Impact of CP-violation on neutrino lepton number asymmetries revisited

Gabriela Barenboim\textsuperscript{1} and Wan-Il Park\textsuperscript{1}\textsuperscript{†}

\textsuperscript{1} Departament de Física Teòrica and IFIC, Universitat de València-CSIC, E-46100, Burjassot, Spain
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We revisit the effect of the (Dirac) CP-violating phase on neutrino lepton number asymmetries in both mass- and flavor-basis. We found that, even if there are sizable effects on muon- and tau-neutrino asymmetries, the effect on the asymmetry of electron-neutrinos is at most similar to the upper bound set by BBN for initial neutrino degeneracy parameters smaller than order unity. We also found that, for the asymmetries in mass-basis, the changes caused by CP-violation is of sub-% level which is unlikely to be accessible neither in the current nor in the forthcoming experiments.

INTRODUCTION

If the three active neutrinos are Dirac particles, all the possible sources of CP violation are restricted to only one so-called Dirac phase in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [1, 2]. This phase can cause an asymmetry between conversion probabilities associated with a pair of neutrino flavors and their anti-particles [3–5]. Being this phase also present in the case neutrino species that might be helpful for a better fit of cosmological data, it is worth to revisit the effect of CP-violation on neutrino asymmetries.

In this paper, we revisit the effect of CP-violation on neutrino lepton number asymmetries in both flavor- and mass-basis by solving numerically the quantum kinetic equations (QKEs) of neutrino/anti-neutrino density matrices.

QUANTUM KINETIC EQUATIONS WITH CP-VIOLATION

Lepton number asymmetries of neutrinos can be defined as

$$L_\ell = \frac{\rho - \bar{\rho}}{n_\gamma}$$

where $\rho/\bar{\rho}$ and $n_\gamma$ are the density matrix of neutrino/anti-neutrino and the number density of photons, respectively. For a mode of momentum $p$, the Fourier component of $\rho/\bar{\rho}$ can be expressed in terms of polarization vectors $P/P$ and Gell-Mann matrices $\lambda_i$ ($i = 1 \sim 8$) as

$$\rho_p = \frac{1}{3} \sum_{i=0}^{8} P_i \lambda_i, \quad \bar{\rho}_p = \frac{1}{3} \sum_{i=0}^{8} \bar{P}_i \lambda_i$$

where $\lambda_0$ is the $3 \times 3$ identity matrix. The evolution equations of $\rho_p$ and $\bar{\rho}_p$ are given by [10, 11]

$$i \frac{d \rho_p}{dt} = \left[ \Omega + \sqrt{2} G_F (\rho - \bar{\rho}), \rho_p \right] + C [\rho_p]$$

$$i \frac{d \bar{\rho}_p}{dt} = \left[ -\Omega + \sqrt{2} G_F (\rho - \bar{\rho}), \bar{\rho}_p \right] + C [\bar{\rho}_p]$$

In the above equations,

$$\Omega = \frac{\mathbf{M}_\ell^2}{2p} - \frac{8\sqrt{2} G_F p E_\ell}{3 m_W^2}$$

where $\mathbf{M}_\ell^2$ is the mass-square matrix of neutrinos in flavor-basis, $G_F$ the Fermi constant, $m_W$ the mass of $W$-boson, $E_\ell = \text{diag}(E_{ee}, E_{\mu\mu}, E_{\mu\mu}, 0)$ the energy density.
of charged leptons, \( \rho = (1/2\pi^2) \int_0^\infty \rho_0 \rho^2 dp \) (and similarly for \( \bar{\rho} \), and \( C[\ldots] \) is the collision term. We take \( C[\rho_\alpha] = -i D_{\alpha\beta}[\rho_\alpha][\rho_\beta] \) for \( \alpha \neq \beta \) only, and similarly for \( C[\bar{\rho}_\alpha] \) [12]. Also, the initial condition for the evolution equations Eqs. (3) and (4) can be set as

\[
\rho_p = f(y, 0)^{-1} \text{diag}(f(y, \xi_e), f(y, \xi_\mu), f(y, \xi_\tau)),
\]

and similarly for \( \bar{\rho}_p \) but with \( \xi_\alpha \rightarrow -\xi_\alpha \), where \( f(y, \xi_\alpha) = (e^{\nu_{\xi_\alpha}} + 1)^{-1} \) is the occupation number for a mode \( y \equiv p/T \) with \( \xi_\alpha \) being the initial degeneracy parameter of \( \nu_\alpha \).

