Collective movement in alarmed animals groups: a simple model with positional forces and a limited attention field

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Abstract.

We perform a numerical analysis of a recent introduced model for describing collective movement in alarmed animals groups. This model, derived from a position-based interaction and a limited attention field, displays a non-equilibrium phase transition between a crystal-like phase, characterised by a single densely packed cluster of individuals, which occurs for small attention field angles, and a disordered phase, constituted of fragmented clusters, which emerges at large angles. The transition is quantitatively characterised, measuring its critical point and describing its scaling behaviour, which suggests that the nature of the transition is discontinuous. Finally, we study the correlations in the directions of motion, which show strong positive values at short distances. This fact suggests that individuals’ effective perception range goes far beyond the model interaction range.

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1. Introduction

A particularly interesting aspect of collective animal motion is the spontaneous and abrupt aggregation manifested by alarmed group of social animals [1, 2, 3, 4]. This problem gained a renewed interest with some recent works on actual herds [5]. Such field studies undertook quantitative analysis under controlled experimental conditions. The work of Ginelli et al. [6] shown an intermittent behaviour between two conflicting tasks: foraging, realised by dispersed individuals, and protection, offered by aggregation in a compact group. This second aspect was previously studied by detecting sheep movements in response to the approach of a dog in a field setting [7]. Trajectories were obtained by using a GPS technology and shown that animals manifest a strong attraction towards the centre of the threaten flock. As described by modelling and theory [1, 8], as a consequence of a possible predation, animals aggregate by means of a fast collective movement. A classical hypothesis explaining this gregarious behaviour is the so called “selfish herd hypothesis”, which suggests that cohesive swarms are generated because each member of the group moves towards neighbours for minimising its own predation risk [1]. Unfortunately, the implementation of this reasonable idea into a specific and practical algorithm is not so straightforward. The original Hamilton’s algorithm [1], where the focal individual moves towards the nearest neighbour, was a first tentative in this direction but it failed in describing compact and dense groups. In fact, in two dimensions, fragmentation into multiple sub-groups of a few individuals occurs [3, 9]. For this reason, discovering an elementary movement rule which can produce compact aggregation is still an unsolved problem, denominated the “dilemma of the selfish herd”.

This movement rule must satisfy some natural constraints, and the solution of the problem must be reached constructing a model built up on perceptive bases. This means that individuals movements should be based exclusively on the instantaneous visual information they obtain and process. Documented physiological and cognitive facts must be taken into account. For example, some type of selection of the interaction partners
must be introduced. In fact, it has been shown that animals generally interact with a small number of neighbours \[10, 11, 12\] and a limited attention field is a general well-known characteristic of cognitive processes. An extreme example of how much visual attention is selective is the phenomenon called “inattentional blindness”, which suggests that humans perceive only those objects and events that receive focused attention \[13\].

Moreover, the model must be assembled from basic mimetic rules which operate just for the punctual duration of the stimulus. The considered individual movement rule must be as simple as possible. This issue is particularly important because the rule must satisfy two biological considerations. The first is that even very simple animals, with limited cognitive ability, like arthropods, must be able to follow it. The second is that many different species should be able to act in accordance with it, from Crustacea \[2\] to Mammals \[4\]. Finally, a behavioural ecology perspective, suggests that the rule must be simple enough for allowing natural selection to favour and fix it \[14\]. Usually considered interactions, as for example molecular type forces, with a weighted sum of a large number of neighbours, clearly do not satisfy these assumptions. Not even models based on local velocity, which imply a memory based mechanism, are coherent with these considerations.

