Completing constrained flavor violation: lepton masses, neutrinos and leptogenesis

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Constrained flavor violation is a recent proposal for predicting the down-quark Yukawa matrix in terms of those for up quarks and charged leptons. We study the viability of CFV with respect to its predictions for the lepton mass ratios, showing that this remains a challenge, and suggest some possible means for improving this shortcoming. We then extend CFV to include neutrinos, and show that it leads to interesting predictions for hierarchical heavy neutrinos, and leptogenesis dominated by decays of the second heaviest one (“N2 leptogenesis”), as well as the possibility of low-scale leptoquark-mediated exotic decays.

1. INTRODUCTION

Minimal flavor violation (MFV) [1, 2] has been an extremely useful framework for parametrizing effects of new physics in which flavor symmetry is assumed to be spontaneously broken. Recently ref. [3] proposed a more predictive version of MFV in which there are only two fundamental Yukawa matrices, \( Y_e \) and \( Y_u \) (for charged leptons and up-type quarks), while the third one \( Y_d \) (for down-type quarks) is predicted to be the product,

\[
Y_d = \eta Y_e Y_u^\dagger
\]

at tree level, where \( \eta \approx 10^3 \) to fit the observed lepton masses. The structure (1) is a consequence of a spontaneously broken flavor symmetry SU(3)\(_1\) × SU(3)\(_2\) × SU(3)\(_3\), under which the SM fields \( Q, L \) (left-handed doublets) and \( u, d, e \) (right-handed singlets) transform, as shown in fig. 1\(^1\). It is argued that the charged lepton mass ratios are predicted almost correctly in this framework. Clearly, if it were possible to reduce the number of free parameters in the fundamental theory by the elimination of \( Y_d \), this could have profound implications for the ultimate explanation of the flavor structure of the standard model. The authors dub this scenario “constrained flavor breaking;” here we call it “constrained flavor violation” (CFV) in analogy to MFV.

Our goal in this paper is two-fold. First we examine the prediction of the lepton mass ratios more closely, since ref. [3] found that \( m_\mu/m_\tau \) is too low except in a few models based upon large fluctuations of the quark masses and mixings away from their measured central values. We will show that this is not an easy problem to solve, and that without any additional caveats it is more severe than suggested by ref. [3]. We find it necessary to implement the predictions at the GUT scale rather than at \( m_Z \), and to suppose that there is some means for altering the prediction for the up-to-down quark mass ratio at this scale relative to the weak scale. In particular we suggest that model-dependent threshold corrections, coming from integrating out heavy flavon scalars, could improve the situation. (Recently ref. [4] showed that alternate possible relations between the Yukawa matrices could also give some modest improvement in the predicted lepton mass ratios.)

Our second goal is to make a proposal for how to...
### 2. LEPTON MASS RATIOS

In this section we reexamine the prediction of CFV for ratios of the charged lepton masses. This is a fundamental test since these arise directly from the prediction (1), by solving for $Y_e$,

$$Y_e = \eta^{-1}(Y_d^{-1}Y_u)^\dagger = \eta^{-1}\text{diag}(Y_d)\text{V}^\dagger_{\text{CKM}}\text{diag}(Y_u)^{-1}$$  \hspace{1cm} (2)

where the last expression is written in the basis where $Y_d$ is diagonal and $Y_u = V^\dagger_{\text{CKM}}\text{diag}(Y_u)$. By inputting the measured quark masses and CKM mixings, within experimental uncertainties, one can generate $Y_e$ from (2), find its eigenvalues, and compute the mass ratios $m_u/m_d$, $m_d/m_s$, independently of the adjustable parameter $\eta$.

Ref. [3] carries this out for a large ensemble of randomly generated models, taking 1-, 2- and 3-$\sigma$ variations in the input parameters; only for a small fraction near the edge of the 3$\sigma$ allowed region is $m_u/m_s$ as large as its measured value.

In ref. [3]'s implementation, rather generous ranges are taken for the quark masses at the scale of $m_Z$, whose origin is not explained. Here we adopt the running quark masses at the scale $m_Z$ along with uncertainties as given in ref. [8], and ranges given for the CKM matrix elements by the Particle Data Group [9], reproduced in appendix A. With these inputs, a scan over $3 \times 10^5$ models fails to produce any with $m_\mu/m_\tau > 0.045$ even at 3$\sigma$, whereas the observed value is close to 0.06. This result is plotted in fig. 2(a) (upper left). The discrepancy is worse than found in ref. [3], due to their larger and unexplained estimates of the experimental errors.

One might question whether the prediction (1) is valid at the scale $m_Z$, whereas the UV flavor physics is expected to come in at a higher scale. To assess the effect of going to higher scales, we take advantage of the running Yukawa couplings (represented as running masses) calculated in ref. [8] to also test (1) at the GUT scale, taken to be $2 \times 10^{16}$ GeV. For consistency, one also needs the CKM parameters at this scale, which we take from ref. [10]. The result is shown in fig. 2(c) (middle left), giving considerable improvement, though the observed lepton mass ratios still remain at the very edge of the 3$\sigma$ region where the scatter plot is sparsely populated.

In an attempt to address the shortfall in $m_u/m_\tau$, ref. [3] makes a parametric estimate $m_\mu/m_\tau \sim (m_u/m_s)(m_u/m_c)\lambda$ (where $\lambda = \cos \theta_C$ is the Wolfenstein parameter), suggesting that a larger value of $m_u/m_s$ could ameliorate this problem. According to lattice determinations of the light quark masses, there is no latitude, beyond the usual error estimates, for increasing $m_u/m_s$ (for a recent review see [24]). However, there are still no direct lattice determinations of this ratio. Instead, the up and down quark are always represented by the same field, having a mass of $m_{ud} = (m_u + m_d)/2$. Phenomenological input using chiral perturbation theory (ChPT) is required to estimate the isospin breaking effects from $m_u \neq m_d$.