The evolution of the density matrices is governed by various effects as shown in Eqs. (3) and (4). Hence, generally, it is difficult or highly non-trivial to predict analytically the asymmetries at the final equilibrium. However, it may be possible to get a hint by resorting to two properties of the evolution: (i) The evolution of a mode near the average momentum in the absence of the neutrino self-interaction term (i.e., the terms of \( ^F \rho \) in Eqs. (3) and (4)) can mimic the collective behavior of \( \rho \) (or \( \bar{\rho} \)) including the self-interaction term [13], (ii) Since all the contributions other than vacuum one in the right-hand side of Eqs. (3) and (4) eventually diminish and become negligible, at the final equilibrium the shape of density matrices should be determined by the vacuum contribution. These properties imply that the structure of \( M^2_T \) may provide an idea of the eventual configuration of lepton number asymmetries. For a mass-square matrix of neutrinos \( (M^2_m = \text{diag}(m_1^2, m_2^2, m_3^2)) \) in mass-basis, the corresponding one in the flavor basis is given by,

\[
M^2_T = U_{\text{PMNS}} M^2_m U_{\text{PMNS}}^\dagger
\]

where the PMNS matrix is

\[
U_{\text{PMNS}} = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 0 & 0 \\
-s_{13}e^{i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

with \( s_{ij}/c_{ij} = \sin \theta_{ij}/\cos \theta_{ij} \) and \( \delta \) being the Dirac CP-violating phase. The entries of \( M^2_T \), denoted as \( m^2_{\alpha\beta} \), are found to be

\[
m^2_{ee} = m^2_1 - c^2_{13} \Delta m^2_{32} - c^2_{12} c^2_{13} \Delta m^2_{21}
\]

\[
m^2_{\mu\mu} = m^2_2 - (c^2_{23} + s^2_{13} s^2_{23}) \Delta m^2_{32} - (s^2_{12} c^2_{23} + c^2_{12} s^2_{13} s^2_{23}) \Delta m^2_{21} - 2c_{12}s_{12}s_{13}c_{23}s_{23}c_4 \Delta m^2_{21}
\]

\[
m^2_{\tau\tau} = m^2_3 - (s^2_{23} + s^2_{13}s^2_{23}) \Delta m^2_{32} - (s^2_{12}s^2_{23} + c^2_{12} s^2_{13} s^2_{23}) \Delta m^2_{21} + 2c_{12}s_{12}s_{13}c_{23}s_{23}c_4 \Delta m^2_{21}
\]

\[
m^2_{\mu\tau} = c_{12}s_{12}c_{13}c_{23} \Delta m^2_{21} + c_{13}s_{13}s_{23} \Delta m^2_{32} + c^2_{12} \Delta m^2_{21} \ e^{-i\delta}
\]

\[
m^2_{\tau\mu} = -c_{12}s_{12}c_{13}s_{23} \Delta m^2_{21} + c_{13}s_{13}s_{23} \Delta m^2_{32} + c^2_{12} \Delta m^2_{21} \ e^{-i\delta}
\]

\[
m^2_{\mu\tau} = c_{23} s_{23} [c_{13} \Delta m^2_{32} + (s^2_{12} - c^2_{12}s^2_{13}) \Delta m^2_{21}] + c_{12}s_{12}s_{13} \Delta m^2_{21} \ (s^2_{23} e^{i\delta} - c^2_{23} e^{-i\delta})
\]

where \( \Delta m^2_{ij} \equiv m^2_i - m^2_j \). From Eqs. (9)-(14), one can see that for the measured central values of \( \theta_{ij} \) and \( \Delta m^2_{ij} \) the effect of a non-zero CP-phase is driven dominantly by the \( \delta \)-dependence of \( m^2_{\mu\mu} \) and \( m^2_{\tau\tau} \). It is worth to note that, if \( \delta = (\pi/2, 3\pi/2) \) and \( \theta_{23} = \pi/4 \),

\[
m^2_{\mu\mu} = m^2_{\tau\tau}
\]

and

\[
\text{Im}[m^2_{\mu\mu}] \gg \text{Re}[m^2_{\mu\mu}] = -\text{Re}[m^2_{\tau\tau}]
\]

As discussed in Ref. [8], for \( \delta = (0, \pi) \) if \( \theta_{23} = \pi/4 \) and either \( \theta_{12} \) or \( \theta_{13} \) are zero, \( m^2_{\mu\mu} = m^2_{\tau\tau} \) and \( \text{Re}[m^2_{\mu\tau}] = |\text{Re}[m^2_{\tau\mu}]| \) which results in \( L_\mu = L_\tau \). Similarly, Eqs. (15) and (16) may be hinting towards \( L_\mu = L_\tau \) again or at least a tendency to such an equalization as \( |\delta| \rightarrow \pi/2 \). On the other hand, it is difficult to get an insight on the dependence of \( L_\mu \) on \( \delta \) from the structure of \( M^2_T \).