A possible solution to this problem was recently introduced in \[15\], with the adoption of a new algorithm. This algorithm added a simple ingredient to the Hamilton’s rule: the fact that animals can exercise their attention only over a limited horizon \[16\]. The analysis of this model demonstrated that, if animals try to approximate the first neighbour enclosed in a specific attention field, it is possible to produce the densest aggregation in a centrally compact swarm \[15\]. These results are obtained for selected values of the angle which defines the attention field and the compact swarms are robust to the introduction of noise. The general results produced by this model can be connected with some reported behaviours of animals in the wild. In \[15\], a qualitative comparison of the model output with some empirical observations of groups of crabs \[2\] was presented. These results were obtained calibrating the model parameters with realistic values measured in \[2\]. For the opportune values of the attention field lively shrunk configurations qualitatively very similar to the observed real data were obtained. Moreover, for this set of parameters, the typical time necessary to reach the compact configurations resulted to be comparable with the experimental one. In fact, the calibration of the animals velocity implied fixing the unit of the simulation time step. It followed that a significant reduction of the domain of danger was obtained after a few seconds, a result that can be realistically compared with field data \[15\], validating the model as capable of describing an actual process in ecology.

It is important to note that this model of collective movement is based on an attraction-repulsion interaction which depends on the instantaneous position of neighbouring individuals and not on their velocity. The simple introduction of an attention cone is sufficient to generate a very rich behaviour. These two fundamental ingredients were introduced for the first time for solving the “dilemma of the selfish herd” in \[15\],
and they have recently gained a lot of attention in the literature. Different works investigated the impact of the introduction of a vision cone in other type of models [17], and, more recently, a positional-based force with a vision cone have been applied in the classical framework of flocking models without cohesion in a study written by Barberis et al. (see [18]).

The principal purpose of this work is to quantitatively explore the model first introduced in [15] throughout an accurate numerical analysis. First, we outline the general model dynamical behaviour. Second, we characterise the process of compact aggregation in term of a phase transition. Finally, we studied the properties of the correlations in the direction of motion of the individuals.

2. The model

We build up a model which considers only the movements of the individuals relative to the centre of mass of the group. This means that the contribution of the global translation and rotation of the swarm is subtracted from the individuals movements.

The basic rule which governs our model is very simple: every agents seek for an optimal distance from the nearest topological neighbour encompassed in the attention field. This is implemented by means of the following algorithm: an agent $X$ and its gazing direction are randomly selected. The gazing direction is the bisector of the angle $\Theta$, which defines the cone (attention field) where neighbours are perceived. The nearest topological neighbour $Y$ contained in the attention field is identified. Agent $X$ gives a step $\epsilon$ towards $Y$, trying to reach an optimal distance $D$. In detail, considering $x_t$ the position of agent $X$ and $y_t$ the one of $Y$, if $|x_t - y_t| > D$, $X$ moves in the direction of $Y$ a step $\epsilon$: $x_{t+1} = x_t + \epsilon v$, where $v$ is a unit vector. If this movement causes the stress zone invasion, $X$ stops at a distance $D$ from $Y$. If $|x_t - y_t| < D$, the motion is: $x_{t+1} = x_t - \epsilon v$. If this motion leaves $X$ farther than $D$ from its neighbour, $Y$ stops at a distance $D$. Movements are determined asynchronously and the time step $t$ is incremented after the updating of all the agents’ positions. Figure 1 shows an example of these rules.

The system is composed by $P$ agents which move continuously on a square of linear size $L$ and which do not cross the given boundaries. As initial condition, individuals are uniformly and randomly distributed on the square. In the following, we set, without loss of generality, $D = 1$ and $\epsilon = 0.1$, and we express all scales in terms of these units. We run different simulations varying the number of individuals but maintaining fixed the density ($\delta = 0.3$). This was obtained modifying the square size ($L = \sqrt{P/0.3}$). This is a standard approach in finite size scaling analysis, as the model behaviour depends on the density, as suggested in [15].
3. Results and discussion

The dynamics behaviour of the model is described introducing three different order parameters which can characterise the degree of order and cohesion of the system.

