} = \frac{d\ln(R_{L2})}{d\ln M_2} \equiv \zeta_{R_{L2}}. \tag{1}$$

For deeply convective secondary stars ($M_2 \lesssim 0.5 M_\odot$) polytropic models represent a reasonable approximation, i.e. $\zeta_{ad} = -1/3$. For more massive stars, the size of the convective envelope decreases and $\zeta_{ad}$ increases steeply (Hjellming 1989). The right hand side of Eq.(1) sensitively depends on the assumed CAML. We therefore performed BPS models with three versions of CAML.

3.1 The fully conservative case

We first assume conservation of both angular momentum and the total mass of the binary, i.e. we assume only systemic angular momentum loss through magnetic braking and gravitational radiation but no CAML. This assumption is inconsistent with mass loss due to nova eruptions, which is known to be of the same order as the mass accreted by the WD between two eruptions, but the fully conservative model has been previously used in BPS models and we therefore use it as a reference model (despite the obvious inconsistency).

Using the fitting formula for the secondaries Roche-lobe provided by Eggleton (1983), Eq. (1) can be converted to:

$$\zeta_{ad} = \frac{1}{2} \ln(1 + q^{1/3}) - \frac{1}{2} \left(\frac{1}{1 + q^{1/3}}\right) \ln(1 + q) + 2(q - 1) \tag{2}$$

which can be solved to obtain the critical mass ratio above which mass transfer is dynamically unstable. For the fully conservative case we obtain $q_{\text{crit}} = 0.634$ for $\zeta_{ad} = -1/3$ and a steep increase in the range $M_2 = 0.5 - 0.7 M_\odot$. The results of the corresponding BPS model are illustrated in Fig. 1 (left panels). We have plotted the predicted CV population in a mass ratio ($q = M_2/M_1$) versus secondary mass ($M_2$) diagram (top panel) where the regions forbidden either due to dynamically unstable mass transfer or the WD mass exceeding the Chandrasekhar limit are gray shaded. The predicted current CV population is represented by the cyan dots while the black squares indicate the position of observed CVs with relatively robust measurements of both stellar masses. The observed sample consists of the fiducial systems listed in ZSG11 and three more recently identified systems with robust mass estimates for both stars: KISJ 1927+4447 (Littlefair et al. 2014), HS0220+0603 (Rodriguez-Gil et al. 2015) and PHL 1445 (McAllister et al. 2015).

In the bottom left panel of Fig. 1 we compare the simulated (light gray) and observed WD mass distribution and find the expected disagreement: the model predicts far too many CVs containing low-mass WDs.

3.2 The classical non-conservative model

In more consistent evolutionary models of CVs, mass loss due to nova eruptions is taken into account. In binary evolution models this can be incorporated as a continuous effect as shown by Schenkner, Kolb & Ritter (1998). Usually it is assumed that the specific angular momentum of the ejected mass equals the specific angular momentum of the WD. This
has a stabilizing effect on the mass transfer. The corresponding CAML is

\[
\frac{\dot{J}\text{CAML}}{J} = \nu \frac{M_2}{M_2}
\]

with \(\nu = M_2^2/(M_1(M_1 + M_2))\) (see e.g. King & Kolb 1995). This CAML has to be taken into account when evaluating the right hand side of Eq. (1). Compared to the fully conservative case, CAML brings the system closer to dynamically unstable mass transfer as mass transfer generates additional angular momentum loss and reduces the Roche-volume of the secondary. However, in the classical CAML case this effect is more than compensated by the mass loss of the primary which significantly reduces the decrease of the secondaries Roche-lobe. The critical mass ratio is again defined by requesting \(\zeta_{\text{ad}} = \zeta_{\text{Ro},2}\) which results in:

\[
\zeta_{\text{ad}} = \frac{2}{3} \left( \ln(1 + q^{1/3}) - \frac{1}{2} \frac{q^{1/3}}{(1 + q^{1/3})} \right) + 2\nu + \frac{M_2}{(M_2 + M_1)} - 2.
\]

We performed BPS calculations using the stability limit imposed by Eq. (4) and taking into account the corresponding mass and angular momentum loss. The results are shown in the right panel of Fig. 1. The disagreement between observations and theoretical prediction is even worse than in the fully conservative case (left panel), because even more CVs with low-mass WD and low-mass secondary star \((\lesssim 0.35M_\odot)\) are predicted to exist. Such systems are not found in the observed sample. As outlined in the introduction, neither observational biases nor our ignorance of the details of CE evolution, nor WD mass growth during or prior to CV evolution can solve this WD mass problem (see ZSG11 and Wijnen, Zorotovic & Schreiber 2015 for details).

Given the large uncertainties affecting the limits of dynamically unstable mass transfer in models of CV evolution and benefiting from the improved observational data, we present in the next section an empirical model for CAML aiming to reproduce the observed WD masses.

### 3.3 An empirical CAML model

To investigate if and which form of CAML could solve the WD mass problem, we keep the standard assumptions used in the previous sections but interpret the angular momentum loss associated with mass loss as a free parameter. Given that the disagreement between the observed and predicted WD masses in CVs is mainly caused by the large number of predicted CVs with low-mass WDs below the period gap (see Fig. 1), we expect significant improvement if \(\dot{J}\text{CAML}\) is stronger for low-mass WDs.

Simulating CV populations for different forms of CAML and comparing with the observations we find good agreement if we assume:

\[
\nu(M_1) = \frac{C}{M_1}
\]

with \(C = 0.3-0.4\). In the top left panel of Fig. 2 we show the observed and predicted CV populations in the \(q\) versus \(M_2\) diagram for a simulated population of CVs assuming this empirical CAML (eCAML from now on) model using \(C = 0.35\). The right panel shows the observed (top) and predicted (bottom) WD mass distribution while the bottom left panel shows the cumulative distribution of WD masses in CVs for the simulated samples (blue, cyan and red for the fully conservative, classical non-conservative and eCAML model, respectively) and for the observed systems (green).