**NUMERICAL RESULTS**

For the numerical integrations of Eqs. (3) and (4), we take the single mode approach as in Ref. [8]. The presence of self-interaction term in Eqs. (3) and (4) causes
FIG. 1: Evolutions of $\mathbf{L}_f$ for $\delta = 0$ with $(\xi, \xi_\mu, \xi_\tau) = (-1.1, 1.6, 0.2)$, $\theta = (\theta_{12}, \theta_{13}, \theta_{23})$, and normal mass-hierarchy. Colored/gray lines are with self-interaction turned-on/off.

synchronized oscillations of density matrices, which becomes manifest for certain sets of $\xi_\alpha$. However, as shown in Fig. 1 for example, for the sets of parameters we are considering here, the evolution of density matrices is essentially the same as the case without self-interaction except the oscillatory feature which will be averaged out. Hence, for simplicity and clarity of the result, we turn off the self-interaction. Neutrino mass-square differences and mixing angles are taken as [14, 15]

$$\Delta m^2_{21} \approx 7.5 \times 10^{-5} \text{eV}^2$$

$$|\Delta m^2_{32}| \approx 2.67 \times 10^{-3} \text{eV}^2$$

and

$$(\theta_{12}, \theta_{13}) = \left( \frac{\pi}{5.5}, \frac{\pi}{20} \right), \quad \theta_{23} = \frac{\pi}{3.4}, \frac{\pi}{4}, \frac{\pi}{4.6} \quad (19)$$

In Fig. 2, the evolution of the entries of $\mathbf{L}_f$ are shown for $\delta = 0$ (dashed lines) and $\pi/2$ (solid lines). $\xi_\alpha$ is chosen to have $L_e$ within the BBN constraint for $\delta = 0$. From the top-left panel where diagonal entries are shown, one can see that, if $\delta = 0$, although $L_\mu$ and $L_\tau$ are completely equilibrated across $x \approx 0.06 - 0.07$ (or $T \approx 15 \text{MeV}$) due to the large $\Delta m^2_{23}$ and $\theta_{23} \approx \pi/4$, they are separated across $x \approx 0.2$ (or $T \approx 5 \text{MeV}$). On the other hand, if $\delta = \pi/2$, such a separation does not take place, keeping $L_{\mu e} = L_{\tau e}$ once it is achieved. This maintenance of equalization of $L_\mu$ and $L_\tau$ seems to be due to the specific patterns of Eqs. (15) and (16). For $\delta = \pi$, $L_\mu$ and $L_\tau$ exchange their positions relative to the case of $\delta = 0$. For $\delta = 3\pi/2$, the result turns out to be the same as that of $\delta = \pi/2$, as could be expected.

In the same figure, the top-right panel shows how $L_e$ depends on $\delta$ in a limited (narrower) plot-range for a clear view of the difference. Again, the results for $\delta = 0$ and $\pi$ are the same, and so are $|\delta| = \pi/2$ and $3\pi/2$. As $|\delta| \rightarrow \pi/2$, $L_e$ and $L_{\mu, \tau}$ become closer, maximizing the shift of $L_e$ at $\delta = \pi/2$ and $3\pi/2$ relative to the case of $\delta = 0$ (or $\pi$). For $\xi_\alpha \lesssim 1$, the shift is comparable to or smaller than the upper bound of $|L_{ee}|$. Such a change can be easily compensated by a different choice of initial $L_\alpha$. So, practically it is difficult to constrain $\delta$ by the BBN constraint on $L_e$. On the other hand, as the BBN constraint on $L_e$ becomes tighter, for a given $\delta$ the case of $L = 0 = L_e$ but $|\xi_{\mu, \tau}| \gtrsim 1$ as an initial configuration can be excluded since neutrino oscillations can cause non-zero $L_e$ at the final equilibrium, depending on $\delta$.

The bottom-left panel is showing the real off-diagonal entries of $\mathbf{L}_f$. While the changes of $L_{e\mu}$ and $L_{e\tau}$ are manifest, the change of $L_{\mu\tau}$ is negligible. If $\delta = \pi$, relative to the case of $\delta = 0$, the roles of $L_{e\mu}$ and $L_{e\tau}$ are reversed with opposite signs for both of them, but $L_{\mu\tau}$ is not affected. For $\delta = 3\pi/2$, the result is the same as the case of $\delta = \pi/2$.