Kaplan and Manohar [5] (KM) pointed out that at second order in ChPT there is an operator

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<th>$\rho$</th>
<th>$\eta$</th>
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TABLE I: Upper rows: best-fit models from random scan over GUT-scale parameters to lepton mass ratios, including KM variations of $m_u/m_d \in [0,1]$. $\lambda, A, \rho, \eta$ are Wolfenstein parameters for the CKM matrix. Lower rows: standard model central values and errors used for the scan.
FIG. 2: Scatter plots for the lepton mass ratios $m_\mu/m_\tau$ versus $m_e/m_\tau$, with 1-, 2-, and 3-$\sigma$ variations of the quark mass and mixing input parameters around their measured central values. Shaded dot shows experimental value. In the upper two rows, top (middle) row is for inputs at the $m_Z$ (GUT) scale; left (right) column is without (with) the Kaplan-Manohar (KM) ambiguity in the light quark masses. Third row shows 3-$\sigma$ allowed regions for different assumptions about the experimental errors on $m_{ud} = (m_u + m_d)/2$ and $m_s$, as well as the range of $r = m_u/m_d$ probed by KM transformations.

$$(\det M)\text{tr}(M^{-1}\Sigma)$$

that effectively transforms the quark masses by

$$
\begin{align*}
m_u &\rightarrow m_u + \alpha m_d m_s \\
m_d &\rightarrow m_d + \alpha m_u m_s \\
m_s &\rightarrow m_s + \alpha m_u m_d
\end{align*}
$$

(3)

where $\alpha$ is a parameter of order $1/\Lambda_{\overline{MS}}$. This could shift the apparent quark masses as deduced from ChPT away from the true values, with a much bigger effect on $m_u$ and $m_d$ than on $m_s$. In principle, $\alpha$ can be determined by comparing enough measured quantities to their second order ChPT predictions (thus determining all the second order coefficients), and this procedure would thus resolve the KM ambiguity. On this basis, the isospin breaking effects are considered to be well understood and the ratio

$$r \equiv m_u/m_d$$

is known to high precision $0.46 \pm 0.02 \pm 0.02$ from simulations with $2 + 1$ flavors ($2$ denoting degenerate $u$ and $d$, and $1$ denoting $s$). However this requires the implicit assumption (not usually stated) that third order ChPT contributions are negligible. This assumption can only be rigorously tested by doing a full $1 + 1 + 1$ lattice simulation, leaving room for some doubt about the
true value of $r$.\textsuperscript{2} In fact such simulations have recently been performed \cite{6, 7}. This would seem to close the door on this loophole. Nevertheless it inspired us to consider the possibility of allowing $m_u/m_d$ to differ from its standard value. Below we will suggest an alternative possible justification for doing so.

Hence we define $r \equiv m_u/m_d$ and allow it to vary away from its standard value, while keeping the errors on $m_{ud}$ and $m_d$ consistent with ref. \cite{8}. We allow $r$ to vary in the interval $[0, 1]$ to obtain the augmented allowed regions shown in fig. 2(b,d). This does not improve the situation for Yukawa couplings at the $m_Z$ scale, but it does improve it somewhat at the GUT scale. Allowing $r$ to vary more widely, $r \in [0, 2]$, can further improve the overlap, as shown in the fig. 2(e,f) (bottom row).\textsuperscript{3} Parameters of the three best-fit models from this scan are given in table 1, along with the central values and errors for the varied parameters. In Fig. 3 we show the distributions of quark mass and mixing parameters resulting from the scan at the GUT scale.

Before declaring a modest victory however, it should be noted that many lattice practitioners consider the errors on the light quark masses like those quoted in \cite{8, 9} to be overestimates that do not reflect the state-of-the-art lattice results. In figs. 2(c,d), we took the errors to be $\delta m_{ud} = 0.25$ MeV, $\delta m_s = 6.5$ MeV. The rare points in our scans that agree with the lepton mass ratios rely upon large upward fluctuations in $m_{ud}$ and downward ones in $m_s$. Figs. 2(e,f) demonstrate that these rare fluctuations are eliminated by taking the smaller estimates $\delta m_{ud} = 0.045$ MeV and $\delta m_s = 1$ MeV inferred from the lattice results \cite{8} after rescaling to account for the running of the masses to lower values at the GUT scale. We can afford to take the tighter error bar on either $m_{ud}$ or $m_s$, but not both, and still get agreement with the lepton masses. Taking the smaller errors on $m_{ud}$ and $m_s$, even with compensating large values of $r \lesssim 2$, although $m_{u}/m_{r}$ can be large enough, a new problem arises in that $m_{e}/m_{r}$ is predicted to be too small.

We have seen that renormalization effects are quite important in the interpretation of the CFV prediction (2). It is conceivable that the flavons whose VEVs give rise to the Yukawa couplings have masses over some range of scales, which could induce threshold corrections in the running of the Yukawas and perhaps explain a larger value of $r$ at the GUT scale than at low scales, avoiding the need for invoking the KM ambiguity. We do not attempt any such model-building here, but this could motivate giving further consideration to CFV. In the following we suggest an extension of CFV that encompasses the neutrino sector.

\textsuperscript{2} We thank D.B. Kaplan for discussion on this point.

\textsuperscript{3} Proton stability could be consistent with $r > 1$ if $r$ runs to smaller values in going from the GUT to the QCD scale.
3. COMPLETING CFV WITH NEUTRINOS

To incorporate neutrinos into CFV, one needs to make some assumption about how the right-handed neutrinos $\nu_R$ transform under the flavor symmetries. In CFV, two species transform nontrivially under each one of the SU(3) subgroups, with the exception of SU(3)$_1$, as depicted in fig. 1. It is therefore natural to assign $\nu_R$ to the SU(3)$_1$ node of the moose diagram. To predict the neutrino masses and mixings, we must introduce an additional symmetric spurion field $Y_R$ for the right-handed Majorana neutrino mass matrix, that transforms in the $\bar{6}$ (the symmetric part of $\bar{3} \times \bar{3}$) representation of SU(3)$_1$.

Then, the Lagrangian invariant under flavor symmetry includes following Yukawa interactions and the right-handed Majorana neutrino mass term,

$$
\mathcal{L} = -H \bar{Q}_L Y_d d_R - \tilde{H} \bar{Q}_L Y_u u_R - \tilde{H} \bar{L}_L Y_e e_R - \tilde{H} \bar{L}_L Y_\nu \nu_R - \frac{1}{2} v_R \bar{v}_R Y_R \nu_R + \text{h.c.}
$$

(4)

where $v_R$ is a large mass scale and $v_R Y_R$ gives the Majorana mass matrix. $H$ gets VEV $v/\sqrt{2}$ in its neutral component, with $v = 246$ GeV. The flavor symmetries imply that

$$
Y_\nu = \eta' Y_e Y_u^\dagger
$$

(5)

We will study the consequences of this choice for the spectrum of heavy right handed neutrinos, and the resulting implications for leptogenesis and low-energy lepton flavor violation.