Not too surprisingly, adding a new free parameter can solve the WD mass problem. However, before the eCAML model can be considered a viable option for CV evolution, we need to investigate how its predictions compare with other models.

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**Figure 1.** Top: Observed (black squares) and predicted (cyan dots) CV populations in the \(q\) versus \(M_2\) diagram for the fully conservative case (left) and for the classical non-conservative model (right). The gray shaded areas represent the forbidden regions either due to dynamically unstable mass transfer or because the WD mass exceeds the Chandrasekhar limit. The black solid lines represent the average of the WD masses measured in CVs \((M_{WD} = 0.83M_\odot)\) and the mass limit for He-core WDs \((0.47M_\odot)\). The dashed line represents the limit for thermally unstable mass transfer. Bottom: Comparison between the observed (dark gray) and simulated (light gray) WD mass distribution. Apparently, both models can not reproduce the observations.
observed properties of CVs and we need to find a physical mechanism that might be responsible for eCAML.

4 ORBITAL PERIOD DISTRIBUTION AND SPACE DENSITY

As described in the introduction, previous binary population models of CVs failed to reproduce both the predicted space density of CVs and the orbital period distribution. We now show that incorporating eCAML not only solves the WD mass problem but also brings into agreement the predicted and observed space density and orbital period distribution of CVs.

The orbital period distribution of all discovered CVs with measured orbital period is heavily biased because CVs with more massive secondaries, i.e. those with longer orbital periods, are significantly brighter than short orbital period CVs. The sample of CVs least affected by this bias has been presented by Gänsicke et al. (2009), thanks to the depth of SDSS. In Fig. 3 we compare the orbital period distribution of SDSS CVs with those predicted by the three models discussed in this paper. The classical CAML and the fully conservative model do not only dramatically disagree with the observed period distribution predicting the existence of too many CVs at short orbital periods and too few above.

The eCAML model we used to bring into agreement observations and theory.

5 PHYSICAL INTERPRETATION OF ECAML

The eCAML model presented in this paper can simultaneously solve the three biggest problems of CV evolution. However, there must be a physical mechanism behind this parameterised model.

The most obvious mechanism that may generate significant CAML in CVs are nova eruptions. In fact, the classical CAML model represents a weak form of CAML caused by novae. This model assumes the expelled material to take away the specific angular momentum of the WD which represents a reasonable assumption if friction between the ejecta and the secondary star is negligible. If, on the other hand, friction contributes significantly, CAML might be much stronger than predicted by the classical CAML model.

According to a detailed study of Schenker, Kolb & Ritter (1998), frictional angular momentum loss produced by novae only weakly depends on the mass ratio but is very sensitive to the expansion velocity of the ejecta, i.e. assuming slow expansion velocities, the consequential (frictional) angular momentum loss can be significant (see Schenker, Kolb & Ritter 1998, their Figure 5). Interestingly, nova models predict significantly faster expansion velocities for CVs containing high-mass WDs than for those containing low-mass WDs (see e.g. Yaron et al. 2005, their Fig. 2c). The CAML generated by slow nova eruptions in CVs containing low-mass WDs might therefore be the physical cause behind the eCAML model we used to bring into agreement observations and theory.

To test this hypothesis, the velocity of nova ejecta at the position of the secondary as a function of WD mass needs to
Loss. To test this hypothesis realistic models for nova eruptions because CVs with low-mass WDs produce slower novae is frictional angular momentum loss following nova eruptions and the resulting frictional angular momentum loss need to be developed.

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6 CONCLUSION

Since the work of Zorotovic, Schreiber & Gánscicke (2011), the WD mass problem has become the most serious limitation of our understanding of CV evolution. In Wijnen, Zorotovic & Schreiber (2015) we have shown that neither assuming WDs in CVs to grow in mass nor assuming a preceding phase of thermal time scale mass transfer for many CVs can solve the problem without violating other observational facts. Here we have presented a new model for CV evolution assuming consequential angular momentum loss (CAML) that increases with decreasing WD mass. In this model the previously predicted CVs with low mass WDs at short orbital periods suffer from dynamically unstable mass transfer and will disappear as CVs. This empirical CAML (eCAML) model is currently the only available model that can reproduce the observed WD mass distribution. Simultaneously and without any (further) fine-tuning, the model also solves the long standing disagreement between the predicted and observed orbital period distribution and space density of CVs. This makes eCAML a promising candidate for being the missing piece in the CV evolution puzzle. The best candidate for the physical mechanism behind eCAML is frictional angular momentum loss following nova eruptions because CVs with low-mass WDs produce slower novae which might lead to increased frictional angular momentum loss. To test this hypothesis realistic models for nova eruptions and the resulting frictional angular momentum loss need to be developed.

Figure 3. Orbital-period distributions for our three models of CV population: eCAML (top), classical CAML (middle), and the fully conservative (bottom). In grey the observed orbital period as measured by Gánscicke et al. (2009). The eCAML model is not only the first model able to reproduce the observed WD mass distribution but also the first one to be in agreement with the observed orbital period distribution.

be determined and incorporated in detailed models of frictional angular momentum loss. observationally, this would require to perform spectroscopy at the onset and throughout a nova which is probably unrealistic as currently the occurrence of nova eruptions can not be predicted. Thus, testing the eCAML hypothesis will probably rely on improving theoretical models of novae for some time to come.
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