The degree of positional orientational order of a configuration is obtained introducing, for each organism [19]: \( \psi_j = \frac{1}{N_j} \sum_{k=1}^{N_j} \exp(i6\phi_{jk}), \) where \( N_j \) is the number of topological neighbours of individual \( j \), which are the animals whose Voronoi polygons share an edge with \( j \). The neighbours’ index is \( k \) and \( \phi_{jk} \) is the angle relative to the bond between \( j \) and \( k \) and an arbitrary fixed reference axis. A positional orientational order parameter is estimated calculating the norm of \( \psi_j \) averaged over all individuals \( j \):

\[
\psi_6 = \frac{1}{P} \left| \sum_{j=1}^{P} \psi_j \right| .
\]

Translational order is described counting the number of individuals contained in a circle of radius \( D + 0.001 \) centred on a given animal. The translational ordering parameter \( N \) is obtained averaging this quantity over all individuals.

Finally, the level of swarm cohesion is measured evaluating the quantity:

\[
C(t) = \frac{a(t)}{a(0)}
\]  

where \( a(t) \) is the convex hull of the finite points set corresponding to the given group.
at time $t$. The convex hull of a set of points is the area of the smallest convex polygon encompassing all the points and it is a good approximation of the area covered by the swarm at a given time $t$. It has been already used for characterising group cohesion in previous works \[13, 6\]. $C(t)$ estimates what in the classical ecological literature is called "the reduction of the overall domains of danger" \[1\]. The domain of danger indicates the area closer to a given individual than to any other group member and it is estimated considering the Voronoi polygon constructed around the group member \[1\]. The overall domains of danger is obtained summing all the Voronoi polygons, excepted the ones corresponding to the edge individuals \[2\]. A more efficient evaluation of this quantity is obtained measuring the convex hull of the group.

Figure 2 shows the time evolution of these order parameters for two typical $\Theta$ values. For $\Theta = 75^\circ$, after a rapid transient, the system enters in a state characterised by some level of cohesion and order. This is shown by higher values of $\psi_6$ and $N$. Even if in the case of $\psi_6$ the parameter value is very noise, we can distinct a plateau which corresponds to higher level of order. Looking at the parameter $C$ we can observe a clear plateau with a reduction in the area occupied by individuals which reaches 80%. A high cohesion of the swarm is rapidly obtained and maintained. At very long times, the system reaches an absorbing state where all the individuals’ distances with all their topological neighbours are equal to $D$. The interaction arranges the swarm in the densest way, a perfect sixfold ordered structure (see Figure 3). This is reported by an abrupt jump of the parameters to the values $\psi_6 = 1$ and $N = 5.27$. In contrast, $C$ maintains a value comparable with the one reached after leaving the first transient period.

For $\Theta = 330^\circ$ the parameters values do not change significantly. In fact, neither ordering, nor a general group contraction is possible. As can be seen in Figure 3, a fragmented swarm appears, where the initial group splits in different clusters and cohesion is lost.

The outlined deep differences present in the system dynamics in dependence of the attention field angle $\Theta$ can be studied also looking at the final configurations attained by the groups (see Figure 3). This analysis was systematically carried on in a previous paper \[15\]. Those results can be summarised highlighting four different possible final phases. For $\Theta < 60^\circ$ a disordered, highly dense, connected phase appears. For $60^\circ < \Theta < 220^\circ$ it changes to a crystal-like phase, with hexagonal patterns. A low ordered phase with the presence of holes and ruptures is attained for $220^\circ < \Theta < 300^\circ$. Finally, for $\Theta > 300^\circ$, a fragmented swarm emerges, where the initial group splits into different clusters and cohesion is lost.

The transition between these different phases can be characterised by looking at the $\psi_6$, $N$ and $C$ values. In this work, we are interested in unfolding an in deep analysis of the transition between the crystal-like phase and the low ordered phase. At sufficiently long times, this shift is characterised by the passage from an absorbing state, characterised by a single densely packed cluster of individuals, to an active, but slow, stationary state of disordered and/or fragmented clusters. This transition is particularly relevant from the biological point of view, because it discriminates between attention field angles
**Figure 2.** Time evolution of the order parameters $N$, $\psi_6$ and $C$ for $\Theta = 75^{\circ}$ and $\Theta = 330^{\circ}$; $P = 61$.