After integrating out the heavy neutrino, the light neutrino mass term is

$$
\frac{v^2}{2v_R} \bar{\nu}_L Y_\nu Y_\nu^T Y_\nu \nu_R + \text{h.c.}
$$

(6)

Let $\nu_L = L_\nu \nu_m$ denote the relation between the weak eigenstates $\nu_L$ and the mass eigenstates $\nu_m$. If $Y_\nu$ was already diagonal, then $L_\nu$ would coincide with the PMNS matrix. However in a basis where $Y_\nu$ is not diagonal, this is not the case. Suppose that the mass term $(v/\sqrt{2})\bar{e}_L Y_e e_R$ is diagonalized by taking $e_L \rightarrow L_e e_L$, $e_R \rightarrow R_e e_R$. Then the PMNS matrix is given by

$$
U_{PMNS} \equiv U = L^\dagger_\nu L_\nu
$$

(7)

where $L_\nu$ diagonalizes the neutrino mass matrix via

$$
\mu_\nu = \frac{v^2}{v_R} L^\dagger_\nu Y_\nu Y_\nu^{-1} Y_\nu^T L_\nu = \frac{\eta^2 v^2}{\eta^2 v_R} L^\dagger_\nu Y_\nu Y_\nu^{-1} Y_\nu^T L_\nu
$$

(8)
3.1. Heavy neutrino mass spectra

An interesting feature of the above scenario, also anticipated by ref. [3], is that even if $L_\nu$ is close to the identity matrix, the factor $L_\nu^T$ generates large mixing angles in $U_{PMNS}$. This means that the Yukawa matrix $Y_R$ for the sterile neutrino Majorana masses can be very hierarchical despite the large neutrino mixing angles.

Since $Y_L = L_Y (\tilde{\eta} \eta)^T$, it is fixed by the down quark masses, up to an overall normalization factor. For definiteness, we choose its largest matrix element to have unit magnitude, anticipating our application below to leptogenesis where this choice is advantageous. It means that the heavy neutrino mass spectra we derive here represent the maximum sizes consistent with perturbative values of $Y_\nu$. Rescaling $Y_\nu$ by a factor of $\lambda < 1$ implies a reduction in $M_i$ by the factor $\lambda^2$, for fixed values of the light neutrino masses.

The Majorana matrix $Y_R$ is constrained only by the experimental values of the neutrino masses and mixing angles. We can solve eq. (8) for $Y_R$, using (7):

$$Y_R = \frac{\eta^2 v^2}{\eta^2 v_R} (Y_d L_e U)^* m^{-1}_\nu (Y_d L_e U)^\dagger$$

where $m_\nu$ is diagonal and $L_e$ is a unitary transformation such that $L_\nu^T (Y_\nu Y_\nu^T)^T L_e$ is diagonal, using eq. (2) for $Y_\nu$. However this determines $L_e$ only up to multiplication on the right by a diagonal matrix of phases, $L_e \rightarrow L_e \text{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$, of which two relative phases are physically significant. In addition, $U$ can be multiplied on the right by two undetermined Majorana phases. Therefore $Y_R$ is a function of the down-to-up quark mass ratios, the CKM parameters, the light neutrino masses, the PMNS parameters, and four additional phases, as well as the overall scaling factor.

We perform a scan over models where the neutrino mass differences and mixing angles vary randomly within their allowed ranges, as specified in appendix A, but the quark parameters are constrained to be close to values needed to get the right lepton mass ratios, as described in section 2. This requires a choice of the lightest neutrino mass $m_{\nu 1}$, as well as whether the neutrino mass hierarchy is normal or inverted. We also scan over the four phases mentioned above. The magnitudes of the heavy neutrino masses $M_i$ are then fixed, being given by

$$M_i = Y_i v_R$$

where $Y_i$ are the eigenvalues of $Y_R$ in eq. (9). We obtain distributions of $M_i$ as shown in fig. 4, varying $m_{\nu 1}$ from $10^{-5}$ to $10^{-1}$ eV, and also allowing for normal or inverted mass hierarchy. For the majority of models, there is a clear separation between the mass eigenvalues, and a hierarchy that becomes more pronounced for smaller values of $m_{\nu 1}$. $M_1$ tends to be several orders of magnitude smaller in the inverted compared to normal hierarchy.

The prediction of a hierarchical sterile neutrino spectrum is not unique to our model. For example the Altarelli-Feruglio model [14] which explains tribimaximal mixing as a consequence of the discrete $A_4$ symmetry, predicts that the heavy neutrino spectrum has the form $M_i = \{ A, B - A, B + A \}$, while the light neutrino masses go as $m_i = c/M_i$, so that any hierarchy in the latter is a direct consequence of hierarchy in the former. Similarly the Frampton-Glashow-Yanagida ansatz [15], which has only two sterile neutrinos, allows for a strong hierarchy between their masses (although it is not required). A notable exception is the class of models based upon minimal lepton flavor violation [16] where the flavor symmetry imposes a nearly degenerate spectrum of heavy neutrinos.

4. Leptogenesis

As an application of CFV extended to include neutrinos, we explore the consequences for thermal leptogenesis from taking the ansatz (5) for the neutrino Dirac Yukawa matrix. Thermal leptogenesis assumes that the heavy neutrino whose decays produce the asymmetry is initially not present in the thermal bath, but only gets generated by its Yukawa interactions [17, 18]. This assumption allows for definite predictions, which would otherwise be ambiguous due to dependence upon the initial conditions.

It is important to notice that the CFV ansatz only fixes the ratios of right-handed neutrino masses $M_i$ but not their overall scale. On the other hand the asymmetry produced by leptogenesis can depend strongly on the actual scale of $M_i$. To gain a qualitative understanding of possible correlations between the lepton asymmetry and the structure of couplings predicted by CFV, free from uncertainties from the undetermined overall scale, we first focus on the simplest scenario of thermal leptogenesis (for recent reviews see [19, 20]). Namely, we assume the reheating temperature $T_{rh}$ is much higher than $M_i$, and $N_i$ decays in the “single lepton flavor regime.” This means that the linear combination of flavors produced by the decay of heavy neutrino $N_i$ does not decohere due to Yukawa interactions. In other words, it is assumed that the temperature is sufficiently high for these interactions to not yet have come into equilibrium.

For the hierarchical $N_i$ mass spectrum, it is often assumed that interactions mediated by the lightest neutrino $N_1$ wash out any lepton asymmetries produced by decay of $N_{2,3}$, in which case the one produced by $N_1$ decay is relevant. However under certain conditions, $N_1$ decays can only destroy a particular linear combination of lepton number, that only partially overlaps with the combinations created in $N_{2,3}$ decays [22, 23].

For an initial estimate, we will calculate the asymmetries that can be produced by any of the three decays in the simplest scenario in section 4.1, ignoring washout effects mediated by the lighter neutrinos. We then comment on the flavor effects in section 4.2 when $N_i$ decays at lower temperature. We discuss the pure $N_1$ contribution in section 4.3. The case that $N_{2,3}$ decay dominates is
FIG. 5: Distributions of effective neutrino mass $\tilde{m}_i$, eq. (14), for different choices of the lightest neutrino mass $m_{\nu_1} = 10^{-5}, 10^{-4}, \ldots, 10^{-1}$ eV. Left (right): $\tilde{m}_2$ ($\tilde{m}_3$) versus $\tilde{m}_1$. Top (bottom) Normal (inverted) mass hierarchy. Smaller values of $m_{\nu_1}$ correspond to smaller values of $\tilde{m}_3$.

discussed in section 4.4, with results summarized in section 4.5. In section 4.6 we compare our results to those of other scenarios for leptogenesis. In this work we focus on the qualitative features and are not concerned with $O(1)$ uncertainties.