The bottom-right panel is for the imaginary off-diagonal entries of $\mathbf{L}_f$. In the panel, the line for $L_{e\mu}$ was completely overlapped with the line of $L_{e\tau}$. Note that, while these off-diagonal entries become zero in case of $\delta = 0$ (or $\pi$) (i.e., CP-conservation), they are non-zero for $\delta \neq 0$ (or $\pi$) (CP-violation). For $\delta = 0$, the result is the same as the case of $\delta = 0$. For $\delta = 3\pi/2$, the result is the same as the case of $\delta = \pi/2$, but with signs flipped for both of $L_{\mu\tau}$ (for $x \gtrsim 0.1$) and $L_{e\mu, \tau}$.

The matrix of the late time asymmetries shown in Fig. 2 turned out to be an Hermitian matrix containing imaginary off-diagonal entries. We found that irrespective of CP-phase the diagonalization matrix is given by the PMNS matrix and the diagonalized matrix has only real entries, as it should be. That is, again the asymmetries in mass-basis are obtained as [9]

$$\mathbf{L}_m = U^{-1}_{\text{PMNS}} \mathbf{L}_f U_{\text{PMNS}}$$

and $\mathbf{L}_m$ is real and diagonal. In Fig. 3, we show the late time asymmetries of neutrino flavor- and mass-eigenstates for the parameters of Fig. 2. As can be seen from the figure, even if CP-violation can cause sizeable changes in $L_\alpha$ (especially $L_\mu$ and $L_\tau$), its impact on $L_i$ causes changes of less than 1% relative to the CP-conserving case.

The case of inverted mass-hierarchy is shown in Fig. 4 and 5. As shown in the top-left panel of Fig. 4, contrary to the case of normal mass hierarchy, in this case all $L_\alpha$s approach to the complete equalization across $x \sim 0.5$ (or $T \sim 2 \text{MeV}$). However, top-right panel shows that they do not reach complete equalization. It may mean that, if BBN bound on $L_e$ becomes tighter in the future, the possibility of $\xi_\alpha \sim \mathcal{O}(1)$ as the initial degeneracy parameters of neutrino flavors at very high energy might be ruled out even if the total lepton number asymmetry is zero.

Comparing top-right panel to bottom panels, we found that the real components of off-diagonal entries are larger than diagonal entries by a factor of a few. This fact
FIG. 2: Evolutions of $L_f$ for $\delta = 0$ (dashed lines) and $\pi/2$ (solid lines) with $(\xi_e, \xi_\mu, \xi_\tau) = (-1.1, 1.6, 0.2)$, $\theta = (\theta_{12}, \theta_{13}, \theta_{23})$, and normal mass-hierarchy. Top-left: Diagonal entries. Top-right: $L_e$. Bottom-left: Real components of off-diagonal entries. Dashed red line was overlapped by solid red line. Bottom-right: Imaginary components of off-diagonal entries. Green line was overlapped by blue line.

FIG. 3: The late time lepton number asymmetries of flavor- and mass-eigenstates (i.e., diagonal entries of $L_f$ and $L_m$ as a function of $\delta$ for normal mass-hierarchy. The “×” mark indicates actual data points obtained from numerical simulations. Left/Right: $L_\alpha$s and $L_i$s at different scales for clearer views of $\delta$-dependence.

results in mass-basis asymmetries much larger than ones in flavor basis, as shown in Fig. 5 where $\delta$-dependence of asymmetries in both bases is depicted. Similarly to the case of normal mass-hierarchy, CP-phase can cause sizable changes in $L_\mu$ and $L_\tau$, but the change of $L_e$ is at most of $O(1)\%$. The smallness of the change seems to be
from the smallness of the late time $L_{\mu,\tau}$. The changes in the asymmetries in mass-basis is less than 1% as the case of normal hierarchy. Also, the pattern of the evolutions of $L_{\bar{t}}$ for $\delta = \pi (3\pi/2)$ relative to the case of $\delta = 0 (\pi/2)$
is the same as the case of normal mass-hierarchy.