**Figure 3.** Typical configurations of the stationary states for different $\Theta$ values. The red dots represent individuals' positions, the lines the corresponding Voronoi tessellation.
which efficiently allow for a single densely packed cluster, with a sensitive reduction of the domains of danger, from the ones which cannot generate effective cohesion. For this reason, we analyse the transition looking at the $C$ value, the order parameter which better characterises the level of cohesion. $C$ is measured in the plateau region, which can be considered as a stationary state. We do not wait to reach the final configuration. In principle, the final configurations of the system are more interesting from a theoretical point of view, as they allow a precise description of the ordering property of the model. On the other hand, from an empirical point of view, the living configurations present all along the plateau region are the ones that can be directly compared with the behaviour of real swarms. With this choice, we are also able to speed up our measures, even if the simulation time continues to be very large. In fact, the system presents very slow relaxation time close to the transition. For this reason, we analyse the state reached after running $10^6$ Monte Carlo steps. Moreover, the algorithm for identifying the nearest neighbour in the attention field is costly. Finally, for estimating the $\langle C \rangle$ value, we must realise ensemble averages, and not time averages. Such slow numerical analysis forces us to adopt $P$ values limited between 91 and 721. Besides the $\langle C \rangle$ values, we estimate their fluctuations $\chi$, which are obtained by evaluating $\chi = L^2(\langle C^2 \rangle - \langle C \rangle^2)$.

In Figure 4 we exhibit the behaviour of the order parameter and its fluctuations as a function of $\Theta$ for different system sizes. One can observe the typical behaviour of a phase transition: the order parameter presents an evident shift near a critical value $\Theta_c$, which corresponds to a clear peak in the fluctuation values. These peaks $\chi_{\text{max}}$ grow following a power-law for increasing values of $L$, a behaviour typical of a classical phase transitions [20] (see Figure 4). Finally, as in a genuine phase transition, the curves exhibit the characteristic steepest aspect of the order parameter shift when the system size is increased. In contrast, the expected dependence of the critical value of $\Theta$ on the system size is not clearly identifiable. This is probably due to the fact that its dependence is on a sharp scale of angles, which we are not able to identify with the discretisation of our points and for the relative small difference among the considered system sizes.

We try to characterise the transition order looking at the Binder cumulant $U = 1 - \frac{\langle C^4 \rangle}{3(\langle C^2 \rangle)^2}$. As expected for discontinuous phase transitions [21], the Binder cumulant exhibits a sharp drop toward negative values (see Figure 5). This minimum is generated by the coexistence of two phases near $\Theta_c$.

Following this indication, we verify if the order parameter $\langle C \rangle$ presents the typical scaling of a discontinuous transition near the transition point, a standard procedure for equilibrium finite-size scaling analysis [21]. The scaling plot should be obtained by simply considering the rescaled control parameter $(\Theta - \Theta_c)L^d$, where $d$ is the system dimension. As shown in Figure 5, it is possible to obtain a reasonable collapse which satisfies this relation. In fact, data roughly collapse to a single curve, strongly suggesting the validity of the finite-size scaling ansatz expected for a discontinuous transition. In the case of discontinuous phase transitions in equilibrium systems, near the transition point, fluctuation peaks should present power-law scaling of the form: $\chi_{\text{max}} \sim L^d$. We
Figure 4. Top: $\langle C \rangle$ in dependence of $\Theta$ for different system sizes. Bottom: The fluctuation $\chi$ in dependence of $\Theta$. In the inset, logarithmic plot of the scaling of the maxima of $\chi$ in dependence of $L$. The fluctuation peaks grow following a power-law (continuous line). We remember that the density is fixed to 0.3, which implies that $L = \sqrt{P/0.3}$. Each point is averaged over 100 simulations.

verified the power-law scaling of the fluctuation peaks, but, for our out-of-equilibrium system, the exponent is intriguing close to 3.4 (see the inset in Figure 4).