### 4.1. Decoupled asymmetries

Essential quantities for estimating the lepton asymmetry from heavy neutrino $N_i$ decays are the CP asymmetries $\epsilon_i$. These are expressed in the basis where $Y_R$ is diagonal (with positive real entries), $\nu_R = R_{\nu_i} N_i$, implying $R_{\nu_i} (Y_R^T Y_R) R_{\nu_i}$ is diagonal, and its eigenvalues $Y_i^2$ determine the heavy neutrino masses through eq. (10). In the special basis assumed for eq. (2), $Y_d$ and $Y_{\nu}$ are both diagonal. In the basis where $Y_R$ is diagonal, the neutrino Yukawa matrix is $h = R_{\nu_i} Y_R^T$. Then the CP-asymmetry for decay of heavy neutrino $N_i$ is

$$\epsilon_i = \frac{1}{8\pi (hh^\dagger)_{ii}} \sum_{j \neq i} \text{Im} (hh^\dagger)_{ji}^2 g(M_j^2/M_i^2)$$

where $g(x) = \sqrt{x} [1/(1-x) + 1 - (1+x) \ln(1+1/x)]$. It is maximized when $h$ is as large as possible, hence for our initial estimates of the maximum possible asymmetries we scale $Y_{\nu}$ so that the largest element of $h_{ij}$ is unity (as in section 3.1), still consistent with a perturbative analysis.

The baryon asymmetry is conveniently expressed as $Y_B$, the baryon-to-entropy ratio. Big bang nucleosynthesis and the cosmic microwave background give consistent determinations, $Y_{B, \text{obs}} \approx 8 \times 10^{-11}$ \cite{11,12}. The initially-produced asymmetry can be parametrized as \cite{13}

$$Y_B = \sum_i Y_{B,i} \sim 0.4 \frac{c_{\text{sph}}}{g_*} \sum_i \kappa_i \epsilon_i$$

where $c_{\text{sph}}$ depends upon how sphalerons redistribute the lepton asymmetry ($c_{\text{sph}} = 28/79$ in the standard model), $g_* = 106.75$ degrees of freedom in the SM plasma, and $\kappa_i$ is an efficiency factor taking account of washout of the produced lepton asymmetry, due to rescatterings medi-
FIG. 6: Top row: distributions of baryon asymmetries $Y_{B2}$ versus $Y_{B1}$ produced by $N_2$ versus $N_1$ decays for normal hierarchy (left) and inverted hierarchy (right). Colors correspond to light neutrino mass values as in previous figures. $Y_{B3}$ versus $Y_{B1}$ (not shown) is similar. Bottom row: distributions of abundance ratios $Y_{B3}/Y_{B2}$ versus $Y_{B2}/Y_{B1}$, before accounting for washout by $N_1$ and $N_2$ interactions.

The efficiency factor $\kappa_i$ is a function of the ratio of the $N_i$ decay rate to the Hubble rate,

$$K_i = \frac{\Gamma_{N_i}}{H(M_i)} = \frac{M_{pl}}{1.66/(8\pi\sqrt{2})} \frac{(hh^\dagger)_{ii}}{M_i}$$  \hspace{1cm} (13)

where $\Gamma_{N_i} = \frac{1}{\pi} (hh^\dagger)_{ii} M_i$, $H = 1.66\sqrt{g_\ast}T^2/M_{pl}$, and $M_{pl} = 1.22 \times 10^{19}$ GeV is the Planck mass. The rates are proportional to the effective light neutrino masses $\tilde{m}_i$, defined as

$$\tilde{m}_i = \frac{(hh^\dagger)_{ii} v^2}{M_i}$$  \hspace{1cm} (14)

The respective conditions $K_i \ll 1$ and $K_i \gg 1$ for weak and strong washout correspond to $\tilde{m}_i \ll m_\ast$ and $\tilde{m}_i \gg m_\ast$, with $m_\ast \approx 10^{-3}$ eV.

In general, we have the freedom to scale $M_i$ and $Y_\nu^2$ (or $hh^\dagger$) by a common factor while keeping the light neutrino mass spectrum fixed. The washout factors $K_i$ are invariant under this scaling, while the asymmetries $\epsilon_i$ have a linear dependence. Hence we have chosen $M_i$ and $hh^\dagger$ to be as large as is consistent with a perturbative treatment, $\max|h_{ij}| = 1$, for our estimate of the initially produced asymmetries. This freedom can be used to dial down the asymmetry in cases where it may be larger than the observed one. For our initial estimates, we follow ref. [21] in estimating the washout factor as

$$\kappa_i \cong \min \left( 2 \times 10^{-2} \left( \frac{0.01 \text{eV}}{\tilde{m}_i} \right)^{1.1}, 1 \right)$$  \hspace{1cm} (15)$$

In the weak-washout regime, the produced asymmetries are sensitive to the initial values of the heavy neutrino abundances. The approximation $\kappa_i = 1$ in this case assumes a thermal distribution.

As before, we scan over quark parameters consistent with the lepton mass ratios, and over neutrino masses...
and mixing angles as given in appendix A. Distributions of the effective neutrino masses \( \bar{m}_i \) are shown in fig. 5. For almost all cases, \( \bar{m}_{1,2} \sim 0.01 - 1 \text{ eV} > m_{\ast} \), corresponding to some washout of the \( N_{1,2} \) asymmetries, while \( \bar{m}_3 \) can be in the weak washout regime, if \( m_{\nu_3} \lesssim 10^{-4} \text{ eV} \). In fig. 6 we show the contributions to the baryon asymmetry \( Y_{B2} \) versus \( Y_{B1} \) from the same decays. (The results for \( N_3 \) decays look similar to those from \( N_2 \).) For the normal hierarchy, \( Y_{B1} \) can be marginally big enough to account for observations, but it falls short in the inverted hierarchy. On the other hand \( Y_{B2} \) tends to give the dominant contribution (assuming it does not get washed out by subsequent \( N_1 \) scatterings), motivating us to give more careful consideration to this scenario below.

We have also performed scans using values of the neutrino mixing angles at the GUT scale. The exact values depend upon the structure of the lepton and neutrino Yukawa couplings, and the heavy neutrino masses, which give thresholds in the renormalization group equations \([25]\). It is beyond the scope of this work to carry out the RGE evolution for each model in our scans; instead we adopted typical GUT scale inputs \( \sin^2 \theta_{12} = \sin^2 \theta_{23} = 0.5, \sin^2 \theta_{13} < 0.01 \) (while the running of the light neutrino masses is small enough to neglect) found in a survey of neutrino models \([26]\). The results do not differ markedly from those presented above from the low-energy neutrino parameters.