For a normal mass hierarchy, the late time asymmetries at the final equilibrium depend on the mixing angles critically. For example, in the case of $\delta = 0$, for a given set of initial flavor asymmetries (of $L_e \neq L_{\mu, \tau}$) which end up in a value of $L_e$ satisfying the BBN bound for a set of $\theta_{ij}$, the smaller $\theta_{12}$ or $\theta_{13}$ is, the larger $|L_{\mu} - L_e|$ or $|L_{\tau} - L_e|$ becomes, respectively, as expected. On the other hand, the smaller $\theta_{23}$ is, the larger $|L_{\mu} - L_{\tau}|$ becomes. The $\delta$-dependence for different values of $\theta_{23}$ is shown in Figs. 6 and 7. In Fig. 6, we find that at the final equilibrium, if $L_{\tau} > L_{\mu} \gg |L_e|$ for $\delta = 0$ with a given set of $\theta_{ij}$, nonzero $\delta$ makes $L_e$ and $L_{\mu}$ shifted toward each other by an equal amount in addition to an overall shifting toward $L_e$, resulting in a shift of $L_e$ to keep the conservation of the total asymmetry. Such a shift of $L_e$ is maximized when $L_{\mu} = L_{\tau}$. Hence, the largest effect of non-zero $\delta$ appears when $L_{\mu}$ becomes the closest to $L_{\tau}$ as happens when $\delta = \pi$. If $L_{\mu} = L_{\tau}$ for a certain set of $\theta_{ij}$ with $\delta = 0$, non-zero $\delta$ pushes $L_e$ away from $L_{\mu, \tau}$, as shown in Fig. 7. The changes in the final lepton number asymmetries of flavor- and mass-eigenstates relative to the case of $\theta_{23} = \pi/4$ are minor and do not affect our conclusions.

**CONCLUSIONS**

In this paper, we revisited the effect of CP-violation (i.e., nonzero Dirac CP-phase ($\delta$) in the PMNS matrix) on the neutrino lepton number asymmetries in both flavor- and mass-basis by solving the evolution equations of neutrino/anti-neutrinos density matrices in a simplified single mode approach as taken on our recent work, Ref. [8].

In the evolution equations of neutrino density matrices, the effect of CP-violation appears in the mass-square matrix $M_i^2$ in the flavor basis. In particular, the effect is dominated by $\delta$-dependence of the off-diagonal entries of $M_i^2$ associated with $\nu_e - \nu_\mu$ and $\nu_e - \nu_\tau$ mixings. For $\theta_{23} \sim \pi/4$ as one of the mixing angles of the PMNS ma-

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**FIG. 6:** Evolutions of $L_\ell$ for $\delta = 0$ (dashed lines), $\pi/2$ (solid lines), and $\pi$ (dotted lines) with $(\xi_e, \xi_\mu, \xi_\tau) = (-1.1, 1.6, 0.2)$, $\theta = (\theta_{12}, \theta_{13}, \theta_{23})$, and normal mass-hierarchy. *Left:* Diagonal entries. *Right:* $L_e$. $\delta = 3\pi/2$ gives the same result as $\delta = \pi/2$.

**FIG. 7:** The same as Fig. 6 but $\theta_{23} = \pi/3.4$ instead of $\pi/4.6$. Again, $\delta = 3\pi/2$ gives the same result as $\delta = \pi/2$. 
trix, it turned out that the $\delta$-dependence of $L_\mu$ and $L_\tau$ at the final equilibrium is nearly equal and opposite. As a result, the change of $L_e$ appears to be much smaller than the change of $L_{\mu,\tau}$ since the total lepton number asymmetry should be conserved.

The effect of CP-violation turned out to be maximized for a $\delta$ such that the values of $L_\mu$ and $L_\tau$ are the closest or farthest from each other. In normal mass hierarchy, the change of $|L_{\mu,\tau}|$ is about a few tens of % at most. However, the change of $|L_e|$ is within the range of the upper-bound set by BBN as long as $\xi_\alpha \lesssim 1$ with $\xi_\alpha$ being the initial degeneracy parameters of neutrino flavors. In inverted hierarchy, one finds $|L_e| \sim |L_\mu| \sim |L_\tau|$ at late time. Hence, $L_{\mu,\tau}$ is constrained to be small to satisfy the BBN constraint on $L_e$. CP-violation in this case can significantly increase $L_{\mu,\tau}$, with a raise up to more than 100%, but the change in $L_e$ appears to be negligible.

Contrary to the cases of flavor eigenstates, the effect of CP-violation on asymmetries of mass-eigenstates, the ones relevant to CMB, turns out to be practically negligible. It causes sub-% changes of asymmetries which are difficult to be explored in current or future experiments, even for initial asymmetries as large as $\xi_\alpha \sim 1$.

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* Gabriela.Barenboim@uv.es
† Wanil.Park@uv.es