The hallmark of a collective process can be characterised measuring correlations. In fact, we expect that, for generating a collective response, individuals must be able to influence their interaction partners to drive behavioural changes on a well defined spatial scale. This measure is particularly relevant for our model where it is not easy to perceive a clear directional order in individuals movements, as we are describing only relative movements, and the gazing directions are taken randomly. Despite these facts, a clear strong correlation can be detected.

To accomplish this, we measure correlation in the directions of individuals' motions. The correlation function measures to what extent the motion of individual $i$ is correlated to that of $j$. We used a definition inspired by the study of Attanasi et al. \cite{22}:

$$F(r) = \frac{\sum_{i \neq j} P \delta(r - r_{ij}) \delta v_i \cdot \delta v_j}{\sum_{i \neq j} P \delta(r - r_{ij})}$$

(2)

where $\delta v_i = \frac{x_i^t - x_i^{t+\Delta t}}{\Delta t} - \mathbf{V}$, and $\mathbf{V}$ is the overall velocity of the group; $\delta(r - r_{ij})$ is the Kronecker's delta, which is equal to 1 if $r < r_{ij} < r + dr$ and zero otherwise, and $dr$ is the selected space discretisation unit.
Figure 5. Left: Rescaled plot for $\langle C \rangle$. Top right: Binder cumulant as a function of $\Theta$ for $P=721$ showing the characteristic well-defined minimum near the transition. Results are averaged over 100 samples. Bottom right: a typical example of the shape of the probability distribution function of $C$ near the minimum of the Binder cumulant.

The behaviour of $F(r)$ is reported in Figure 6 measured at different times of the simulation. We can observe how the correlation is practically null for very small time steps, and it grows, for higher times, towards strong positive values at short distances. The correlation crosses zero at a distance equal to five $D$. This means that neighbours inside a circle of radius $5D$, tend to align their directions of motion. As we are analysing a simulation near $\Theta_c$, in general, we expect that individuals have a mean nearest neighbour distance close to $D$. This fact suggests that the influence between individuals’ directions of motion goes far beyond the first neighbour, even if the model interaction operates only with the first topological neighbour. For distances greater than five, $F(r)$ relaxes to very small negative values, which describe a general tendency to compact the group, manifested by a direction of motion tending to tighten individuals’ distances.

In summary, we have presented a numerical study of a simple model, which describes collective movement in alarmed animals groups, with the intent of clearly characterising the nature of the transition displayed by this system. Our analysis shows that the model effectively presents a non-equilibrium phase transition between a perfectly regular absorbing state and a low ordered active one. In the first phase swarms are able to attain a single densely packed cluster of individuals which can well describe the behaviour of some group of alarmed animals. The analysis is carried out looking at a control parameter which measures the degree of cohesion of the swarms. A clear critical value of the angle which controls the attention field is identified. This value is consistent with a previous qualitative analysis realised looking at other order parameters. The nature of the phase transition has been studied varying the system size. The fluctuations of the order parameter show a power-law behaviour. The form of the Binder cumulant,
the coexistence of different phases near the critical point and the collapse of the scaling plot of $\langle C \rangle$ suggest that the system undergoes a discontinuous transition. However, the unexpected behaviour of the fluctuations scaling behaviour suggests prudence.

Finally, we described the correlations in the directions of individuals’ motions, which show strong positive values at short distances. Hence correlations are relevant enough to prove that the individuals’ effective perception range goes far beyond the model interaction range. Despite our model implements random gazing directions, a clear strong influence between individuals can be measured. All these elements suggest that a stimulus generated by an environmental perturbation can be perceived at a collective level, causing a general change in the group behaviour. These results are not only relevant for this specific model but also, they show, more generally, how a model purely based on a repulsion/attraction positional force can generate correlations analogous to the ones measured in field observations [22, 23]. This is an important result, which shows the relevance of positional interaction models for describing swarms collective movements and it sheds a new light on the possible nature of the interaction present in such systems.

References