### 4.2. Flavor effects

The simple description given above needs to be modified in a more quantitative treatment taking account of flavor effects. We so far assumed that \( N_i \) decays in the single lepton flavor regime, where the lepton state produced by \( N_i \) decay propagates coherently. This is true at sufficiently high temperatures such that interactions involving the charged lepton Yukawa couplings are out of equilibrium. These come into equilibrium as the temperature decreases: the equilibration temperatures for the \( e, \mu, \tau \) co couplings are given by \( T_e \sim 4 \times 10^8 \text{ GeV}, T_\mu \sim 2 \times 10^9 \text{ GeV}, T_\tau \sim 5 \times 10^{11} \text{ GeV} \) respectively \([20]\).

Therefore for \( T \lesssim 10^9 \text{ GeV} \), the flavor basis \( \{\ell_e, \ell_\mu, \ell_\tau\} \) gets fully resolved by scattering processes involving the Higgs. On the other hand, in the temperature window \( 10^9 \lesssim T \lesssim 10^{12} \text{ GeV} \), the lepton state produced from \( N_i \) decay gets projected onto the \( \ell_\tau \) direction and some linear combination of light flavors \( \ell_{e+\mu} \). Since \( N_j \) (with \( j \leq i \)) can only completely wash out the flavor direction to which it couples, some parts of these asymmetries are partially protected from washout.

When the Boltzmann equations are modified to take account of this effect, the net washout is reduced relative to the naive treatment, and the final asymmetry is typically enhanced. When only the \( y_\nu \) Yukawa coupling is important, the symmetry is generally enhanced by a factor of 2 relative to the single flavor case \([20]\). In parts of parameter space, it is possible to get an asymmetry purely from flavor effects even when \( \epsilon_i = 0 \) \([27]\). For our more quantitative numerical study of \( N_i \) leptogenesis for CFV, rather than solving the exact Boltzmann equations for the case \( M_i < 10^{12} \text{ GeV} \), we roughly estimate the tau flavor effect by multiplying the asymmetry in (12) by a factor of 2.

### 4.3. \( N_1 \) leptogenesis

Since our heavy neutrino mass spectrum is very hierarchical, it is plausible that the reheating temperature \( T_{\text{rh}} \) is in between \( M_1 \) and \( M_{2,3} \), so that \( N_1 \) has a thermal distribution but \( N_{2,3} \) are highly suppressed in their abundances. However in this case the likelihood of sufficient baryogenesis from \( N_1 \) decays alone is quite small, as can be seen from fig. 6. Only with the normal mass hierarchy is it possible. For \( m_{\nu_1} = 10^{-5} \text{ eV} \), 1 in 1000 random models have \( Y_{B1} \) as large as the observed value, and this fraction remains constant with increasing \( m_{\nu_1} \), until \( m_{\nu_1} \sim 0.01 \text{ eV} \) when it drops to zero.

The heavy neutrino masses \( M_1 \) cluster around their maximum values in the interval \((0.5 - 2) \times 10^{11} \text{ GeV}\) for these models, which also have relatively large values of \( |\epsilon_1| \) compared to the average. The preference for large \( M_1 \) and the scarcity of viable models can be understood in terms of the strong washout effect from the \( N_1 \) neutrinos. Higher efficiency requires smaller values of \( \bar{m}_1 \sim M_1^{-1} \).

From fig. 4 it is clear that all models have \( M_1 < 10^{12} \text{ GeV} \). Hence the \( N_1 \)-generated asymmetry estimated in eq. (12) may get enhanced by a factor of \( O(1) \) as mentioned in section 4.2, due to the \( \tau \) flavor effect. This small enhancement does not change the qualitative unlikelihood of successful \( N_1 \) baryogenesis in this model.

### 4.4. \( N_{2,3} \) leptogenesis

On the other hand, a large fraction of models in our scans can produce a sufficient baryon asymmetry through the decays of \( N_2 \) or \( N_3 \) neutrinos. We will first describe that coming from \( N_2 \), then explain its generalization to \( N_3 \).

As first pointed out by ref. \([22]\)\([23]\), a large fraction of the asymmetry produced by \( N_2 \) can survive the effects of \( N_1 \)-mediated scatterings even if the latter have not decoupled, but are in fact fast enough to induce decoherence of the \( N_2 \)-produced lepton state. In this case, it is only a projection of the original state that is washed out by \( N_1 \) interactions.

The conditions for strong \( N_1 \) washout (decoherence) are

\[ M_2 \gg M_1, \quad \bar{m}_1 \gg m_{\ast}, \quad \frac{M_1}{M_2} \ll \frac{m_{\ast}}{\bar{m}_2} \quad (16) \]
where we choose $M_2 > 10 M_1$ and $\tilde{m}_i > 10 m_s$ in practice. The third condition ensures that $N_2$-mediated interactions play a negligible role at $T \sim M_1$ compared to the $N_1$ decoherence effect. A further condition for protecting a direction in flavor space from $N_1$ washout is that $M_1 > 10^9$ GeV, so that it does not decohere fully into its $\{\ell_e, \ell_\mu, \ell_\tau\}$ flavor components. The lepton doublet produced in decays of $N_i$ is initially in the flavor superposition

$$|\ell_i\rangle = (hh^\dagger)^{-1/2} \sum_\alpha h_{\alpha i} |\ell_\alpha\rangle \equiv \sum_\alpha c_{\alpha i} |\ell_\alpha\rangle \quad (17)$$

where $i = 1, 2, 3$, $\alpha = e, \mu, \tau$ and $c_{\alpha i} = \langle \ell_\alpha | \ell_i \rangle$. In the following we use roman index $i$ for the heavy neutrino mass eigenbasis, and greek index $\alpha$ for the flavor basis.

As mentioned before, the lepton flavor effect from $y_\tau$ Yukawa interactions becomes relevant when the temperature is below $10^{12}$ GeV. This is always true for $N_1$ decays, since our preliminary scan (fig. 4) shows that $M_1 < 10^{12}$ GeV in all cases. Thus only the projection of $|\ell_i\rangle$ orthogonal to $|\ell_\tau\rangle$ remains coherent. Of this, the part that is orthogonal to $|\ell_i\rangle$ is untouched by $N_1$ washouts, while the parallel component is reduced by the efficiency factor $\kappa_1$.

To quantify the resulting baryon asymmetry, we define an orthonormal basis $\{\ell_e, \ell_0, \ell_1\}$,

$$|\ell_0\rangle = N'_1 (c^\dagger_{\mu 1} |\ell_\mu\rangle - c^\dagger_{e 1} |\ell_e\rangle)$$

$$|\ell_1\rangle = N'_1 (c^\dagger_{e 1} |\ell_e\rangle + c_{\mu 1} |\ell_\mu\rangle) \quad (18)$$

where $N'_1 = (|c_{e 1}|^2 + |c_{\mu 1}|^2)^{-1/2}$, so that $|\ell_2\rangle$ decomposes as

$$|\ell_2\rangle = c_{e 2} |\ell_e\rangle + c_{\mu 2} |\ell_\mu\rangle + c_{1 2} |\ell_1\rangle \quad (19)$$

with $c_{e 2} = \langle \ell_0 | \ell_2 \rangle$, $c_{1 2} = \langle \ell_1 | \ell_2 \rangle$. Then the corrected asymmetry is

$$Y_{B2} \equiv Y_{B2,0} \left( |c_{1 2}|^2 \kappa_1 + |c_{0 2}|^2 \right) \quad (20)$$

where $Y_{B2,0}$ is the naive estimate given in eq. (12), if $M_2 > 10^{12}$ GeV. If $M_2 < 10^{12}$ GeV, we include an extra factor of 2 to estimate the $y_\tau$ Yukawa effect as discussed above. For this quantitative study we use the more exact approximations for the efficiency factors from ref. [21], given in appendix B.

An analogous procedure can be carried out to include the contribution from $N_3$ decays to the surviving asymmetry. According to fig. 4, if $M_1$ is no smaller than $10^9$ GeV, we always have $M_3 > 10^{12}$ GeV. Thus the initial $N_3$-generated asymmetry $Y_{B3,0}$ is given by eq. (12). If $N_2$ interactions are fast, they will wash out the part of $|\ell_2\rangle$ that is parallel to $|\ell_3\rangle$. The analogous condition to eq. (16) is

$$M_3 \gg M_2, \quad \tilde{m}_2 \gg m_s, \quad \frac{M_2}{M_3} < \frac{m_s}{\tilde{m}_3} \quad (21)$$

Again, we demand that $M_3 > 10 M_2$ and $\tilde{m}_2 > 10 m_s$, similarly to the conditions (16). Depending upon whether $M_2 > 10^{12}$ GeV, the appropriate expansions are

$$|\ell_3\rangle = \begin{cases} 
  c_{23}\ell_2 + c_{03}\ell_0, & M_2 > 10^{12} \text{ GeV} \\
  c_{-3}\ell_3 + c_{23}\ell_2', + c_{03}\ell_0', & M_2 < 10^{12} \text{ GeV}
\end{cases} \quad (22)$$

where $c_{03}' = (1 - |c_{23}|^2)^{1/2}$ (we are free to define the phase of $\ell_0'$ such that $c_{03}'$ is real and positive) and

$$|\ell_0'\rangle = (|\ell_3\rangle - c_{23}|\ell_2\rangle) / c_{03}'$$

$$|\ell_0''\rangle = N_2 (c_{\mu 2}' |\ell_\mu\rangle - c_{e 2}' |\ell_e\rangle)$$

$$|\ell_2'\rangle = N_2' (c_{e 2}' |\ell_e\rangle + c_{\mu 2}' |\ell_\mu\rangle) \quad (23)$$

with $N_2' = (|c_{e 2}'|^2 + |c_{\mu 2}'|^2)^{-1/2}$, and $c_{03}'' = (|\ell_0''| |\ell_3\rangle, c_{23}' = (|\ell_2''| |\ell_3\rangle$.

The components $|\ell_2\rangle$, $|\ell_2'\rangle$ get suppressed by $\kappa_2$, while $|\ell_0', \ell_0''\rangle$ are unaffected by $N_2$ washouts. If the $N_1$ strong washout condition (16) is satisfied, all of these basis vectors must then be expanded in terms of $|\ell_0\rangle$ and $|\ell_1\rangle$ to determine the effect of $N_1$ washouts. This was already done for $|\ell_2\rangle$ in eq. (19). For the rest,

$$|\ell_0''\rangle = c_{00}' |\ell_0\rangle + c_{10}' |\ell_1\rangle$$

$$|\ell_0'''\rangle = c_{10}'' |\ell_0\rangle + c_{00}' |\ell_1\rangle$$

$$|\ell_2''\rangle = (c_{e 2}' |\ell_e\rangle + c_{\mu 2}' |\ell_\mu\rangle) \quad (24)$$

where $c_{00}' = \langle \ell_0 | \ell_0'' \rangle$, $c_{10}' = \langle \ell_1 | \ell_0'' \rangle$, $c_{10}'' = \langle \ell_1 | \ell_0''' \rangle$, $c_{00}'' = \langle \ell_0 | \ell_0''' \rangle$, $c_{10}'' = \langle \ell_1 | \ell_0''' \rangle$, $c_{12}'' = \langle \ell_1 | \ell_2'' \rangle$ with the explicit expressions for the state vectors in (18, 23, 24). The naive contribution to $Y_{B3}$ gets reduced analogously to (20) as $Y_{B3} \equiv P_3 Y_{B3,0}$ where

$$P_3 = \frac{\kappa_2 |c_{23}|^2 (|c_{02}|^2 + \kappa_1 |c_{12}|^2)}{+ (1 - |c_{23}|^2) (|c_{00}''|^2 + \kappa_1 |c_{10}''|^2)} \quad (25)$$

for $M_2 > 10^{12}$ GeV, and

$$P_3 = \kappa_2 |c_{23}|^2 (|c_{02}|^2 + \kappa_1 |c_{12}|^2) + |c_{03}''|^2 (|c_{00}''|^2 + \kappa_1 |c_{10}''|^2) \quad (26)$$

for $M_2 < 10^{12}$ GeV.

4.5. Results for $N_{2,3}$ leptogenesis

Here we present results for successful $N_{2,3}$ leptogenesis from CFV model samples based on the formalism discussed in previous subsection. First let us estimate how readily the two strong washout conditions (16) and (21) can be satisfied. These conditions are independent of an overall rescaling of neutrino Yukawa couplings and heavy masses (in which the light neutrino masses remain fixed), so we can analyze them prior to doing such a rescaling, which we will use to renormalize any too-large baryon asymmetry down to the observed value. As shown by the parameter scan in figs. 4 and 5, we find
\[ \tilde{m}_i \gg m_\nu, M_{i+1} \gg M_i \text{ with } i = 1, 2 \text{ hold always. Hence the fraction of models meeting the strong washout requirements is mainly determined by the last condition in each case. For the five representative values of } m_\nu, \text{ considered, we find that more than 90\% of samples pass conditions (21), while roughly 70\% satisfy (16). The fraction with both conditions satisfied is } \sim 60 \textcolor{red}{- } 70\%. \]

We select samples that successfully generate observed baryon asymmetry by \( N_{2,3} \) decay in following way. Assuming \( M_2 > 10^{12} \text{ GeV} \) initially, we first check the validity of the \( N_2 \) strong washout condition (21). If it is satisfied, we make the \( \langle \ell_3 \rangle \) projection as in (22), otherwise \( N_3 \) decay does not contribute in the final asymmetry. Next we check \( N_1 \) strong washout condition (16). If it is satisfied, we have \( Y_B = Y_{B2} + Y_{B3} \) with projections (20, 25), otherwise we reject the sample model.

In any case, we require \( r = Y_B/Y_{B,\text{obs}} \geq 1 \) since we have found the largest possible asymmetry, corresponding to a prescribed value of the maximum allowed neutrino Yukawa coupling \( |h_{ij}^\nu| \). Two choices are considered, max \( |h_{ij}^\nu| = 1 \text{ and } \sqrt{4\pi} \), the latter being the largest allowed by perturbative unitarity. We then rescale \( y_\nu \) by \( 1/\sqrt{r} \) and \( M_i \) by \( 1/r \) to bring \( Y_B \) into agreement with the observed value. The above consistency requirements with respect to the heavy neutrino masses must be checked following this rescaling before declaring the sample model to be successful.

We present the fraction of successful \( N_{2,3} \) baryogenesis models in fig. 7, as a function of the lightest neutrino mass, and for the two choices 1, \( \sqrt{4\pi} \) of max \( |h_{ij}^\nu| \). Generally, the fraction increases with \( m_\nu \), but drops abruptly above \( \sim 0.01 \text{ eV} \). The negligible fraction at 0.1 eV is correlated with the stronger \( N_1 \) washout effect indicated in fig. 5. The yield is consistently greater in the case of normal mass hierarchy, where the models tend to have larger values of \( M_1 \) than for the inverted hierarchy. \( N_2 \) decay almost always gives the dominant contribution to the final asymmetry. Only when \( m_\nu < 10^{-5} \text{ eV} \), the weak \( N_3 \) washout effect becomes important and \( N_3 \) decay contributes significantly in the selected samples5. In summary, the CFV extension to neutrinos provides a framework for a very hierarchical right-handed neutrino mass spectrum, where the observed baryon asymmetry mainly comes from \( N_{2,3} \) decay. The fraction of successful models, taken from random samples consistent with the charged lepton mass spectrum, can be significant, 20 – 50\%, especially if \( m_\nu \sim 10^{-2} \text{ eV} \).

### 4.6. Comparison to other frameworks

To conclude our study of leptogenesis, we compare our results with previous literature based on other ansatzes for the neutrino Yukawa couplings. In the Altarelli-Feruglio model, it was found that the CP-asymmetry vanishes at leading order as the consequence of the \( A_4 \) symmetry for tribimaximal mixing [28]. Hence the lepton asymmetry can only be generated by subleading corrections. An extension of the Altarelli-Feruglio model was recently studied in ref. [29], where it was shown that with the right-handed neutrino decaying in the single lepton flavor regime, a sufficient lepton asymmetry can be generated by next-to-leading order terms. However washout effects mediated by the lighter neutrino were not considered, which are crucial in our estimates for the CFV framework.

For the Frampton-Glashow-Yanagida model, a systematic study was recently done in ref. [30], where it was shown that with renormalization group running, the normal neutrino mass hierarchy is disfavored. In the inverse hierarchy case, successful leptogenesis can be realized with \( M_1 \sim 10^{13} \text{ GeV} \). The naturalness of the 125 GeV Higgs boson prefers a much lower mass of heavy neutrino [31]. Taking this into account, the authors of [30] studied resonant leptogenesis with a nearly-degenerate mass spectrum, finding that sufficient baryon asymmetry can be generated if \( M_1 < 4 \times 10^7 \text{ GeV} \).

A class of minimal seesaw models involving two right-handed neutrinos and Yukawa matrices with one texture zero was recently studied in ref. [32] with respect to its predictions for leptogenesis. There is only one important phase in this scenario, that controls both leptogenesis and low-energy CP violation; the predicted CP phase is still consistent with current experimental data. An example of an SU(5) SUSY GUT model was studied with the lepton asymmetry coming mainly from the lightest

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5 When \( M_4 \) is quite large, the gauge and top interactions might not be in equilibrium. Ref. [13] (see Table 1) shows the corrections to the yield of leptogenesis assuming certain spectator processes are in equilibrium. Generally they induce \( \mathcal{O}(1) \) uncertainty for the estimation based on eq. (12) and (15).
right-handed neutrino decay. This is in contrast to the dominance of $N_2$ leptogenesis in our case.

For realizations of leptogenesis based upon minimal lepton flavor violation [16], the heavy right-handed neutrino masses are degenerate at tree level, with mass differences only generated radiatively by loop corrections. The lepton asymmetry is then derived from a different mechanism, resonant leptogenesis, which is unlikely to be realized in CFV. Successful leptogenesis was obtained with the right-handed neutrino mass scale above $10^{12}$ GeV. This implies sizable neutrino Yukawa couplings and the possibility to probe the additional new physics scale up to 100 TeV via lepton flavor violation in low energy experiment [33][34].

5. LEPTON FLAVOR VIOLATION

By the assumed symmetries of constrained flavor violation, there can be exotic dimension-6 operators that are suppressed only by heavy mass scales and no Yukawa couplings,

$$\frac{1}{\Lambda_1^2} |\bar{e}_\mu \gamma^\mu u^\dagger_u|^2, \frac{1}{\Lambda_2^2} |\bar{e}_\mu L^\dagger_e|^2$$

where color indices are implicit. These have the structure of leptoquark-induced interactions, which have been previously studied in the context of MFV interactions in refs. [35, 36]. Both operators lead to lepton-flavor violating and quark $\Delta F = 1$ decays, $D \to e\mu$ from the first operator and $K \to e\mu, B_d \to e\tau$ and $B_s \to \mu\tau$ from the second one. Ref. [3] chose not to consider these interactions; we do so here.

Of the rare leptonic decays, the most highly constrained is $K^0_L \to e\mu$ with branching ratio $< 5 \times 10^{-12}$. Corresponding constraints on the new physics scale have been worked out in ref. [37]. Updating their result with the current experimental limit reported in PDG [9], we find

$$\Lambda_2 > 260 \text{ TeV}$$

which is comparable to the bound derived from MFV leptoquarks that have Yukawa structure [36]. The corresponding constraint from $D^0 \to e\mu$ with branching ratio $< 2.6 \times 10^{-7}$ is much weaker, since it is helicity suppressed by $m_\mu/m_D$, and the constraint on the branching ratio is also weaker:

$$\Lambda_1 > 1.7 \text{ TeV}$$

In addition to the purely leptonic channels, there are semileptonic decays such as $K^0_L \to \pi\mu e$ that are also strongly constrained by experiment. However ref. [37] finds these generally less constraining on the scales $\Lambda_i$ than the purely leptonic ones.

6. CONCLUSIONS

Evidence for a simple constraint $Y_d \sim Y_u Y_e^\dagger$ between the fermion Yukawa matrices of the standard model would be very interesting, for gaining insights into the origin of flavor. The biggest challenge for this hypothesis is that charged lepton masses are known extremely well, and do not agree with the naive predictions coming from this relation. We have shown that if the prediction actually applies at a high scale such as the GUT scale, and if the up-to-down quark mass ratio is somewhat larger at this scale than at low energies, the problem with lepton masses can be overcome. It is conceivable that the effects of some scalar associated with flavor violation affects the running of the Yukawa couplings in such a way, as the renormalization group scale crosses its mass threshold.

One of the hints that the $Y_d \sim Y_u Y_e^\dagger$ relation might be correct is that it naturally leads to large mixing angles in the leptonic sector. In this paper we have suggested a completion of the framework that includes the neutrino Yukawa matrix, such that $Y_\nu \sim Y_\nu Y_u^\dagger$. This is not as predictive as the original relation, because it does not specify the structure of the heavy neutrino mass matrix. The latter we have fixed (up to phases) using experimental constraints on neutrino masses and mixings.

As an application, we studied leptogenesis within this framework, whose heavy neutrino masses are very hierarchical. It was found to give an example where decays of the intermediate heavy neutrino $N_2$ give the dominant contribution to the baryon asymmetry. In a random scan, the models with the highest probability of giving large enough asymmetry are those with normal mass hierarchy for the light neutrinos, and mass $m_{\nu_1} \sim 0.01$ eV for the lightest state. This could be interpreted as a loose prediction of the model. It is an interesting mass from the point of view of neutrinoless double beta decay searches, since for $m_{\nu_1} \sim 0.01$ eV, there is still a reasonable chance of being sensitive to the effective $|\langle m_\beta\beta \rangle|$ measured in $0\beta\beta$ coming from the normal hierarchy, while being able to distinguish it from that predicted in the inverted hierarchy.

We also noted that the flavor symmetry of the CFV scenario allows for vector and scalar leptoquarks, constrained respectively at the scales of 2 and 260 TeV. The former is clearly in an interesting range for the Large Hadron Collider, where ATLAS [39] and CMS [40] have set lower limits near 1 TeV for leptoquark masses. In CFV they are predicted to have equal couplings to all three generations.

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Appendix A: Quark and neutrino masses and mixing angles

In performing random scans to produce model realizations, for the quark sector, we use the quark masses and CKM matrix elements, along with uncertainties, at the scale $m_Z$ as given by the Particle Data Group [9]. For the models generated at the GUT scale, we use the following central values and $1\sigma$ uncertainties for quark masses from ref. [8], in GeV:

$$m_{u,c,t} = (0.48 \pm 0.18) \times 10^{-3}, \quad 0.235 \pm 0.04, \quad 74 \pm 3.9$$
$$m_{d,s,b} = (1.14 \pm 0.5) \times 10^{-3}, \quad 22 \pm 7 \times 10^{-3}, \quad 1.00 \pm 0.04$$

(A1)

The uncertainties are derived assuming that $\Delta m(m_Z) = \Delta m(m_Z) \times (m(m_Z)/m_Z)$. For the CKM matrix, we take the PDG values and uncertainties at scale $m_Z$, using the exactly unitary parametrization (12.3-12.4) of http://pdg.lbl.gov/2014/reviews/rpp2014-rev-ckm-matrix.pdf. At the GUT scale we take the central values of [10]. In the Wolfenstein parametrization,

$$\lambda = 0.22045 \pm 0.00061, \quad A = 0.8797 \pm 0.024, \quad \rho = 0.0 \pm 0.021, \quad \eta = 0.371 \pm 0.013$$

(A2)

still using the PDG errors from the $m_Z$ scale.

For the neutrino sector, we do parameter scans using the $U_{PMNS}$ mixing angles and neutrino mass differences [38],

$$\sin^2 \theta_{12} = 0.304^{+0.013}_{-0.012}, \quad \sin^2 \theta_{23} = 0.452^{+0.025}_{-0.026}, \quad \sin^2 \theta_{13} = 0.0218^{+0.0010}_{-0.0000},$$
$$\Delta m_{21}^2 = 2.457^{+0.047}_{-0.047} \times 10^{-3} \text{eV}^2, \quad \Delta m_{31}^2 = 7.50^{+0.19}_{-0.17} \times 10^{-5} \text{eV}^2.$$  

(A3)

for normal hierarchy, and

$$\sin^2 \theta_{12} = 0.304^{+0.013}_{-0.012}, \quad \sin^2 \theta_{23} = 0.452^{+0.025}_{-0.026}, \quad \sin^2 \theta_{13} = 0.0218^{+0.0010}_{-0.0000},$$
$$\Delta m_{21}^2 = 2.457^{+0.047}_{-0.047} \times 10^{-3} \text{eV}^2, \quad \Delta m_{32}^2 = -2.449^{+0.048}_{-0.047} \times 10^{-3} \text{eV}^2$$  

(A4)

for inverted hierarchy. These show the $1\sigma$ allowed regions, while in our scans we vary to $3\sigma$. The Dirac phase $\delta_{CP}$ is not constrained at $3\sigma$.

Appendix B: Efficiency factors

Analytic fits to the efficiency factors quantifying washout of the lepton asymmetries are given in ref. [21], for varying degrees of complexity in the initial conditions. For the case of thermal leptogenesis, where the initial abundances of heavy neutrinos is assumed to vanish for solving the Boltzmann equations, the efficiency for decays of a given species with $K_i = \bar{m}_i/m_\nu$ can be expressed as

$$\kappa = \kappa^+ + \kappa^-, \quad \kappa^+ = \frac{2}{\bar{z}} \left(1 - e^{-\frac{\bar{z}}{2} N}\right), \quad \kappa^- = -2e^{-\frac{\bar{z}}{2} N} \left(e^{\frac{\bar{z}}{2} N} - 1\right)$$

(B1)

where

$$\bar{z} = K + \frac{K}{2} \ln \left(1 + \frac{\pi K^2}{1024} \ln \left[\frac{3125\pi K^2}{1024}\right]\right)$$
$$N = \frac{9\pi}{16} K, \quad \bar{N} = N \left(1 + \sqrt{\frac{4}{3} N}\right)^{-2}$$

(B2)

Unlike eq. (13), here $\kappa$ vanishes in the weak washout limit $K \to 0$, due to the initial condition of vanishing $N$ abundance at early times.

References: